K. Srinivasa Rao

Srinivasa Ramanujan

Life and Work of a Natural Mathematical Genius, *Swayambhu*

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Dedicated to my parents

Sri Killampalli Vallabheswar Rao
(22 December 1889–10 Janunary 1983)
B.A., B.L., Advocate

and

Srimathi Dhanalakoti Lakshmikanthamma
(15 May 1907–19 December 2003)

and siblings,

with gratitude for their unbounded affection.

Foreword

For many decades, I have kept the portraits of three scientists I admire behind my office chair for inspiration. One of them is the great Indian mathematical genius Srinivasa Ramanujan, the others being those of Sir C.V. Raman and Dr. Homi J. Bhabha. The results of the self-taught Ramanujan's research and his conjecures, jotted down in his famous *Notebooks*, are still being analysed, and sometimes used in applications Ramanujan had not dreamed of, or could have been interested in. These Notebooks, including his 'Lost' and found Notebook, representing work carried out during his short lifespan of a little over 32 years, contains about 4000 entries/theorems. He did not provide proofs for them; he perhaps thought they were obvious, or he followed the methodology of the *Sutra*s in Hindu scriptures. It is incredible that there are almost no errors in them.

This book includes a brief biographical account of his life. It also gives glimpses into his work on number theory, partitions, continued fractions, 'mock' theta functions and especially hypergeometric functions. The author's own work in hypergeometric functions, as inspired by the work of Ramanujan, is also presented.

The book provides complete lists of all the papers of Ramanujan in the Wren Library of Trinity College, the letters and other material about him with the National Archives, Chennai. The 75th, 100th and 125th birth anniversaries of Ramanujan

have also been celebrated in different ways the world over. Some of the significant events in those years are mentioned in the book.

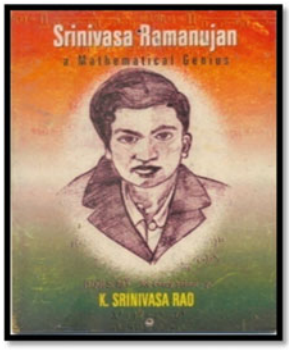
The author has earlier created two CD ROMs on the life and work of Srinivasa Ramanujan during these events and these are also described in the book.

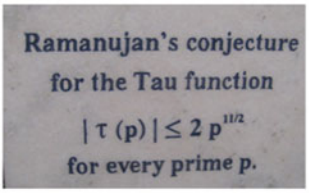
Over the years, there have been other books on Ramanujan, but Dr. Killampalli Srinivasa Rao, a well-known Mathematical Physicist himself, has been studying and researching on the life and work of Ramanujan over decades and has lectured on the Entries in Ramanujan's Notebooks. Add to that his Indian cultural background, and we may say that there is perhaps no person today more qualified than Srinivasa Rao to write the biography of Ramanujan.

Srinivasa Ramanujan (1887–1920), who has been compared to all time greats like Euler, Gauss and Jacobi, has been an eternal source of inspiration, especially to students and researchers in Mathematics.

The author has expressed the hope that this book will add to our existing knowledge about Ramanujan and appreciation of his profound work. I am sure that his very readable book will.

R. Chidambaram
Former Chairman Department of Atomic Energy,
Govt. of India
Former Principal Scientific Advisor to the Govt. of India
New Delhi
March 2020





Rear of the pedastal.

Preface

This work on Srinivasa Ramanujan, who for a natural genius has been compared with Euler and Gauss, is the outcome of my lectures on the life and work of Srinivasa Ramanujan at several institutes and universities in India and abroad. I was an Alexander von Humboldt Fellow at the Institut für Theoretische Physik der Universität Bonn, Germany, for 2 years (1977–1980). During a four-year collaboration project (1992–1996) funded by the European Economic Commission, with me as Principal Investigator from the Institute of Mathematical Sciences (IMSc), India, and Prof. Guido Vanden Berghe as Principle Investigator from the University of Ghent, Belgium, I had the privilege of addressing the Flemish Royal Academy at Brussels on *Gauss, Ramanujan and Hypergeometric series*.

My lectures on the life and work of Srinivasa Ramanujan started during the Ramanujan birth centenary year in December 1987. I had the pleasure and privilege of accompanying the delegation of Prof. Richard Askey, Prof. George Andrews, Prof. Bruce Berndt, Prof. Robert Rankin, Prof. Don Zagier, Prof. E.C. George Sudarshan and Prof. G. Bhamathi and also giving the opening lecture on the life and work of Srinivasa Ramanujan at an international conference held in honour of Ramanujan at the Institute of Fundamental studies, Kandy, Sri Lanka, that year.

My first detailed acquaintance with the "romantic" story of Ramanujan's life and glimpses into his work was through *Ramanujan: Letters and Reminiscences*, and *Ramanujan: An Inspiration*, two volumes edited by (late) P. K. Srinivasan, Muthialpet High School, Madras (1968), published to mark the 75th birth anniversary of Ramanujan. I am thankful to my research guide, mentor and Founder Director of IMSc, Prof. Alladi Ramakrishnan, who directed me to acquire copies of this two volume compilation. I acquired a copy of the same for myself and one for my fellow student at IMSc at that time, Ms. P.K. Geetha. In 1976, she became Dr. Geetha Srinivasa Rao my better-half, a mathematician of repute in functional analysis and approximation theory, who retired as Professor from the Ramanujan Institute for Advanced Study in Mathematics (RIASM), University of Madras, India. Among the many achievements in her illustrious career, she became, in 2012, President of the Indian Mathematical Society. Many facts and most of my writings on Ramanujan were scrutinized by her meticulously for

which I am grateful to her. A better understanding of the genius of Ramanujan came to me with my acquaintance with the special functions group led by Prof. Ratan P. Agarwal, Professor of Mathematics and later Vice Chancellor of Lucknow University and Rajasthan University, India. I was directed by Prof. Sudarshan, Director, MATSCIENCE, the Institute of Mathematical Sciences, in March 1986, to go to Gorakhpur University and participate in a National Seminar on Special Functions. The subsequent year's birth centenary celebrations brought me into contact with Professors Richard Askey, George Andrews and Bruce Berndt, the ardent followers of the work of Ramanujan.

My area of research shifted from theoretical nuclear physics to the quantum theory of angular momentum and then to theory of generalized hypergeometric series, their transformations as well as their group theoretical aspects, from the mid-1980s. This enabled me to make a few contributions to number theory, in the area of multiplicative Diophantine equations (what may be considered as an unstated part of the tenth Problem of David Hilbert), and special functions—in particular, generalized hypergeometric functions, ordinary and basic, summation theorems for hypergeometric series, and group theory for transformations of hypergeometric functions. This stimulated my interest in the work of Ramanujan, which has been propagated through the works of George Andrews on the 'Lost' notebook of Ramanujan and the five volumes work of Bruce Berndt on *Ramanujan's Notebooks*.

The following were my sources of inspiration: a biography titled *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, by Robert Kanigel, a bound copy of which was graciously sent to me by Prof. Bruce Berndt in June 1991; visits to Smt. Janaki Ramanujan, often accompanying visiting mathematicians who wanted to pay their respects to the wife of the mathematical genius Ramanujan; several discussions with her in the company of my journalist friend, (late) Mr. A. Ranganathan, over the years and the gentle persuasions of Mr. C. A. Reddi, culminated in my visiting the National Archives in New Delhi and the Wren Library of Trinity College. Thanks to Prof. Robert Rankin, a table was reserved for me for 2 days (in October 1995), to see for myself the originals in the Archives of the Wren Library.

J.M. Whittaker's statement, in February 1979, that "Rankin and I thought that Trinity was the right place for it (Ramanujan's notebooks), rather than India which had done nothing for him",[1] has been refuted by Berndt and Rankin,[2] in their *Ramanujan Letters and Commentary*, who assert that "it is clear that his Indian colleagues had done all that was within their powers to assist him."

This work is an attempt to provide a brief introduction to the life and work of Srinivasa Ramanujan; my study led to a few new results and a new summation theorem, directly as a consequence of combining two of his entries in his chapter on hypergeometric series. It is also written with the hope that it will stimulate the

[1] J.M. Whittaker's statement, to G.E. Andrews, 15 August 1979, in *Ramanujan: Letters and Commentary*, Bruce C. Berndt and Robert A. Rankin, AMS-LMS (1995).

[2] Also, an Indian Edition with a Preface, Additions to the Indian Edition and Errata, by K. Srinivasa Rao, published by Affiliated East West Press Pvt. Ltd. (1997).

interest of the mathematics student, on what Ramanujan did to become renowned as a natural mathematical genius.

Dr. Samuel Johnson said: "Knowledge is of two kinds. We know a subject ourselves, or, we know where we can find information about it."

In that spirit, the author hopes that the reader will find pointers to the original sources of his work for further study. This is the Ramanujan Remembrance Centenary Year and the author wishes to stress the need for a National Science Museum, in a spacious area, with all amenities and facilities, which will portray to the common man, as well as the interested student and specialist in science and technology, the evolution and growth of physical laws and their foundation which has mathematical beauty; depict the works of our great scientists like Srinivasa Ramanujan, Sir C.V. Raman (Nobel Prize for Physics, 1930), Satyendranath Bose, and Nobel Laureates Rabindranath Tagore (first Indian, for Literature, in 1913), Har Gobind Khorana (Physiology or Medicine, shared with Robert W. Holley and Marshall W. Nirenberg, 1968), S. Chandrasekhar (Physics, 1983, with Dr. William A. Fowler), Mother Theresa (Peace, 1979, born in Skopje, Ottoman Empire), Amartya Sen (Economic Sciences, 1998), Venkataraman Ramakrishanan (Chemistry, shared with Thomas A. Steitz and Ada Yonath, 2009), Kailash Satyarthi (Peace, shared with Malala Yousafzai, 2014), Abhijit Banerjee (Economic Sciences, shared with Esther Duo, his wife and Michael Kremer, 2019). The Museum should collect and collate artefacts and memorabilia of great scientists of India or of Indian origin, who brought credit to the nation by their discoveries and achievements. And, needless to say, the Museum will be a source of inspiration and will instill a sense of pride and kindle the urge to a better understanding of nature through sciences in the visitor.

Chennai, India
August 2020

K. Srinivasa Rao

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The author thanks Mr. P. Ramkumar, Secretary, and Mr. C. S. Parthasarathy, Director, of the Board of Management of the Muthialpet Higher Secondary School for graciously granting permission for use of photos compiled in the publication: *Ramanujan: Letters & Reminiscences*, Memorial Number, Volume 1, Edited by P.K. Srinivasan, M.Ed., The Muthialpet High School, Number Friends Society, Old Boys' Committee, Madras-1 (1968).

The author places on record his thanks to the Board of Governors and Directors, of the Institute of Mathematical Sciences, especially Founder Director, Prof. Alladi Ramakrishnan, and his successors Prof. E.C.G. Sudarshan, Prof. R. Ramachandran, Prof. R. Balasubramanian and Prof. K. Arvind, all his former colleagues, including his collaborators in research Prof. K. Ananthanarayanan, Prof. R. Jagannathan, Prof. R. Parthasarathy, Prof. V. Rajeswari, Prof. T.S. Santhanam, Prof. R. Sridhar, and Prof. A. Sundaram (who retired from the Agricultural University, Madurai).

I am grateful to Prof. Bruce C. Berndt for sending me copies of the books: *Ramanujan Revisited* (Ed. vol.), *The Man Who Knew Infinity: A Life of the Genius Ramanujan* by Robert Kanigel; his *Ramanujan Notebooks*, Part III and *Ramanujan: Letters and Commentary* edited by him and Robert A. Rankin and for innumerable correspondences on various aspects concerning Ramanujan including the Foreword to a book of mine in 1998, over the past decades.

I am also grateful to Prof. Richard Askey, Prof. George Andrews, Prof. Subrahmanyan Chandrasekhar and his sister Mrs. Savithri, Prof. Richard Dalitz (on his visit in March 1986 to IMSc), for their enthusiasm to share information with me on topics of mutual interest.

To my journalist friend Mr. Airavatham Ranganathan, I am indebted for many stimulating conversations, discussions and motivation to get copies of two letters from Dr. S. Chandrasekhar to the Prime Minister Jawaharlal Nehru and the latter's reply from the Nehru Memorial Museum and Library, New Delhi. Mr. C.A. Reddi's efforts in getting the papers on Ramanujan from the Tamil Nadu Archives, and it was his persuasion which made me visit the National Archives in New Delhi and the Wren Library of Trinity College, Cambridge. I am thankful to Prof. Rankin for his spontaneity in writing to Dr. David McKitterick, Librarian at the Wren Library, and getting for me a Reader's Table at the Wren Library for two days in October 1995.

I wish to place on record my heartfelt thanks to Dr. R. Chidambaram, for encouragement throughout my career, when he was on the Board of Governors of IMSc, and later when he was the Chairman of the DAE, and more recently Principal Scientific Advisor to the Government of India, but for whom the life-size Bronze statue for Ramanujan (sculpted by K.G. Ravi, a student of Mani Nagappa, whom I approached with the opening sentence, that I am the son of Mr. Vallabheswar Rao, which made him say "How can I forget your father, Sir?", since my father was his Legal Advisor) at the entrance to the Society for Electronic Transactions and Security (SETS), in 2010, would not have become a Dream come true! In this context, I wish to acknowledge the help of Mr. S. Thiagarajan, former Registrar of SETS and its Executive Director M.S. Vijayaraghavan.

I am indebted to Dr. V.S. Ramamurthy, Secretary DST, Director of and now Emeritus Professor at the National Institute for Advanced Study, Bengaluru, ever since our first meeting at the annual Nuclear and Solid State Physics Symposium in 1970, for his encouragement especially for the creation of the *πie Pavilion and the Ramanujan Gallery* for the Indian Science Congress Exhibition at the Engineering College, Guindy, in the Hall for DST (in December–January 1988), followed by a 2 and a 1/2 year DST CD ROMs Project on the Life and work of Ramanujan (December 2002–May 2005) at the NMRC of C-DAC, Pune. The website and the digitization of the Notebooks as a part of the Project was done by the author as its Principal Investigator. This led to the creation of the first Distinguished DST-Ramanujan Chair Professorship at the Srinivasa Ramanujan Center, SASTRA University, where I was encouraged to set and reorganise a Museum for Ramanujan by its Founder Vice Chancellor Prof. Sethuraman and guided three students for their Ph.D. thesis. With the continued support of DST, the author was able to set up a permanent *πie Pavilion and the Ramanujan Gallery*, thanks to a dynamic Director of the Science City, Dr. C.K. Gariyali, and the Executive Director, Dr. E. Sarguru Murthy of Periyar Science and Technology Center, Kotturpuram, Chennai, first in 1998 and enlarged, more recently into a full-fledged Ramanujan Mathematics Museum in 2012.

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About the Author

K. Srinivasa Rao, former Senior Professor, IMSc, is Director (Honorary) of the Srinivasa Ramanujan Academy of Maths Talent (SRAMT), Chennai, India. Earlier, he served as Senior Professor of the Institute of Mathematical Sciences (IMSc), Chennai, India, which he joined in 1964 as a research trainee with an M.Sc. (Physics) from the University of Madras, India. He completed his Ph.D. in theoretical nuclear physics at the IMSc, Chennai, India, in 1972, under the supervision of Prof. Alladi Ramakrishnan. His research areas include quantum theory of angular momentum, special functions and orthogonal polynomials, group theory, numerical analysis and computational physics. His research papers have been published in various national and international journals of repute. He is the editor and author of a number of books, including *Srinivasa Ramanujan: A Mathematical Genius* (1998, 2004), *Quantum Theory of Angular Momentum: Selected Topics* (1993) and *Introduction to Computers and Programming* (1974). He was an Alexander von Humboldt Foundation Fellow, a recipient of the Ramanujan Award of the Ramanujan Mathematics Academy and Mathematics Library (2013), awarded in recognition of his research on *Gauss, Ramanujan and Hypergeometric Series* and *Life and Works of Ramanujan*; the Tamil Nadu National Science Academy Award (TANSA 2000) for Mathematical Sciences; the TANSA 2000 Award for Popularisation of Science; and the Lifetime Achievement Award of SRAMT, Chennai.

Chapter 1
Life of Srinivasa Ramanujan

Srinivasa Ramanujan,[1] the brilliant twentieth-century Indian mathematician, has been compared with all-time greats Leonhard Euler,[2] Carl Friedrich Gauss[3] and Carl Gustav Jacob Jacobi,[4] for his natural mathematical genius.

It may be impossible to define who a mathematical genius is, or, genius for that matter. But that does not prevent us from recognizing the work of a genius, for the rarest of the rare—like the Himalayan peaks or the Niagara falls—stand out in any field of human activity.

Despite his short lifespan of 32 years, 4 months and 4 days, Ramanujan has left behind an incredibly vast and formidable amount of original work, which has greatly influenced the development and growth of some of the best research work in mathematics of the twentieth century.

If we call Euler and Gauss as mathematicians who were the most outstanding in the eighteenth and nineteenth centuries, to which they belonged, then Ramanujan is undoubtedly an outstanding mathematician of the twentieth century, whose work continues to be relevant in the new millennium.[5]

[1] Srinivasa Ramanujan, Dec. 22, 1887–April 26, 1920.

[2] Leonhard Euler, April 15, 1707–September 18, 1783.

[3] Carl Friedrich Gauss, April 30, 1777–February 23, 1855.

[4] Carl Gustav Jacob Jacobi, December 10, 1804–February 18, 1851.

[5] "Relevance of Srinivasa Ramanujan at the Dawn of the New Millennium", K. Srinivasa Rao, (2002) in A.K. Agarwal, B.C. Berndt, C.F. Krattenthaler, G.L. Mullen, K. Ramachandra, M. Waldschmidt (eds), *Number Theory and Discrete Mathematics*, Hindustan Book Agency, Gurgaon (2002) 261–268.

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1.1 Early Years

Ramanujan was born on December 22, 1887, a Thursday, at Erode, the parental home of his mother Komalathammal, as traditionally the needs of the mother-to-be are taken care of by her mother. His father, K. Srinivasa Iyengar,[6] was a clerk (*Gumaastha*) to a cloth merchant, in Kumbakonam, earning a monthly salary of about Rs. 25/- per month which was barely adequate for his growing family. So, to augment the family income, Komalathammal used to go to the Sarangapani Temple, in Kumbakonam, and she was a part of a group which used to gather every evening to sing songs in praise of the God Narayana.

Komalathammal knew by heart 1000 of the 4000 verses in the renowned *Naalaayira Divya Prabhandam*,[7] devotional songs, sung as "Bhajans" (with cymbals), in Temples. This phenomenal memory of the mother was inherited by Ramanujan, whose sharp memory, enabled him to recall, in later years, at will any one of the thousands of Mathematical formulas, just like that, whenever it was necessary.

Very little is known about the involvement of the father in the bringing up of the children in his family. Unfortunately, three children born after Ramanujan did not survive—their names and dates of birth and death are given in the family tree of Srinivasa Ramanujan (see, in the figures pages). These births and deaths were a strain on the family in every sense, especially financially.

Komalathammal would bring home from the Sarangapani Temple *prasadam* (lintel cooked with rice called pongal and/or curd rice, offered to the God first and later) distributed to the devotees. This would be the staple food for Ramanujan, on some days. The lady Venku Ammal in whose Bhajan party, Komalathammal was a member, would receive some money from voluntary contributions of the listeners and devotees on regular visits to the temple, amounting to about Rs.100/- per month, out of which Komalathammal would get her share of about Rs.5/- to Rs.10/-. This augmented the family income.

Ramanujan had his *Aksharaabhyaasam*,[8] on October 1, 1982, the Vijayadasami day.[9] He stood first in the Tanjore (or, Thanjavur) District Primary Examinations, held in November 1897. This entitled him to a half-fee concession in the Town High School at Kumbakonam, where he was a student from 1898 to 1903. In those days

[6]It is unfortunate that there is no photograph of the father of Srinivasa Ramanujan. A close look at the photographs of Ramanujan and his mother Komalathammal reveals the strong resemblance between the mother and son.

[7]"Naalayiram" in Tamizh means four thousand, "Divya Prabhandam" means Divine collection of hymns in praise of the god Narayana, the sustainer of all creations in the Universe—composed and sung by His devotees, referred to as Azhwars, in the Tamizh language.

[8]In Sanskrit, "Akshara" means letter and "abhyasa" means practising (to read/write).

[9]"Vijayadashami", also known as Dasara or Dusshera, reveres victory of good over evil—Rama's victory over Ravana, in the epic Ramayana, and the day the Pandava's came out of their 13th year of banishment, spent incognito, in the epic Mahabharata.

the calendar year was also the academic year for the students. Ramanujan passed the Matriculation examination of the University of Madras, by the end of December 1904.

During his school days, he impressed his classmates, senior students and teachers with his extraordinary intuition and astounding proficiency in several branches of mathematics: namely, arithmetic, algebra, geometry, number theory and trigonometry. Thanks to his access[10] to S.L. Loney's book on *Trigonometry*, Ramanujan mastered it in his Fourth Form[11] itself.

In later years, Gopala Rao, a friend of Ramanujan recalled the following incident: In an arithmetic class on division, the teacher said that if three bananas were given to three boys, each boy would get a banana. The teacher was trying to generalize this idea to say that n divided by n equals 1. Ramanujan is said to have stood up and asked:

Sir, if no banana is distributed to no student,

will everyone still get a banana?

With hindsight it is clear that Ramanujan was intuitively aware of exceptional cases and wanted to know the answer for zero divided by zero.

K.S. Viswanatha Sastri[12] who took private tuition from Ramanujan recalled that Ramanujan used to ask about the value of zero divided by zero and then himself volunteer his answer that it can be anything, since the zero of the denominator can be many times the zero of the numerator and vice versa and that the value[13] can therefore not be determined.

Once a senior school student, C.V. Rajagopalachari,[14] who "became an admirer of Ramanujan because of his fame of having acquired an astonishing knowledge in mathematics", said

to T.K. Rajagopalan – then reading in the sixth Form and known as the most brilliant student in the class – that Ramanujan was a very great mathematician. Rajagopalan gave me a question in a sheet of paper and asked me to give it to Ramanujan to work it out. The question was:

$$\text{if } \sqrt{x} + y = 7;\ x + \sqrt{y} = 11, \text{ what are } x \text{ and } y? \tag{1.1}$$

[10] At the beginning of the twentieth century, India was under the British rulers. Indians tried to master the English language to impress their superior officers, to gain an advantage in the form of favours and recommendations to positions of importance. Some of the important books published in England were available within a short time in Indian libraries also.

[11] The Fourth Form corresponds to the current IX standard in schools.

[12] *Reminiscences of my esteemed Teacher*, K.S. Viswanatha Sastri, in *Ramanujan Letters and Reminiscences*, Edited by P.K. Srinivasan, (1968), Vol. I, pp. 89–93.

[13] To this intriguing question of $0/0$, the Hindu ancient mathematician Bhaskara proved the answer to be infinity.

[14] *My Association with Ramanujan*, in *Ramanujan: Letters and Reminiscences*, Memorial Number 1 (1968), Ed. P.K. Srinivasan, The Muthialpet High School, Madras-1, Volume 1, p.83–88.

This question could not obviously be tackled by a student in the IV Form. I gave the paper to Ramanujan and asked him to work it out. To my astonishment, Ramanujan worked it out in half a minute and arrived at the answer by two steps – and gave the answer as $x = 9$ *and* $y = 4$. *I ran to Rajagopalan and gave the answer. He said that it was indeed a marvelous achievement for a IV Form student and this won for Ramanujan the friendship of C.V. Rajagopalachari, who in later years took him to the Collector of Nellore, Diwan Bahadur R. Ramachandra Rao.*

The senior mathematics teacher, Ganapathy Subbier, had such confidence in Ramanujan's ability, that year-after-year, he entrusted the arduous task of preparing the conflict-free Time Tables for the whole school, which had at that time about 1400–1500 students studying in Forms I to VI, and for each Form there were several sections and each section had about 30–40 students, on an average.

One can conjecture that, it is perhaps, at this time that Ramanujan came across 3×3 and 4×4 magic squares. A magic square is one in which the numbers in each one of the rows, the numbers in each one of the columns and the numbers along the leading diagonal and the skew-diagonal all add up to the same given number. It is easy to verify that there can be no 2×2 magic square and the smallest magic square is indeed a 3×3 magic square in which the nine integers $1, 2, \cdots, 9$ are arranged to give a magic square for the number 15. This is explicitly given below:

4	3	8
9	5	1
2	7	6

The 4×4 magic square for 34, with the numbers $1, 2, \cdots 16$ can be:

9	16	05	04
07	02	11	14
12	13	08	01
06	03	10	15

If one forms a 4×4 magic square for Ramanujan's date of birth, 22-12-1887, with the numbers 22, 12, 18 and 87 in the first row of this Date Magic Square, they add to 139. The twelve other numbers (with no number being repeated and zero not used to fill any square), can be found in a multitude of ways.

Given below are two Ramanujan's Date of Birth Magic Squares, for 139:

22	12	18	87
62	20	31	26
41	07	81	10
14	100	09	16

and

22	12	18	87
88	17	09	25
10	24	89	16
19	86	23	11

It is interesting to note that the second 4×4 magic square has the following in-built 2×2 squares with 139 as the sum of the four numbers in each square:

22	12
88	17

18	87
09	25

10	24
19	86

89	16
23	11

Equivalently, if we indicate by D, M, C and Y the date, month, century and the year, respectively, and form a magic square for D+M+C+Y, then the interested reader can easily write down the Date Magic Square for his date of birth, from the following magic square:

D	M	C	Y
$Y+1$	$C-1$	$M-3$	$D+3$
$M-2$	$D+2$	$Y+2$	$C-2$
$C+1$	$Y-1$	$D+1$	$M-1$

The magic squares for a given number are not unique. The only chapter which has a title in the Notebooks of Ramanujan[15] is Chap. 1 of the first Notebook and the title is *Magic Squares*. Ramanujan has written three pages in his first Notebook on magic squares, and in his second Notebook, Chap. 1 has seven pages devoted to this topic and ends with an 8×8 magic square (see figure at the end of Chap. 1), with four in-built 4×4 magic squares.

This is perhaps the only chapter in his Notebooks which belongs to the realm of recreational mathematics. The remaining chapters in his Notebooks contain thousands of entries each of which has been considered a theorem but most of them without proofs, only a few with a hint of a proof, thereby providing opportunities for generations of Mathematicians the world over, an opportunity to provide the proofs. More details about the contents of the Notebooks of Ramanujan are in Chap. 3.

Ramanujan won prizes for proficiency in English and in Mathematics from the Town High School. The following are the certificates he received:

- II Form B Division, 1st Prize presented to S. Ramanujan as a reward of merit and an incentive to further improvement (dated 5 October 1900): *Wallace and Bruce: Heroes of Scotland*, by Mary Cochrane, W. & R. Chambers, Limited, London and Edinburgh (1897).
- IV Form A Division, 2nd Prize awarded to S. Ramanujan as a reward of merit and an incentive to further improvement (dated October 1902): *Poems of England: A Selection of Patriotic Poetry*, London Macmillan and Co., Limited (1900).

[15] *Notebooks of Srinivasa Ramanujan*, (facsimile edition) 2 volumes, Tata Institute of Fundamental Research, Bombay, 1957; Narosa, New Delhi, 1987. Reprinted in 2012 by the National Board for Higher Mathematics (NBHM).

- VI Form C Division, Special Prize by Mr. S. Sivagurunatha Chettiar Awarded to S. Ramanujan for Proficiency in mathematics: *The Poetical Works of William Wordsworth*, Oxford University Press Warehouse (1895).
- *VI Form C Division, 5th Prize Awarded to S. Ramanujan as a reward of merit and an incentive to further improvement* (dated 18 November 1905): *Critical and Historical Essays* by Lord Macaulay, Longmans, Green, and Co. (1899).

It is surprising that *The Poetical Works of William Wordsworth*[16] was the *Special Prize* for proficiency in Mathematics! However, it has to be noted that Ramanujan won also as a prize S.L. Loney's *Trigonometry*, Part II, a classic mathematical textbook, in use even today, in his IV Form at School. It is known that he mastered the contents of that book immediately.

The first public recognition of Ramanujan was on the occasion of the Annual Prize Distribution, at the Porter Hall, in Kumbakonam. Another certificate dated *18 November 1905, VI Form C Division*, and *Awarded to S. Ramanujan as a reward of merit and an incentive to further improvement*. Ramanujan was also the recipient of Sri K. Ranganatha Rao Prize for mathematics. While giving this prize to Ramanujan, a classmate of Ramanujan at the Government Arts College N. Hari Rao[17] recalls that Sri Krishna Iyer, the Head Master of the Town High School said: *Here is a student, who, my Mathematics assistant says, has displayed in his answer papers a remarkable ability and deserved many more marks than even the maximum itself.*

To augment the family income, Komalathammal took in a couple of students from Tirunelveli and Tiruchirapalli as boarders. Noticing Ramanujan's precocity in mathematics, these undergraduate college students are purported to have given him an elementary introduction to all the branches of mathematics. In 1903, through these friends from the Kumbakonam Government College, Ramanujan obtained George Shoobridge Carr's: *A Synopsis of Elementary Results, a Book on Pure Mathematics*, which contained propositions, formulae and methods of analysis with abridged demonstrations, published in 1886. This book was a primary source for all those preparing for the competitive Mathematical Tripos Examination of the Cambridge University.

Professor G.H. Hardy[18] says about G.S. Carr, a former scholar of Gonville and Caius College, Cambridge: *Carr himself was a private coach in London, who came to Cambridge as an undergraduate when he was nearly forty, and was 12th Senior Optime in the Mathematics Tripos of 1880 (the same year in which he published the first volume of his book). He is now completely forgotten, even in his own College, except in so far as Ramanujan has kept his name alive. . . . Carr's book*

[16] Oxford University Press Warehouse, 1895—as pointed out by N. Hari Rao, a classmate of Ramanujan in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School (1968) pp. 120–123.

[17] *We two together in the college*, in *Ramanujan: Letters and Reminiscences, Memorial Number*, Vol. 1, The Muthialpet High School, 1968, p. 120–123.

[18] *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*, G.H. Hardy, Chelsea (1940) and Reprinted by the Am. Math. Soc. and the London Math. Soc. (1999).

covers roughly the subjects of Schedule A of the present Tripos[19] *(as these subjects were understood in Cambridge in 1880), and is effectively a "synopsis" it professes to be. It contains the enunciations of 6165 theorems, systematically and quite scientifically arranged, with proofs which are often little more than cross-references and are decidedly the least interesting part of the book. All this is exaggerated in Ramanujan's famous notebooks (which contain no proofs at all), and any student of the notebooks can see that Ramanujan's ideal of presentation has been copied from Carr's.*

Professor Richard A. Askey has pointed out[20] the fact that in Carr's 'Synopsis', while formulae in each chapter are numbered continuously, the numbering for each chapter starts with 100, 200, etc., discontinuously. In all about 1300 numbers have been skipped and so, though the last Entry bears the number 6165, referred to by Hardy, the total number of formulae is about 4865. These formulae, presented in Carr's Synopsis without Proofs, are in Algebra, Trigonometry, Analytical Geometry, Number Theory and Calculus. Carr's *Synopsis* is similar to the later day compilations like that of, say, *Tables of Integrals, Series, and Products*, by I.S. Gradshteyn and I.M. Ryzhik (5th Edition, Academic Press, New York, 1994). Prof. P.V. Seshu Aiyar and Mr. R. Ramachandra Rao, in their biographies of Ramanujan, in the Collected Papers of Ramanujan, state that: *It was this book which awakened his genius. He set himself to establish the formulae given therein. As he was without the aid of other books, each solution was a piece of research so far as he was concerned.* It is the considered opinion of many[21] that in proving one formula, he discovered many others and thus, Ramanujan laid for himself a foundation for higher mathematics. Also, during the years 1904 to 1907, he started noting the formulae in his Notebooks, which were his treasures with which he wanted to convince all, especially his benefactors and admirers, about his prowess as a mathematician.

Ramanujan's failure to get promoted to the senior F.A. Class marked the beginning of a very trying period in his life. In 1906, Ramanujan set out from Kumbakonam for the first time, and joined the Pachaiyappa's College in Madras, in the F.A. Class again. After about 3 months, he fell ill and discontinued his studies. However, he appeared as a private candidate for the F.A. examinations in 1907 and he failed again.

[19] An excellent exposition of the Mathematical Tripos of Cambridge, a coveted examination which Hardy took but in later years denounced, can be found in Robert Kanigel's *The Man Who Knew Infinity*, Charles Scribner's, New York (1991).

[20] Private communication to the author, when Prof. Askey visited India on the occasion of the Birth centenary of Ramanujan in December 1987.

[21] Ref. Kanigel's *The Man Who Knew Infinity*, Charles Scribner's, New York (1991) p. 57.

1.2 Years of Adversity

Hardy in his book entitled *Ramanujan* states: *The years between 18 and 25 are the critical years in a mathematician's career. ⋯ During his five unfortunate years (1907–1912) his genius was misdirected, side-tracked and to a certain extent distorted.*

Despite the pecuniary circumstances and the stresses and strains of day-to-day existence, Ramanujan started noting down his own results as Entries in different Chapters in his Notebooks. A classmate stated that during his collegiate years, Ramanujan taught him the method of Magic Squares. Ramanujan's experience gained when he was asked to prepare conflict-free time tables for the Town High School, Kumbakonam, perhaps inspired him to fill the squares with numbers instead of names of teachers, room numbers or subject names. His interest in Magic squares should have started in his school days. But it is to be stressed that while this is in the realm of recreational mathematics,[22] it is disconnected with the exceptionally original investigations which led Ramanujan to results in continued fractions and divergent series, during this period.

Ramanujan's betrothal to 9-year-old Janaki was mainly due to an accidental meeting of his mother Komalathammal with this young girl, whom she considered as smart and precocious and without any hesitation, and without even discussing this with her husband, she negotiated the details for the marriage alliance with the parents of Janaki. The wedding took place in Karur, in 1909—Ramanujan was 21 years old and Janaki was only 9 years old and had not even come of age.[23] Robert Kanigel in his comprehensive biography of Ramanujan, constructs a vivid description of this marriage arranged by Komalathammal, not approved by her husband, and dramatizes the foreboding of the impending disaster through the omens preceding the wedding, which was on the brink of being called off, due to the late arrival of the bridegroom's party.

During 1906–1912, Ramanujan was constantly in search of a benefactor and a job to earn enough for his needs. He tutored a few students, even college students, in mathematics and sought employment as a tutor. Disappointed at the lack of

[22]There are several teachers now in our country who are capable of propagating the nuances of creating even and odd magic squares, date magic squares, rectangular magic squares, squares which include negative numbers as well or even fractions and even create 100×100 squares. One such person who is a Railway Station Master, who can be contacted, is T.R. Jothilingam, Retired Railway Superintendent, Madurai. He was selected by the All India Ramanujan Maths Club, New Delhi, for the Ramanujan Award for 2016. The author provided him with all the dates of relevance in the life of Ramanujan, with which he formed a large magic sqaure. The interested reader may contact him at: mailtojothi@gmail.com.

[23]Note: Social reformers like Raja Ram Mohan Roy, Kandukuri Veeresalingam Panthulu and E.V. Ramaswami Naicker were yet to awaken the public to protest against child marriages, which, in later years, became illegal under the Hindu Marriages Act.

recognition, during this trying period, Ramanujan bemoaned to his friend[24] that he was probably destined to die in poverty like Galileo Galelei! Providentially, this was not to be.

In 1910, Ramanujan sought the patronage of Professor Ramaswamy Iyer[25]—the Founder of the Indian Mathematical Society—who was then a Deputy Collector at Tirukkoilur, and asked him for a clerical post in his office. The main recommendation Ramanujan had was his Entries in his Notebooks, which he carried with him, containing by then several results on varied chapters on magic squares, prime numbers, infinite series, divergent series, Bernoulli numbers, Riemann's ζ (zeta) function, hypergeometric series, partitions, continued fractions, elliptic functions, modular equations and approximations to π, etc. A glance at and a scrutiny of a few of these Entries in the Notebooks was sufficient to convince Ramaswamy Iyer[26] that Ramanujan was an extraordinarily gifted mathematician, and he *had no mind to smother his* (Ramanujan's) *genius by an appointment in the lowest rungs of the revenue department* of the Government. So, he sent Ramanujan back to Madras with a letter of recommendation to Prof. P.V. Seshu Aiyar, then at the Presidency College, Madras (Chennai), transferred from the Government Arts College, Kumbakonam, where he had first met Ramanujan as a student. Seeing Ramanujan after a gap of about 4 years, Seshu Aiyar was greatly impressed with the contents of the *well-sized Notebooks* and so gave Ramanujan *a note of recommendation to that true lover of mathematics, Dewan Bahadur Ramachandra Rao, who was then the District Collector of Nellore*.

1.3 A Turning Point

With the help of a friend, R. Krishna Rao, a nephew of Dewan Bahadur Ramachandra Rao, Ramanujan went to Nellore, in December 1910. This was a turning point in the life of Ramanujan. Ramachandra Rao states[27] that *in the plentitude of my mathematical wisdom, I condescended to permit Ramanujan to walk into my presence*. At that time, Ramanujan appeared to Ramachandra Rao as *a short uncouth figure, stout, unshaved, not over-clean, with one conspicuous feature – shining eyes – walked in, with a frayed Notebook under his arm . . . He was miserably poor. He*

[24] K.S. Viswanatha Sastri, whose article can be found in *Ramanujan: Letters and Reminiscences*, ed. P.K. Srinivasan, Publisher: The Muthialpet High School, Madras, Vol. 1 (1968).

[25] Sri Ramaswamy Iyer was a lover of Mathematics, and when he received a reply from London with the cover addressing him as Professor Ramaswamy Iyer, his friends started calling him, and addressing him as 'Professor' from then on, though he never was a Professor in any educational institution!

[26] *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, The Muthialpet High School, Madras, Vol. 1 (1968) p. 129.

[27] Ref.: Extract from late Dewan Bahadur Ramachandra Rao's reminiscences, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, The Muthialpet High School, Madras, Vol. 1 (1968) pp. 126–127.

had run away from Kumbakonam to get leisure in Madras to pursue his studies. He never craved for any distinction. He wanted leisure, in other words, simple food to be provided for him without exertion on his part and that he should be allowed to dream on.

Though Ramachandra Rao gave ultimately a patient hearing, he took a few days to look into the Notebooks. At their fourth meeting, when Ramanujan confronted Ramachandra Rao with a letter from Prof. Saldanha, from Bombay, appreciating the genuineness of his results, Ramachandra Rao began to feel that Ramanujan needed encouragement and that his work must be examined in depth by other eminent mathematicians. He himself was convinced when Ramanujan led him *step-by-step to elliptic integrals and hypergeometric series and at last to his theory of divergent series not yet announced to the world*, and this converted him into a benefactor who undertook to support Ramanujan with a monthly stipend of Rs. 20/-per month.[28]

Prof. Seshu Aiyar also communicated the earliest contributions of Ramanujan to the Journal of the Indian Mathematical Society (JIMS), in the form of either Questions or Answers to Questions. Perhaps, one of the motivations for starting this journal by Ramaswamy Iyer was the incessant flow of mathematical results which baffled the contemporary mathematicians in Madras to whom Ramanujan showed the Entries in his Notebooks. Ramanujan continued to contribute to the Questions and Answers to Questions section of the JIMS from then on till the very end of his brief and illustrious career in which he proposed in all 59 Questions or Answers to Questions, in the JIMS.

The first few Bernoulli numbers[29] are:

$$B_0 = -1, \quad B_2 = \frac{1}{6}, \quad B_4 = \frac{1}{30}, \quad B_6 = \frac{1}{42}, \quad B_8 = \frac{1}{30}, \quad B_{10} = \frac{5}{66}$$

$$B_{12} = \frac{691}{2730}, \quad B_{14} = \frac{7}{6}, \cdots, B_{38} = \frac{2929993913841559}{6}, \&c. \tag{1.2}$$

Any one can discern the irregularity and the lack of a simple pattern among these numbers, and this is perhaps what engrossed Ramanujan's attention, which resulted in his first 15-page article entitled: *Some properties of the Bernoulli numbers*, published in the JIMS in 1911. In it Ramanujan stated eight theorems embodying arithmetical properties of the Bernoulli numbers, indicating proofs for three of them; two theorems are stated as corollaries of two others, while three theorems are stated

[28] Perhaps due to the reluctance with which he was given an audience (on the fifth visit) by the status conscious Dewan Bahadur Ramachandra Rao, Ramanujan initially did not avail himself of this dole. But later reluctantly did accept the same for a short period of time, till he got the first ever research scholarship of the University of Madras.

[29] *Notebooks of Srinivasa Ramanujan*, (facsimile edition), 2 Volumes, Tata Institute of Fundamental Research, Bombay, 1957; reprinted by Noarsa, New Delhi, 1987.

as mere conjectures. In the words of Prof. Seshu Iyer:[30] *Ramanujan's methods were so terse and novel and his presentation was so lacking in clearness and precision, that the ordinary reader, unaccustomed to such intellectual gymnastics, could hardly follow him.*

Mr. M.T. Narayanaiengar, one of the early editors of the JIMS, states that this first article of Ramanujan *attracted considerable attention. It is however a sad confession to say that the Editor's work in connection with Ramanujan's contribution was by no means light. Ramanujan saw intuitively many things and could not bring himself to explain and the first article had to be referred back to him no less than three times*.

Ramanujan lived in a small house called '*Summer House*' in Sami Pillai Street, Triplicane, Madras, accepting reluctantly a monthly financial assistance from the Collector of Nellore for about a year. Later, he declined this help and from January 12 to February 21, 1912, he worked as a clerk in the Accountant General's Office. Not satisfied with this job, Ramanujan applied for and secured a post in the Accounts Section, as a Class III, Grade IV clerk in the Madras Port Trust, with the help of Mr. S. Narayana Iyer, then Manager of the Madras Port Trust and the Treasurer of the Indian Mathematical Society (IMS) and a friend of V. Ramaswamy Iyer and Professor P.V. Seshu Aiyar.

Mr. Narayana Iyer, M.A., was a good mathematician and was a great source of support to Ramanujan. He was not only instrumental in Ramanujan being offered a job in the Madras Port Trust but also in securing for Ramanujan the lifelong support of Sir Francis Spring. Narayana Iyer's son N. Subbanarayanan relates the role his father played in the career of Ramanujan:[31]

To illustrate the depth of his genius, I can quote one of the incidents which happened during the stay of Sri Ramanujan with my father when we were living in No. 580, Pycrofts Road, Triplicane. During that period, every night both Ramanujan and my father used to work Mathematics on two big sized slates sitting on a small parapet upstairs. This used to be carried on till about 11.30 p.m. and was a source of nuisance to other inmates of the house who used to sleep in the adjoining rooms. I distinctly remember the noises of the slate pencils which used to be a background music for my sleep. Several nights I have seen Sri Ramanujan get up at 2 O'clock in the night and note down something in the slate in the dull light of a hurricane lamp. When my father asked him what he was writing, he used to say that he worked out mathematics in his dreams and now he was jotting the results in the slate to remember them. Evidently, this goes to show that he must have had a remarkable power of subconscious working and grasp of intellectual strides in mathematics. Peculiarly enough, my father used to ask Ramanujan some doubts in these jottings. Between one step and another step there used to be a big gap. My

[30] *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras, Vol. 1 (1968) p. 125.

[31] *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras, Vol. 1 (1968) p. 112.

father, being a fairly good mathematician himself, was unable to capture the strides of Ramanujan's discoveries. He used to tell him, "When I am not able to understand your steps, I do not know how other mathematicians of a critical nature will accept your genius. You must descend to my level and write at least ten steps between the two steps of yours." Sri Ramanujan used to say, "When it is so simple and clear to me, why should I write more steps?" But somehow my father slowly got him round, cajoled him and made him write some more, though it used to be a mighty task of boredom for him [Ramanujan].

Mr. Narayana Iyer requested for and obtained the slate used by Ramanujan as a memento (in exchange of his slate) and it is still a heir-loom with the family of Mr. Subbanarayanan. It is interesting to note that Mr. Narayana Iyer used to send his son Subbanarayanan to get a book on Elliptic Functions from the library of the University of Madras.[32]

Dewan Bahadur Ramachandra Rao also wrote to Sir Francis Spring, Chairman of the Madras Port Trust, about Ramanujan. Ramanujan's entry into Port Trust, on March 1, 1912, may well be considered as the turning point in his career prospects. Ramanujan held this post for 14 months. His wife, Janaki, joined him during this period and Ramanujan shifted to Saiva Muthiah Mudali Street in George Town, Madras. This period marked the beginning of the appreciation of Ramanujan's scholarship and research in mathematics.

Dewan Bahadur Ramachandra Rao induced Professor C.L.T. Griffith of the Engineering College, Madras, to take interest in Ramanujan, and he in turn wrote[33] to Sir Francis Spring, the Chairman of the Madras Port Trust about the *very poor accountant* who was *a most remarkable mathematician* and asking him to keep Ramanujan *happily employed until something can be done to make use of his extraordinary gifts.* Prof. Griffith also wrote to Prof. M.J.M. Hill, of University College, University of London, on Ramanujan's work and he received a reply in December 1912. Unfortunately, Prof. Hill[34] could not find time to study the results of Ramanujan. However, he observed that *the book which will be most useful to him is Bromwich's Theory of Infinite Series, published by Cambridge University Press* (*or Macmillan*) and gave advice as to how Ramanujan could get his papers published. In a sequel to this reply, dated 7 December 1912, Professor Hill wrote to Prof. Griffith:[35]

[32] Ref. *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras-1, Vol. 1 (1968) p. 115.

[33] *Griffith to Sir Francis Spring: To keep Ramanujan happily employed*, Letter dated 12 Nov. 1912; in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras Vol. 1 (1968) p. 50.

[34] *M.J.M. Hill to C.L.T. Griffith: Fallen into pitfalls*, dated 3 Dec. 1912; in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras, Vol. 1 (1968) p. 53.

[35] *Ramanujan: Letters and Commentary*, by Bruce C. Berndt and Robert A. Rankin, AMS-LMS (1995). Also, Indian Edition with a Preface, Additions to the Indian Edition and Errata, by K. Srinivasa Rao, published by Affiliated East West Press Pvt. Ltd. (1997).

Mr. Ramanujan is evidently a man with a taste for Mathematics, and with some ability, but he has got on the wrong lines. He does not understand the precautions which have to be taken in dealing with divergent series, otherwise he could not have obtained erroneous results you send me, viz.:

$$\begin{aligned} 1+2+3+\cdots+\infty &= -\frac{1}{12}, \\ 1^2+2^2+3^2+\cdots+\infty^2 &= 0, \\ 1^3+2^3+3^3+\cdots+\infty^3 &= \frac{1}{240}. \end{aligned} \tag{1.3}$$

The sums of n terms of this series are:

$$\frac{1}{2}n(n+1), \qquad \frac{1}{3}n\left(n+\frac{1}{2}\right)(n+1), \qquad \left[\frac{1}{2}n(n+1)\right]^2 \tag{1.4}$$

and they all tend to ∞, *as n tends to* ∞. *I do think you can do no better for him than to get him a copy of the book I recommended, Bromwich's Theory of Infinite Series, published by Macmillan and Co., who have branches in Calcutta and Bombay. Price 15/- net.*

It was soon realized that the lack of formal education was responsible for Ramanujan giving these results which look to any mathematician as absurd. The Riemann zeta function is defined as:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \qquad \text{for } s > 0. \tag{1.5}$$

If one takes negative integer values for s, then the above three series will correspond to $\zeta(-1), \zeta(-2)$ and $\zeta(-3)$, for an analytic continuation of the Riemann zeta function!

Ramanujan published two short notes: The first short note was *On Question 330 of Professor Sanjana* (JIMS IV (1912) 59–61) concerning the summation of the series:

$$\frac{1}{1^n} + \frac{1}{2}\frac{1}{3^n} + \frac{1\cdot 3}{2\cdot 4}\frac{1}{5^n} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{1}{7^n} + \cdots \text{ad inf.}, \tag{1.6}$$

if $n > 3$. Ramanujan in this, his first paper, has summed the series for <u>all</u> values of n.

The second short note of Ramanujan, also published in 1912, was a *Note on a set of simultaneous equations*, in the JIMS IV, pages 94–96. In it, Ramanujan considered the equations:

$$\begin{aligned} x_1 + x_2 + x_3 + \cdots + x_n &= a_1, \\ x_1y_1 + x_2y_2 + x_3y_3 + \cdots + x_ny_n &= a_2, \end{aligned}$$

$$x_1 y_1^2 + x_2 y_2^2 + x_3 y_3^2 + \cdots + x_n y_n^2 = a_3, \tag{1.7}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad \cdots\ \cdots\cdots$$

$$x_1 y_1^{2n-1} + x_2 y_2^{2n-1} + x_3 y_3^{2n-1} + \cdots + x_n y_n^{2n-1} = a_{2n},$$

where $x_1, x_2, x_3, \cdots x_n$ and $y_1, y_2, y_3, \cdots y_n$ are $2n$ unknown quantities. Ramanujan besides giving the general solution illustrated the method by giving an example which had ten equations in 10 unknowns, which were solved using the method of indeterminate coefficients.

When Ramanujan approached Prof. Seshu Aiyar with some theorems on Prime Numbers,[36] his attention was drawn to G.H. Hardy's Mathematical Tract on *Orders of infinity*, which had appeared soon after its publication in England (one of the good things which would not have happened if the British were not ruling our country!). Ramanujan[37] observed that "on p.36 of this Tract 'the exact order of $\zeta(x)$ [defined by the equation:

$$\zeta(x) = \pi(x) - \int_2^x \frac{dt}{\log\ t} \tag{1.8}$$

where $\pi(x)$ denotes the number of primes less than x,] has not yet been determined."

In other words, that "no definite expression has yet been found for the number of prime numbers less than any given number". Ramanujan told Prof. Seshu Aiyar that he had discovered a result which gave the order of $\rho(x)$. This made Prof. Seshu Aiyar suggest the communication of this and other results of Ramanujan in his Notebooks to Mr. G.H. Hardy[38] who was by then a recognized mathematician, a Fellow of the Royal Society of London and a Cayley Lecturer in mathematics at the Trinity College of Cambridge University. It was Ramanujan who contacted Hardy and as Hardy himself stated once, Ramanujan discovered Hardy (and not the other way round!).

[36]The first 25 prime numbers are: 2,3,5,7; 11,13,17,19; 23,29; 31,37; 41,43,47; 53,59; 61,67; 71,73,79; 83,89; 97 and from this one can say that there are 4 prime numbers below 10; 8 below 20; 10 below 30; 13 below 50; 15 below 60; 17 below 70; 20 below 80; 22 below 90 and below 100 there are 25 prime numbers which we can write as: $p(10) = 4$, $p(20) = 8$, $p(30) = 10$, $p(40) = 12$, $p(50) = 15$, $p(60) = 17$, $p(70) = 19$, $p(80) = 22$, $p(90) = 24$ and $p(100) = 25$. One of the significant contributions of Ramanujan was to discover an exact formula for $p(n)$. Only the asymptotic formula $p(n) \sim n/\log n$ was known to Gauss.

[37]*Collected Papers of Ramanujan*, Ed. G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, AMS Chelsea Publishing (2000), p. xxii.

[38]Godfrey Harold Hardy, was born on February 7, 1877, more than 10 years before Ramanujan was born, on December 22, 1887. In early 1947, a confirmed bachelor, Hardy tried to kill himself, by swallowing too many barbiturates, which made him vomit. He was possibly driven to this extreme since he was virtually an invalid for about a year by then and died on December 1, 1947, the day he was to receive the highest honour, the Copely Medal, from the Royal Society of London.

1.4 Ramanujan's First Letter to Hardy

The first 11-page letter of Ramanujan to Hardy was dated January 16, 1913. There is no parallel to such a letter in the History of Mathematics. The letter is reproduced in full in *Ramanujan Letter and Commentary* by Bruce C. Berndt and Robert A. Rankin.[39] In this first letter Ramanujan wrote

I had no University education but I have undergone the ordinary school course... I have made a special investigation of divergent series in general and the results I get are termed by local mathematicians as 'startling'...

"I have got theorems on divergent series to calculate the convergent values corresponding to the divergent series, viz.:

$$\begin{aligned} 1-2+3-4+\cdots &= \frac{1}{4}, \\ 1-1!+2!-3!+\cdots &= .596\ldots, \\ 1+2+3+4+\cdots &= -\frac{1}{12}, \\ 1^3+2^3+3^3+4^3+\cdots &= \frac{1}{240}. \end{aligned} \tag{1.9}$$

Theorems to calculate such values for any given series (say, $1-1^1+2^2-3^3+4^4-5^5+\cdots$) *and the meaning of such values.*

I have also dealt with such questions, 'When to use, where to use, and how to use such values, where do they fail and where do they not?' "

Hardy's prompt reply dated February 8, 1913, was the starting point of Ramanujan's recognition by the Western world of mathematicians, and this most encouraging letter[40] read:

Dear Sir,

I was exceedingly interested by your letter and by the theorems you state. You will however understand that, before I can judge properly of the value of what you have done, it is essential that I should see proofs of some of your assertions.

Your results seem to me to fall into roughly 3 classes:

(1) there are a number of results which are already known, or are easily deducible from known theorems;

(2) there are results which, so far as I know, are new and interesting, but interesting rather from their curiosity and apparent difficulty than their importance;

[39] I am thankful to Mr. Pacha Nambi, s/o Mr. Nambi Iyengar (who was the Personal Assistant of Professor Alladi Ramakrishnan) for sending me a copy of this book in September 1998, which was brought out as an Indian Edition, by M/s. Allied Publisher (India) Pvt. Ltd., after I got the required permission of Prof. Bruce Berndt and all concerned.

[40] Reprinted in *Ramanujan: Letters and Commentary*, by Bruce C. Berndt and Robert A. Rankin, with Additions to the Indian edition and errata, by K. Srinivasa Rao, Affiliated East-West Press Pvt. Ltd. (1997).

(3) there are results which appear to be new and important, but in which almost everything depends on the precise rigour of the methods of proof which you have used.

As instances of these 3 classes I may mention . . .

Ramanujan's second (10-page) letter dated February 27, 1913, was in response to Hardy's reply. In this letter, Ramanujan[41] wrote: *I find in many a place in your letter rigorous proofs are required and you ask me to communicate the methods of proof. I am sure you will follow the London Professor.*[42] *But as a fact I did not give him any proof but made some assertions as the following under my new theory. I told him that the sum of an infinite number of terms of the series*:

$$1 + 2 + 3 + 4 + \cdots = -\frac{1}{12} \tag{1.10}$$

under my theory. If I tell you this you will at once point out to me the lunatic asylum as my goal. I dilate on this simply to convince you that you will not be able to follow my method of proofs if I indicate the lines on which I proceed in a single letter. You may ask how you can accept results based on wrong premises. What I tell you is this. Verify the results I give and if they agree with your results, got by treading on the groove in which the present day mathematicians move, you should at least grant me that there may be some truth in my fundamental basis. So what I now want at this stage is for eminent professors like you to recognize that there is some worth in me. I am already a half starving man. To preserve my brains I want food and this is now my first consideration. Any sympathetic letter from you will be helpful to me here to get a scholarship either from the University or from the Government.

. . . . . .

You may judge me hard that I am silent on the methods of proof. I have to re-iterate that I may be misunderstood if I give in a short compass the lines on which I proceed. It is not on account of my unwillingness on my part but because I fear I shall not be able to explain everything in a letter. I do not mean that the methods should be buried with me. I shall have them published if my results are recognized by eminent men like you.

The passages quoted clearly indicate that Ramanujan was fully aware of the apparently absurd nature of the sums of the series and also the difficulty he had

[41] *Ramanujan: Letters and Commentary*, Bruce C. Berndt and Robert A. Rankin, Am. Math. Soc. and London Math. Soc. (1995) p. 44.

[42] This reference is most probably to Prof. M.J.M. Hill. Ref. M.J.M. Hill to C.L.T. Griffith, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Memorial Number, vol. 1, Muthialpet High School, Chennai-1 (1968) p. 53.

in an unambiguous presentation of his "startling" results. In the words of Bruce Berndt and Robert Rankin:[43]

Evidently, Hill shared Abel's view that 'divergent series are in general deadly', for, not surprisingly, he failed to discern the origin of the three results quoted in his letter. The three claims give the values of $\zeta(-n)$*, for n=1,2,3, respectively.*[44] *It was natural for Ramanujan to write these facts in terms of the divergent series of the Riemann zeta-function to these values. In Ramanujan's theory of the 'constant' of a (convergent or divergent) series, the three values* $-\frac{1}{12}, 0$ *and* $\frac{1}{240}$ *are, respectively, the constants of the three given series. It is likely that this theory was not communicated to Hill. Ramanujan's theory of divergent series is elucidated, albeit rather imprecisely, in Chap. 6 of his second Notebook*[45] *and readers should consult Berndt's book (Ramanujan's Notebooks,Part II, Springer-Verlag, New York (1989)) for an account of this theory.*

1.5 The Years of Fruition

The life of Ramanujan, in the words of C.P. Snow[46] is: *an admirable story, and one which showers credit on nearly every one... Hardy was not the first eminent mathematician to be sent the Ramanujan manuscripts. There had been two before him, both English, both of the highest professional standard. They had each returned the manuscripts without comment. I don't think that history relates what they said, if anything, when Ramanujan became famous.*

As for their identity, Snow adds that "out of chivalry Hardy concealed this in all that he said or wrote about Ramanujan."[47] However, these names were revealed in *Alternative Sciences*,[48] by A. Nandy to be H.F. Baker and E.W. Hobson,[49] Sir Francis Spring, the Chairman of the Madras Port Trust and his trusted Treasurer,

[43] *Ramanujan: Letters and Commentary*, Ed. Bruce C. Berndt and Robert A. Rankin, AMS-LMS (1995) p. 17.

[44] The Riemann zeta-function is defined as: $\zeta(s) = \sum_{n=0}^{\infty} \frac{1}{n^s}$, $Re\ s > 1$. The ζ function has a unique analytic continuation to the point $s = -1$, and it is known that $\zeta(-1) = -1/12$. With hind sight, in the modern day notation, we may say that Ramanujan combined the series definition of: $\zeta(-1) = 1+2+3+4+\cdots$ with its value $\zeta(-1) = -1/12$, equated the two right hand sides into the one equation of his, viz. that the sum of the series: $1+2+3+4+\cdots$ is equal to $-1/12$.

[45] S. Ramanujan, *Notebooks* (2 Volumes), Tata Institute of Fundamental Research, Bombay (1957).

[46] C.P. Snow, in his Foreword to G.H. Hardy's: *A Mathematician's Apology*, Cambridge University Press (1967) p. 30.

[47] p.10 in C.P. Snow's Introduction in *A Mathematician's Apology*, G.H. Hardy, 'Orders of Infinity', Cambridge Univ. Press, Cambridge (1910).

[48] Allied Publishers, New Delhi, 1980.

[49] Also see, Bruce C. Berndt, *Ramanujan Notebooks*, Part I, Springer-Verlag (1985) p. 3.

S. Narayana Iyer[50] who later became the Chairman himself of the Port Trust, gave Ramanujan every conceivable encouragement, which he richly deserved. Dr. Gilbert T. Walker, F.R.S., then Director General of Observatories, Simla, visited the University of Madras, in February 1913, and Sir Francis Spring drew his attention to the Notebooks of Ramanujan.

Dr. Walker, a Senior Wrangler, and a good mathematician was a former Fellow of Trinity College, Cambridge, as well as a lecturer, immediately recognized the intrinsic quality of Ramanujan's work. G.T. Walker wrote to the Registrar of the University of Madras, Mr. Francis Dewsbury, commending the work[51] of Ramanujan to be *comparable in originality with that of Mathematics Fellow in a Cambridge College*, though lacking in precision and completeness, necessary for establishing the universal validity of the results. Walker wrote that it was completely clear to him *that the University would be justified in enabling S. Ramanujan for a few years at least to spend the whole of his time on mathematics without any anxiety as to his livelihood*.

He also wanted the University to correspond with Mr. Hardy, Fellow of Trinity College, Cambridge, since Ramanujan was already in correspondence with him, assuring Hardy of the interests of the University in the well-being of Ramanujan. This recommendation of Walker was accepted by the Board of Studies of the University of Madras, and Ramanujan was granted a special research scholarship of Rs. 75/- per month[52] for 2 years. This was granted with the express consent of the Governor of Madras, and a condition was stipulated that Ramanujan should submit Quarterly Reports on his work. The Madras Port Trust granted Ramanujan 2 months leave, on loss of pay, to enable him to accept this Scholarship from May 1913, as *the first research scholar in mathematics of the University of Madras*. Thus began the research career of Ramanujan as a professional mathematician.

Ramanujan's first letter to Hardy, dated January 16, 1913, contained *the bare statements of about 120 theorems, mostly formal identities from the Notebooks.* This collection obviously represented what Ramanujan himself considered were his important results. Professor Hardy, the professional mathematician, who was aware that he was *the first really competent person who had the chance to see some of his work*, found some of the *series formulae intriguing*, some of the integral formulae *were classical and known, vaguely familiar* and he *could prove some integral formulae with great deal more difficulty*. But these were to him the least impressive. However, some of Ramanujan's formulae were *on a different level*

[50] S. Narayana Iyer, M.A.(Maths.), was one who can be considered the foremost among the benefactors of Ramanujan. He not only supported Ramanujan all through and was indeed a friend, philosopher, guide and perhaps, even a scribe, on occasions for letters written to Hardy, but also continued to support Mrs. Ramanujan throughout his lifetime.

[51] *Ramanujan: Letters and Reminiscences*, Memorial Vol. No.1, Ed. P.K. Srinivasan, The Muthialpet High School, Madras, Vol. 1 (1968), p. 55.

[52] Ramanujan was getting a salary of Rs. 25/- per month when he was in the Accountant General's Office and Rs. 30/- per month when he was at the Madras Port Trust. The salary of a Professor in those days was Rs. 225/- per month.

and obviously both difficult and deep. Hardy states that: *There is more in one of Ramanujan's formulae than meets the eye, as any one who sets to work to verify those which look the easiest will soon discover. In some the interest lies very deep, in others comparatively near the surface; but there is not one which is not curious or entertaining*.[53]

C.P. Snow in his Rectorial Address[54] stated that the life of Ramanujan *is an admirable story, and one which showers credit on nearly everyone*. Sir Francis Spring, the Chairman, and Mr. S. Narayana Iyer, then Treasurer of the Madras Port Trust gave Ramanujan every possible encouragement. Dr. Gilbert T. Walker, F.R.S., Director General of Observatories, Simla,[55] visited the University of Madras, in February 1913, and Sir Francis Spring drew his attention to the Notebooks of Ramanujan. Dr. Walker, immediately recognized the intrinsic quality of Ramanujan's work.

C.P. Snow also wanted the University to correspond with Mr. Hardy, assuring Mr. Hardy of the University's interest in Ramanujan. The recommendation of Dr. Walker was accepted by the Board of Studies of the University of Madras, and Ramanujan was granted[56,57] the **first research scholar of the University of Madras**. Thus began the research career of Ramanujan as a professional mathematician.

Ramanujan's selection of 120 Theorems obviously represented what Ramanujan himself considered as results of importance to him. Prof. Hardy, the professional mathematician has stated that he was aware that he was *the first really competent person who had a chance to see some of his work*. Snow in his Rectorial Address stated: *Hardy gave the manuscript a perfunctory glance, and went on reading the morning newspaper. It occurred to him that the first page was a little out of the ordinary for a cranky correspondent. It seemed to consist of some theorems, very strange-looking theorems, without any argument. Hardy then decided that the man must be a fraud, and duly went about the according to his habits, giving a lecture, playing a game of tennis. But there was something nagging at the back of his mind. Anyone who could fake such theorems, right or wrong must be a fraud of a genius or an unknown Indian mathematician of genius? He went that evening after dinner to argue it out with his collaborator, J.E. Littlewood, whom Hardy always insisted was a better mathematician than himself. They soon had no doubt of the answer. Hardy*

[53]The quotations are from the Introduction to the *Collected Papers of Srinivasa Ramanujan*, Ed. G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, Cambridge the University Press, Cambridge (1927), Second edition, Chelsea, New York (1962).

[54]C.P. Snow in his Foreword to G.H. Hardy's *A Mathematician's Apology*, Cambridge University Press (1967) p. 30.

[55]The Meteorological Observatory has been shifted to Pune, in later years.

[56]Ramanujan was getting a salary of Rs. 25/- per month when he was in the Accountant General's Office and Rs. 30/- per month when he was at the Madras Port Trust.

[57]*Ramanujan's Notebooks*, Bruce C. Berndt, Springer-Verlag, Part I (1985)–Part V (1997).

was seeing the work of someone whom, for natural genius, he could not touch - who, in natural genius, though of course not in achievement, as Hardy said later, belonged to the class of Euler and Gauss.

Hardy made up his mind that Ramanujan should be brought to Cambridge and provided with the necessary education and contact with Western mathematicians of the highest class. So, Hardy wrote to the Secretary of the Indian students, in the India Office, London, suggesting that some means be found to get Ramanujan to Cambridge, and he in turn wrote, in February 1913, to Mr. Arthur Davies, the Secretary to the Advisory Committee for Indian students in Madras conveying the desire of the tutors at Trinity College to invite Ramanujan to Cambridge University.

In quick succession, Ramanujan received in the next 3 months four long letters[58] from Hardy, in which Hardy wrote plainly about what he felt had been proved or claimed to have been proved by Ramanujan. He clearly communicated his genuine anxiety *to see what can be done to give you* [Ramanujan] *a better chance of making the best use of your obvious mathematical gifts*.

At last, Ramanujan found, in Hardy, a sympathetic friend and was willing to place unreservedly in his hands all that he had. He wrote again to Hardy on February 27, 1913, and sent him more formulae and explanations. On April 17, 1913, Ramanujan wrote to Hardy about his having secured the scholarship, of £60 per annum, of the University of Madras, for 2 years. Ramanujan took up residence at Hanumantharayan Koil Lane in Triplicane, Madras, around this time and had access to the Madras University Library's books on mathematics.

Ramanujan was initially reluctant to go abroad, because of the prevalent caste prejudices in those days[59] which were compounded by the extremely orthodox views of his mother to whom he was greatly attached. At the beginning of 1914, Mr. E.H. Neville, a young mathematician and a Fellow of Trinity College, Cambridge, was in Madras as a visiting lecturer to give a series of lectures on Differential Geometry to the Mathematics Honours students of the University of Madras. Mr. Hardy entrusted to him the mission of scrutinizing the Notebooks of Ramanujan and also, persuading Ramanujan to go to Cambridge.

Mr. Neville met Ramanujan and saw the priceless Notebooks and had discussions with him. This was sufficient to convince him of Ramanujan's uncommon abilities in mathematics and took the initiatives required to overcome all the difficulties in arranging a visit by Ramanujan to Trinity College, Cambridge, to work with Prof. G.H. Hardy. Prof. Richard Littlehailes, who was a Professor of Mathematics with the Observatory in Madras, introduced Neville[60] *to everyone who carried*

[58]Ref. Bruce C. Berndt and Robert A. Rankin, *Ramanujan: Letters and Commentary*, AMS-LMS (1999), for these and other letters referred to in this article.

[59]Crossing the oceans was considered a sacrilege by the Hindu Brahmins, and people who did so were, on their return to India, treated as out-castes. All relationships with such foreign returned individual and even their families were shunned!

[60]*From Obscurity to Fame*, E.H. Neville, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968) pp. 138–141.

weight in the University or in the civil administration; everywhere I talked of Ramanujan, explained as I have tried to do now the importance to him of a stay in Cambridge, and urged generosity. In a letter[61] dated January 28, 1914, to the Registrar of the University of Madras, Mr. Francis Dewsbury, Mr. Neville wrote about *the importance of securing to Ramanujan a training in the refinements of modern methods and a contact with men who know what range of ideas have been explored and what have not*. Neville also prophesied that Ramanujan would respond to such a stimulus and that Ramanujan's *name will become one of the greatest in the history of mathematics, and the University and the city of Madras will be proud to have assisted in his passage from obscurity to fame.* The very next day, Prof. Littlehailes also wrote[62] to Dewsbury that Ramanujan *be granted by this University to proceed to Cambridge. Ramanujan is a man of most remarkable mathematical ability, amounting I might say to genius, whose light is metaphorically hidden under a bushel in Madras.*

These letters were written only after Ramanujan was persuaded to put aside the prejudices against crossing the seas. Hardy stated that Ramanujan's *consent was at last got very easily* $\cdots$ *(when) his mother announced that she had a dream in which she saw her son seated in a big hall amidst a group of Europeans and that the goddess Namagiri had commanded her not to stand in the way of her son fulfilling his life's purpose*.

The proposals regarding the scholarship to be granted to Ramanujan by the University of Madras were approved. To the lasting credit of the University of Madras, the Syndicate decided within a week to set aside Rs. 10,000/- to offer Ramanujan a scholarship of £250/- a year plus £100/- for a passage by ship and for initial outfit.[63] At the instance of Professors Neville and Littlehailes, Sir Francis Spring wrote[64] to the personal Secretary (C.B. Cotterell) to the Governor of Madras, Lord Pentland, persuading His Excellency to speedily approve the University's sanction and the Government's sanction arrived within a week.

It is remarkable that while there was no one in the Colleges or the University who could appreciate the depth of Ramanujan's mathematical abilities, the University of Madras awarded its first research scholarship to Ramanujan to study in Cambridge, despite the fact that he had failed to pass the First degree in Arts (F.A.) examinations of the University in 1905, and as a private candidate in 1907. This is undoubtedly due to the courage of conviction of Indian stalwarts of that era—Dewan Bahadur Ramachandra Rao, V. Ramaswamy Iyer, S. Narayana Iyer and P.V. Seshu Iyer—who succeeded in convincing and persuading the British authorities to act immediately on the basis of the merits of the candidate.

[61] *E.H. Neville to Dewsbury: The Discovery of the Genius of Ramanujan*, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968) p.59.

[62] *Littlehailes to Dewsbury: First Research Student in Mathematics*, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968) pp. 61–63.

[63] The second class fare between London and Bombay was £32/- in 1914, or about Rs. 480/-.

[64] Sir Francis Spring to C.B. Cotterell, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School publication, (1968) pp. 64–65.

Ramanujan sent his wife Janaki and mother Komalathammal back to Kumbakonam, and then changed the traditional hair-style of the Brahmin, viz. a tuft, and got his hair cropped in European style before he boarded the ship S.S. Nevasa, at the Madras Harbour on March 17, 1914. Prior to his departure, he arranged with the University of Madras to send £60/- a year to his parents in Kumbakonam, out of the annual £250/- granted to him.

Mr. Arthur Davies and Prof. Littlehailes attended to all the details regarding Ramanujan's passage to England. Except for the first 3 days when he was sea-sick, Ramanujan enjoyed the voyage and reached London through the Channel and the river Thames on[65] April 14, 1914. Ramanujan was received by Mr. E.H. Neville and his brother at the docks, and he stayed at Cromwell Road, London, for a few days before going to Cambridge on the evening of April 18. He remained for a few days in Mr. Neville's house before moving to the college premises to stay, which even though costlier than lodging houses was more convenient for him and the professors. Ramanujan wrote[66] to his friend that *Mr. Hardy, Mr. Neville and others here are unassuming, kind and obliging. As soon as I came here, Mr. Hardy paid £20/- to the college for my entrance and other fees and made arrangements to give me a scholarship of £40/- a year.*

Ramanujan was admitted by Mr. Hardy to Trinity College which supplemented his scholarship with the award of an 'exhibition' of £60/- a year, to augment the £250/- a year scholarship awarded to him by the University of Madras.

Though Ramanujan had access only to George Shoobridge Carr's *Synopsis*—and perhaps, to a few other books[67]—still, in the words of the historian J.R. Newman,[68] he *arrived in England abreast and often ahead of contemporary mathematical knowledge. Thus, in a lone mighty sweep, he had succeeded in recreating in his field, through his own unaided powers, a rich half century of European mathematics. One may doubt whether so prodigious a feat had ever before been accomplished in the history of thought*.

To Mr. Hardy,[69] Ramanujan's friend, philosopher and guide: *The limitation of his* [Ramanujan's] *knowledge was as startling as its profundity. Here was a man who could work out modular equations, and theorems of complex multiplications, to orders unheard of, whose mastery of continued fractions was, on the formal side at any rate, beyond that of any mathematician in the world, who had found for himself*

[65] Traditionally April 14 is the Tamil New Year's day, in India.

[66] Letter to R. Krishna Rao: 'Letter 2. Arrival in London on 14-4-1914 and settling done at Trinity College', dated 11 June 1914, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras-1 publication, (1968) pp. 5–7.

[67] From the article 'My Father and Ramanujan' of Mr. Narayana Iyer's son, N. Subbanarayanan, in *Ramanujan: Letters and reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras-1, publication, (1968) pp. 112–115.

[68] 'Srinivasa Ramanujan', J.R. Newman in 'Mathematics in the Modern World', W.H. Freeman & Co., (1968) pp. 73–76.

[69] *Collected papers of Srinivasa Ramanujan*, Ed. G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, AMS Chelsea Publishing (2000), p. xxx.

the functional equation of the zeta-function, and the dominant terms of many of the most famous problems, in the analytic theory of numbers, and he had never heard of a doubly periodic function or of Cauchy's theorem, and had indeed but the vaguest idea of what a function of a complex variable was. His ideas of what constituted a mathematical proof were of the most shadowy description. All his results, new or old, right or wrong, had been arrived at by a process of mingled argument, intuition and induction, of which he was entirely unable to give a coherent account.

With such a natural genius, Hardy collaborated and tried to teach, as he wrote,[70] the *things of which it was impossible that he should remain in ignorance. . . . It was impossible to allow him to go through life supposing that all the zeros of the zeta function were real. So I had to try to teach him, and in a measure I succeeded though I obviously learnt from him much more than he learnt from me.*

Hardy did not attempt to convert Ramanujan into a mathematician of the modern school but enabled him to go on producing original ideas in his classical mould with rigorous proofs for the theorems he discovered.

Unfortunately, the period of Ramanujan's stay in England almost overlapped with the years of the first World War. Ramanujan wrote[71] to his friend R. Krishna Rao, that one of the lecturers (perhaps, J.E. Littlewood) went to War and that the other professors lost their interest (in mathematics) owing to the war. One of the professors had remarked that Ramanujan was in England at the most unfortunate time. There were about 700 students before the war, but this number was reduced to 150 by November 1915.

Initially he asked for and obtained some South Indian food items, like tamarind, coconut oil, curry powder, etc., by post parcel from his home, as well as from a company in London, but by January 1915, he wrote in a letter[72] to a friend of his that *now as well as in the future I am not in need of anything as I gained control over my taste and can live on mere rice with a little salt and lemon juice for an indefinite time.* His difficulty in getting proper food during those War years was alleviated by the availability of good milk and fruits. Being an adherent to strict vegetarianism, which was not well known or even recognized as a viable alternative to non-vegetarian food, at that time, he had no option but to cook for himself a lintel soup, vegetables, rice and curds.

Ramanujan was attending a lecture by Mr. Berry at the University on elliptic integrals. Mr. Berry was working out some formulae with chalk on the blackboard and a glance at Ramanujan's face, bright with excitement, caused the lecturer to ask Ramanujan whether he was following his lecture and whether he had anything to say. At this opportunity, Ramanujan went to the blackboard and much to everyone's

[70] *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Robert Kanigel, Rupa & Co, 1994, p. 226.

[71] *Outbreak of First World War*, dated Nov.13, 1914, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968), pp. 12–19.

[72] Letter to S.M. Subramanian, (a classmate of Ramanujan during his School days) in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968) pp. 21–27.

surprise wrote down some of the results which were yet to be proved. This anecdote was recalled, in later years, by P.C. Mahalanobis.[73]

Ramanujan wrote a few articles soon after he reached Cambridge and in June 1914, Hardy presented some of the results from Ramanujan's Notebooks at a meeting of the London Mathematical Society. However, in January 1915, Ramanujan wrote to his friend Subramanian: *My notebook is sleeping in a corner for these four or five months. I am publishing only my present researches as I have not yet proved the results in my notebook rigorously.*

Ramanujan was more interested in getting new results (perhaps due to the ongoing war), he decided to publish the old results worked out in his Notebooks after the end of the War. After about a year and half at Cambridge, Hardy wrote to the Registrar of the University of Madras, that Ramanujan *is beyond question the best Indian mathematician of modern times. ...He will always be rather eccentric in his choice of subjects and methods of dealing with them. ...But of his extraordinary gifts there can be no questions; in some ways he is the most remarkable mathematician I have ever known*.

Hardy's letter[74] and official report to the University, as well as an appeal by Sir Francis Spring to the University to continue the assistance extended by it to Ramanujan, made the University, in December 1915, to extend the Scholarship up to March 1919.

During his 5 years in Cambridge, Ramanujan published 21 research papers containing theorems on definite integrals, modular equations and approximations to π, Riemann's zeta function, infinite series, summation of series, analytic number theory, partitions, combinatorial analysis. His paper entitled *Highly Composite Numbers* which appeared in the Journal of the London Mathematical Society (JLMS), in 1915, is 62 pages long and contains 269 equations. This is Ramanujan's longest paper. The London Mathematical Society had some financial difficulties at that time and Ramanujan was requested to reduce the length of his paper to save printing expenses. Five of these 21 research papers were in collaboration with Hardy. Ramanujan also published five short notes in the Records of Proceedings at meetings of the London Mathematical Society and six more in the Journal of the Indian Mathematical Society.

The *Collected Papers of Srinivasa Ramanujan* published in a volume of 350 pages was edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson. The first edition of this book published by the Cambridge University Press, in 1927, resulted in a flurry of research papers by G.N. Watson (25), C.T. Preece (6), W.N. Bailey (4), Robert A. Rankin (3) and some others during the period 1928–1938.

[73] *My friendship with Ramanujan in England*, P.C. Mahalanobis—who became later the eminent mathematician of India who started the Indian Statistical Institute in Calcutta—in *Ramanujan: Letters and Commentary*, Ed. P.K. Srinivasan, Muthialpet High School, Madras-1, (1968) pp. 145–148.

[74] 'Hardy to Dewsbury', in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Madras-1, (1968) pp. 76–77.

Ramanujan was awarded the Bachelor's Degree in Arts, B.A. degree by research, of the Cambridge University, in March 1916, for his work on *Highly Composite Numbers*, published as a long paper in the JLMS. Ramanujan's dissertation bore the same title and included in addition six other published papers of his.

Ramanujan registered as a research student in June 1914 and the prerequisite of a diploma or a certificate, as well as the domiciliary requirement of six terms was waived in his extraordinary case. It is unfortunate that a copy of this dissertation is not to be found in the records[75] of the University. According to Hardy,[76] this long memoir of Ramanujan on Highly Composite Numbers *represents, perhaps, in a backwater of mathematics, and is somewhat overloaded with detail; but the elementary analysis of "highly composite" numbers – numbers which have more divisors than any preceding number – is most remarkable, and shews very clearly Ramanujan's extraordinary mastery over the algebra of inequalities.*

And also, that it is *a very peculiar one, standing somewhat apart from the main channels of mathematical research. But there can be no question as to the extraordinary insight and ingenuity which he has shown in treating it, nor any doubt that the memoir is one of the most remarkable published in England for many years.*

Ramanujan's designated tutor who monitored his progress at Trinity College was E.W. Barnes, who considered Ramanujan as perhaps the most brilliant of all the top Trinity students, which included Littlewood, at that time.[77] Hardy was immensely satisfied with the progress of Ramanujan and wrote so to the Registrar of the University of Madras, supporting an extension of Ramanujan's 2-year scholarship *... until, as I confidently expect, he is elected to a Fellowship of the College. Such an election I should expect in October 1917.*

Later, in June 1916, in an official report on the progress of Ramanujan's work, Hardy wrote:

... it is already safe to say that Mr. Ramanujan has justified abundantly all the hopes that were based upon his work in India, and has shown that he possesses powers as remarkable in their way as those of any living mathematician. ... I have said enough, I hope, to give some idea of his astonishing individuality and power. India has produced many talented mathematicians in recent years, a number of whom have come to Cambridge and attained high academical distinction. They will be the first to recognize that Mr. Ramanujan's work is of a different category.

In spite of the War which was raging, which deprived Ramanujan of the centre stage which he would have held otherwise, with his brilliant research work in the midst of his peers, the confidence he kindled in Hardy was enough to win for him recognition and laurels very soon, but, unfortunately, the first signs of his illness appeared in Ramanujan in the Spring of 1917.

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[75] *Ramanujan: Letters and Commentary, B.C. Berndt and R.A. Rankin, AMS-LMS (1995) p. 137.*

[76] *Collected Papers of Srinivasa Ramanujan*, Ed. by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, AMS Chelsea Publishing, AMS, Providence, Rhode Island (2000) p. xxxiv.

[77] Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Charles Scribner's Sons, New York (1991) p. 233.

1.6 Ramanujan's Medicography

Ramanujan's first biographer, Dr. S.R. Ranganathan[78] stated that by the end of 1918, it was definitely known that Tuberculosis (TB) had set in. More recently, in the acclaimed complete biography, Robert Kanigel,[79] also lays emphasis on this theory of tuberculosis as being the cause of Ramanujan's prolonged, terminal illness. A systematic study of the details of the hospital medical transcripts, by mathematician Dr. Robert A. Rankin,[80] in 1984, led him to point out that the dreaded TB which was in those days considered as a contagious disease which made the doctors confine the patient to a Sanatorium, was not the cause of his death. It is a fact that the warmer climate of Madras and its environment did not show any marked improvement in Ramanujan's health after his return to India. On the contrary, his condition further deteriorated, despite the best medical attention that was available at that time, and he was emaciated to *only skin and bone*, as described by his devoted wife Janaki, in later times.

In 1994, seven decades after the death of Ramanujan, Dr. D.A.B. Young,[81] after researching into the illness of Ramanujan, has given us a better insight into the cause for the deterioration of the health of Ramanujan. We give here a brief summary of the medical biography, which we refer to as medicography.[82]

Ramanujan came to Madras, in 1906, to join the Pachaiyappa's College, to continue his studies in the F.A. class. But, a few months later, he fell ill with dysentery and had to return to his home town Kumbakonam, for a period of about 3 months. Young conjectured that Ramanujan's dysentery was caused by amoebiasis, a tropical infectious disease, widespread in the metropolitan cities of India:
Amoebiasis[83]*, unless adequately treated, is a permanent infection, although many patients may go for long periods with no overt signs of the disease. Relapses, which were the order of the day, occurred when the host-parasite relationship is disturbed. Ramanujan experienced such a relapse, I believe in 1909, when according to his friend R.R. Ayyar, as quoted in Ranganathan's biography:*

"Ramanujan who was living in [Madras], became seriously ill . . . As a patient . . . he was obstinate and would not drink hot water and insisted on eating grapes

[78] S. R. Ranganathan, *Ramanujan: The Man and the Mathematician*, Asia Publishing House (1967).

[79] Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Charles Scribner's Sons, New York (1991), Indian edition published by Rupa & Co. (1994).

[80] R.A. Rankin, *Ramanujan as a patient*, Proc. Indian Acad. Sci., Math. Sci. **93** (1984) 79–100.

[81] Dr. D.A.B. Young, *Ramanujan's illness*, Current Science, **67** (1994) 967–972.

[82] This word "medicography" is coined by the author as an abbreviation for medical biography. While "Discography" is in the Dictionaries, this word Medicography is yet to find its place in the Dictionaries!

[83] The first case of amoebiasis was documented in 1875, and in 1891 the disease was described in detail, resulting in the terms amoebic dysentery and amoebic liver abscess. Amoebiasis, also known as amoebic dysentery, is an infection caused by any of the amoebae of the Entamoeba group. Amoebiasis can be mild and asymptomatic. Symptoms may include abdominal pain, diarrhoea or bloody diarrhoea.

which were sour and bad for him. [The doctor] after examining him, asked to send him to his parents as his condition required constant nursing."

How ill Ramanujan felt at this time is indicated by his giving his host for safe keeping the two large notebooks kept with him all the time and in which he had been recording his mathematical results; the same notebooks that are now famous as a major legacy of his genius.

Later the same year (1909), while still at home with his family, he developed a hydrocele, which was operated on in January 1910 . . . Dr. Shaw's suspicion that the operation was excision of a malignant growth, depending as it must have done on Ramanujan's exact description of the lesion, certainly favours the explanation of a scrotal amoeboma rather than a hydrocele.

These were the illnesses suffered by Ramanujan before and after his marriage to Janaki, in 1909, and before his departure to England. From all accounts available it appears that his health was reasonably good during the 3 years of his stay in Cambridge, despite his strict vegetarian diet, the food shortages and his own admission of "cooking only once a day or two".

This was also the most productive period in the life of Ramanujan in Cambridge. From May 1917, when he was first admitted to the Nursing Hostel in Cambridge for 5 months, he seems to have been in and out of several TB Sanatoria – Mendip Hills in Somerset for 3 weeks in October 1917; Matlock House in Derbyshire from November 1917 to June 1918; Fitzroy House, in London, during June–December 1918; and Colinette House in Putney from the end of December 1918 until his departure to India in March 1919.

Symptoms of improvement showed after considerable treatment for tuberculosis by the autumn of 1918. He was able to meet all his medical expenses incurred during his prolonged illness out of his earnings accumulated through his frugal living. Thanks to the unstinted efforts of Professor Hardy, who did his best to get Ramanujan the recognition which he richly deserved. Ramanujan was elected as a Fellow of the Royal Society (F.R.S.) of London, in February 1918. Reproduced below is an excerpt from the Records.

A copy of this document is an exhibit in the Avvai Kalai Kazhagam (Avvai Academy) in Royapuram, Chennai, which also houses the first Ramanujan Museum created by the late Mr. P.K. Srinivasan,[84] in 1990. The (Avvai Academy) Museum is kept open for visitors and regular events/programmes are conducted by Mr. A.T.B. Bose and Mrs. Meena Suresh, as its Directors. The excerpt from the Royal Society is dated December 18, 1917.

[84]Mr. P.K. Srinivasan, was a Fulbright Scholar for 2 years in the USA, and a Mathematics Teacher at the Muthialpet High School, Chennai, when he formed a Number Friends Society and an Old Boys' Committee, to acquire, from 1962 to 1967, articles which are the contents of two Ramanujan Memorial Numbers: *Ramanujan: Letters and Reminiscences* and *Ramanujan: An Inspiration.* An Appreciation by Bharat Ratna, Dr. S. Radhakrishnan, Former President of India, is a fitting tribute to this stupendous effort, which has been the main source for several later day biographers, including the present author, who acquired copies of these volumes, for the Library of the IMSc, and for himself, in 1967.

1.7 Excerpt from Records of the Royal Society and F.R.S.

Certificate for a Candidate for Election

Name and Title or Designation *Srinivasa Ramanujan*

Profession *Research Scholar in Mathematics*

Usual Place of Residence *Trinity College, Cambridge*

Qualifications (Not to exceed 250 words):

Distinguished as pure mathematician, particularly for his investigations in elliptic functions and the theory of numbers. Author of the following papers, amongst others: 'Modular equations and approximations to π', *Quarterly Journal*, vol. 45; 'New expressions of Riemann's function $\zeta(s)$ and $\Xi(t)$', ibid, vol. 46; 'Highly composite numbers', Proc. London. Math. Soc., 22; 'On the expression of a number in the form $ax^2 + by^2 + cz^2 + dt^2$', Proc. Camb. Phil. Soc., vol. 19.

Joint author with G.H. Hardy, F.R.S., of the following papers: 'Une formulae asymptotique pour le nombre des partitions de n', Comptes Rendus, 2 Jan. 1917; 'Asymptotic Formulae for the distribution of numbers of various types', Proc. London Math. Soc., vol. 16; 'The normal number of prime factors of a number n', Quarterly Journal, vol. 47; 'Asymptotic Formulae in Combinatory Analysis', Proc. London Math. Soc., (awaiting publication).

being desirous of admission into the ROYAL SOCIETY OF LONDON, we the undersigned propose and recommend him as deserving that honour, and as likely to become a useful and valuable Member.

From General Knowledge.	**From Personal Knowledge**.	
(sd.) E.T. Whittaker	(sd.) G.H. Hardy	*Proposer*
(sd.) A.R. Forsyth	(sd.) P.M. MacMahon	*Seconder*.
(sd.) A.N. Whitehead	(sd.) J.H. Grace	
	(sd.) Joseph Larmor	
	(sd.) T.J.P.A. Bromwich	
	(sd.) E.W. Hobson	
	(sd.) H.F. Baker	
	(sd.) J.E. Littlewood	
	(sd.) J.W. Nicholson	

Suspended for 1918.

Delivered at the Apartments of the Society on the 18th day of Dec. 1917.
Read to the Society on the 24th day January 1918.

This certificate on a printed form of the Royal Society has been filled by hand (and the handwriting appears to be that of Mr. Hardy). The citation awarding the Fellowship of the Royal Society to Ramanujan read: *Srinivasa Ramanujan, Trinity College, Cambridge. Research student in Mathematics Distinguished as a pure mathematician particularly for his investigations in elliptic functions and the theory of numbers.*

The renowned Physicist, Prof. R.H. Dalitz (Oxford University), on his visit to the Institute of Mathematical Sciences, in February 1996, spent some time with the author discussing various aspects concerning Raman and Ramanujan, two giants of Indian Science, and Chandra who was a personal friend of Prof. Dalitz and with whom the author had some interesting meetings and correspondence. After Prof. Dalitz returned to Oxford, the author had several interesting exchanges by e-mail and one such is regarding Ramanujan's F.R.S.:[85] *I did not succeed in finding Ramanujan's signature in the Royal Society book. The book is indexed, so it is just not there. The reason undoubtedly is that he was ill in that period and could not go to the Royal Society to sign it. There are other examples of well-known FRS's who somehow didn't get their signatures into the book. That means that he did not ever attend any meeting of the Royal Society; if he had, they would have brought out the book and not let him go until he signed. Of course, it was also war-time, which meant that there were as few meetings as possible.*

Ramanujan was elected to a Trinity College Fellowship, in October 1918, which was a Prize Fellowship worth £250/- a year for 6 years with no duties or conditions. These awards acted as great incentives to Ramanujan who discovered some of the most beautiful theorems in mathematics, subsequently.

Hardy's letter[86] to the Registrar of the University of Madras, Mr. Dewsbury, dated November 26, 1918, struck a hopeful note: *There is at last, I am profoundly glad to say, a quite definite change for the better. I think we may now hope that he has turned the corner, and is on the road to recovery. His temperature has ceased to be irregular, and he has gained nearly a stone*[87] *in weight. The consensus of medical opinion is that he has been suffering from some obscure source of blood poisoning, which has now dried up; and that it is reasonable to expect him to recover his health completely and if all goes well fairly rapidly.*

Ramanujan's symptoms were predominantly night-time fever, loss of weight leading to his emaciated looks and these caused depressions which once drove him to the limit of attempting suicide—a story which was recounted many years after his death, by the Nobel Laureate Dr. S. Chandrasekhar, as told to him by Professor Hardy, and the same reproduced in a later chapter of this book.

[85] Private communication by e-mail, dated 29 March 1996.

[86] In *Srinivasa Ramanujan, 1887–1920: A Tribute*, Ed. K.R. Nagarajan and T. Soudararajan, Macmillan India Ltd. (1988) and articles therein by K. Srinivasa Rao, G.E. Andrews, R. Askey, B.C. Berndt, R.P. Agarwal, A. Verma, S. Bhargava, M.V. Subbba Rao and T. Soundararajan. All these articles were obtained on request by the present author.

[87] One Stone weight is equal to 14 pounds, or about 6.6 kg.

These symptoms made the best of the doctors consider various diagnosis at different times: gastric ulcer, malaria, tuberculosis, cancer of the liver, etc. In recent times, with hindsight, vitamin B_{12} deficiency, something unknown to the world at that time, has been diagnosed as a possibility.[88]

The recovery alluded to by Hardy in his letter to Dewsbury was obviously the reason why Ramanujan was persuaded to return to India, with the hope that he would soon recover and return to take up the 5-year Trinity College Fellowship awarded to him.

1.8 The Beginning of the End

After completing nearly 5 years at Cambridge, early in 1919, when Ramanujan appeared to have recovered sufficiently to withstand the rigours of a long voyage to India, he left England on 27 February 1919 by s.s. Nagoya. Four weeks later on 27 March he arrived at Bombay and soon after at Madras, thin, pale and emaciated, but *with a scientific standing and reputation such as no Indian has enjoyed before*. Professor Hardy who expressed this view[89] also hoped that *India will regard him as the treasure he is*. Hardy urged the University to make a permanent provision for him to enable him to continue his research work. Again, the University rose to the occasion by granting Ramanujan a scholarship of £250/- a year, for 5 years, commencing from April 1919. Ramanujan was sent back to India by Hardy with the fond hope that the warmer climate would help his complete recovery from his illness which was believed to be tuberculosis.

Most unfortunately, his precarious health did not improve on his return to India. Fevers relapsed, and in addition, his wife recalled later on when asked that Ramanujan suffered severe bouts of stomach pain too.[90] Ramanujan was subject to fits of depression and he had a premonition of his death and was a difficult patient. He was for 3 months in Madras, for 2 months in Kodumudi and for 4 months at Kumbakonam. When his condition showed further signs of deterioration, after great persuasion, Ramanujan was brought to Madras, in January 1920, for the best medical treatment possible in those days, suggested by expert medical practitioners.

Despite all the tender attention he could get from his wife who nursed him throughout this period, and his mother, his untimely end came on the 26 of April 1920, in the Forenoon, at about 10 A.M., at Chetpet, Madras, when Ramanujan was 32 years, 4 months and 4 days old. Janaki, his wife who was 9 years old at the time of his marriage, lived with him after she came of age, only for about a year

[88] D.A.B. Young, *Ramanujan's illness*, Current Sci. **67** (1994) 967–972.

[89] Hardy in his letter to Dewsbury, the Registrar of the University of Madras—refer *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, in Ramanujan Memorial No. 1, Muthialpet High School, Chennai, pp. 76–77.

[90] S.R. Ranganathan, *Ramanujan: The Man and the Mathematician*, Asia Pub. House (1967).

before his departure to England. She also looked after him for about a year after his return to India. Even during those months of prolonged illness, she recalls how he kept on working and jotting down his results on sheets of paper. Surprisingly, there was no letter from Hardy nor did Ramanujan write to him about his deteriorating health. However, in the only letter to Hardy after his return, in January 1920, Ramanujan communicated his original work on what he called *mock theta functions*. More about this later, in connection with the discovery of the so-called "Lost" Notebook of Ramanujan, by Professor George E. Andrews of Pennsylvania State University, in the Spring of 1976, more than five decades after Ramanujan's death, which resurrected interest in the life and work of Ramanujan in the later half of the twentieth century.

From all the available records and retrospective diagnosis, Dr. D.A.B. Young[91] makes out the case that Ramanujan's terminal illness was *hepatic amoebiasis*, a tropical disease contacted by Ramanujan in 1906. Medical understanding today is that amoeba which enter into the body due to a viral infection can be controlled by medication but can never be eliminated from the liver completely and that the dormant amoeba multiply to cause additional complications whenever another virus affects the individual.

Dr. Young's reason as to why this was not diagnosed at that time is best recounted in his own words: *Hepatic amoebiasis was regarded, in 1918, as a tropical disease*, ("tropical liver abscess"), *and this would have had important implications for successful diagnosis, especially in provincial medical centers. Furthermore, the specialists called in were experts in either tuberculosis or gastric medicine. Another major difficulty is that a patient with this disease would not, unless specifically asked, recall as relevant that he had had two episodes of dysentery 11 and 8 years before. Finally, there is the very good reason, that, because of the great variability in physical findings, the diagnosis was difficult in 1918 and remains so today: hepatic amoebiasis 'presents a severe challenge to the diagnostic skills* ... [and] *should be considered in any patient with fever and an abnormal abdominal examination coming from an endemic area.'*

Hardy who was unaware of the deteriorating condition of Ramanujan's health, and did not anticipate the end to come so soon, was shocked when the news of his death was communicated to him by the brother of Ramanujan. He was of the view that *a mathematician is often comparatively old at 30*. For, in his roll-call of mathematicians Hardy wrote:[92] *Galois died at twenty-one, Abel at twenty-seven, Ramanujan at thirty-three, Riemann at forty ... I do not know an instance of a major mathematical advance initiated by a man past fifty*.[93]

[91] D.A.B. Young, *Ramanujan's illness*, Current Science, **67** (1994) 967–972.

[92] G.H. Hardy, *A Mathematician's Apology*, Cambridge Univ. Press (1976) p. 71.

[93] A few examples which can be cited which explode *The Myth of the Young Mathematician* are: Newton's *Principia* was written when he was in his mid-40s; Euler, despite his blindness, produced his three volumes on integral calculus when he was in his 60s; Gauss at 34 proposed his theory of analytic functions; and in more recent times, Elie Cartan, Poincaré, Siegel, Kolmogorov and Erdös

1.9 Janaki Ramanujan (Janakiammal)

Ramanujan, at 22, got married on July 14, 1909, to the then 9-year old Janaki, the fourth of six children (including a son), born on March 21, 1899, as the daughter of Rangaswami Iyengar and Ranganayaki Ammal, of Rajendram, a village close to Marudur Railway Station 45 km North-East of Tiruchirapalli city. The young wife went back to live with her parents until she came of age and then joined Ramanujan, in 1912, after he got a clerical job in the Madras Port Trust, on a monthly salary of Rs.30/-. They set up a home in Saiva Muthiah (Mudali) Street,[94] in George Town. In May 1913, Ramanujan joined the University of Madras, as its first research scholar. He first stayed at a house in Hanumantharayan Koil Street, and then went to Thoppu Venkatachala (Mudali) Street, both in Triplicane, renowned for the Parthasarathy Temple—an eighth-century Hindu Vaishnavite temple dedicated to Lord Maha Vishnu. Ramanujan's wife and mother lived with him for some months, at the latter residence, before he left for Cambridge on March 17, 1914.

Ramanujan was alone in England for almost 5 years, from April 14, 1914, to February 27, 1919. He fell ill in the Spring of 1917, as stated earlier. The ongoing (first) World War prevented his early return to India and the possibility of his wife joining him during his 5-year stay abroad. Eventually, when Ramanujan returned to India, in April 1919, with a great reputation, Janaki joined him in Madras and nursed him till his untimely death on April 26, 1920.

During those months, the household was run by Ramanujan's mother Komalathammal and grandmother Rangammal. In later years, after Ramanujan's death, Janaki was happy to state: *I considered it my good fortune to give him rice, lemon juice, butter-milk, etc. at regular intervals and to give fomentation to his legs and chest when he reported pain. The two vessels used then for preparing hot water are alone still with me. These remind me often of those days.*

After April 1920, the young widow spent the next 8 years of her life in Bombay,[95] with her brother, R.S. Iyengar, an Assistant Commissioner of Income Tax. During that period, she learnt tailoring and English, at home. In 1931, after she was deprived of all her *sthree dhanam* (dowry), she chose to return to Madras and spent a year with one of her sisters and another year with her friends, before boldly deciding to live independently. She started to teach tailoring and lived for the next 5 years in a small house in Hanumantharayan Koil Street, Triplicane, having one room upstairs and

exhibited creativity in mathematics in their later years—ref. Susan Landau, Notices of the Am. Math. Soc. **44** (1997) 1284.

[94]The street names in Madras (officially renamed Chennai, in 1966) have been renamed by the caste conscious State Government regimes. So ridiculous were some of the changes that Netaji Subhash Chandra Bose Road, or N.S.C. Bose Road, Krishnamachari Road and Dr. Nair Road, were renamed, respectively as N.S.C. Road, Krishnama Road and Doctor Road! Needless to say, the Government turned a deaf year to appeals, in 1987, by the author and several others, to rename the Wallajah Road, where the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras eventually moved to its own premises, as Ramanujan Road.

[95]Bombay has been renamed as *Mumbai*.

one room on the ground floor. She earned enough from her tailoring and could even save a little, thanks to her frugal living. In 1950, one of her friends, Soundaravalli, died suddenly, after entrusting her 7 year old son, W. Narayanan, to Janaki.

Janakiammal took up the responsibility of bringing up this boy and became a foster mother to him. During his school years, she sent him for some years (1952–1955) to the Ramakrishna Boarding School. She educated him up to the Bachelor's Degree in Commerce, B.Com., at the Sri Ramakrishna Mission Vivekananda College, in Mylapore, Madras. She requested the authorities of the Madras Port Trust, which continued to give her a Pension, to secure a clerical job for him in the State Bank of India. Janakiammal conducted the marriage of Narayanan, in 1972. Janaki chose the daughter-in-law to be and conducted the marriage. Vaidehi Narayanan was also an employee of the State Bank of India. Mr. Narayanan resisted transfers and promotions to take care of Janakiammal all her life, and eventually took voluntary retirement from the State Bank of India, about 6 years before Janakiammal passed away, to take care of her declining health and helped her to converse with the increasing number of visitors who wanted to meet the wife of the legendary Ramanujan, to pay their respects to the living link of the mathematical genius. The Narayanans had a son and two daughters, and in return to Janakiammal taking care of them during their growing years with her pension amounts as supplement to the family income, they took good care of their "grandmother", as she was an integral part of her joint family.

Janakiammal being extremely orthodox and pious, preferred to stay alone in her house in Muthiah (Mudali) Street, under the loving and tender care of the Narayanans, despite her failing health, till a few years before her death, when she moved to 14, Hanumantharayan Koil Street, Triplicane. The Narayanans lived in the same house with their children—N. Sridhar, C.A., ICWA; N. Sripriya, M.C.A. and N. Srividya, in her School final then.

Janakiammal being philanthropic and sympathetic by nature, supported financially not only her foster son's children, but also several children and youngsters. For, when she became renowned, in her later years, her friends, neighbours and acquaintances believed that her blessings would augur well for their children to pass creditably in their exams with high marks, for which getting a token amount of money from her hands was deemed to be a good auspicious beginning for a bright future.

Janakiammal was receiving a pension from the University of Madras, since 1920, the year of her husband's death. This amount was then only Rs.20/- per month. It gradually rose to about Rs.500/- per month, at the time of her passing away. In interviews which appeared in newspapers, soon after the "Lost" Notebook of Ramanujan was discovered by George Andrews, in 1976, Janakiammal lamented that even a bust of Ramanujan was never made, although one had been promised. Professor Richard A. Askey of the University of Wisconsin, Madison, saw these newspaper interviews and decided that a bust of Ramanujan was long overdue since Ramanujan's widow wanted one and that in *Ramanujan's case a permanent memorial is appropriate: one which can be appreciated by those who do not*

understand his mathematics should be added to the memorial Ramanujan made for himself with his work.

Garnering the support of a hundred mathematicians, including Professors George Andrews, Bruce Berndt and S. Chandrasekhar, from the only authentic passport size photograph of Ramanujan, Askey commissioned a Bronze bust, to be made by Paul Granlund, sculptor-in-residence at Gustavus Adolphus College at Saint Peter, Minnesota in the USA. A copy of the bust was presented to Srimathi Janaki Ramanujan, in 1985, at a simple function in the University of Madras.

Janakiammal had cherished memories of her illustrious husband. Even in recent years, though she was hard of hearing, had failing eyesight and was having indifferent health, when I accompanied the mathematician—Professors George Andrews, Richard Askey, Bela Bollobas and others[96]—to call on her at her residence, she recalled with pride the prophetic words of her husband that his mathematics would support her with funds whether he was alive or dead.

Janakiammal also pointed out on such occasions that financial support started coming after the 75th Birth Anniversary celebrations of Ramanujan, in 1962, when a commemorative Stamp was brought out by the Postal Department of the Government of India. During the centenary year of the Madras Port Trust, Janakiammal was honoured and the Pension for life was enhanced. Since then, the Governments of Andhra Pradesh, Tamilnadu and West Bengal, the Indian National Science Academy, the Ramanujan Mathematical Society, founded in 1985, and others also supported her with monthly pensions.

Incidentally, Professor E.C.G. Sudarshan, the Director of the Institute of Mathematical Sciences, asked me to find out the whereabouts of Janakiammal. As my better half Dr. Geetha Srinivasa Rao was with the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras, it was easy for me to locate and meet for the first time the gracious lady, in her abode in Hanumantharayan Koil Street, being cared for by the family of Vaidehi and Narayanan. At the instance of Professor Sudarshan, a function was organized by the author, on the lawns of the Institute of Mathematical Sciences, on August 13, 1987, to honour the lady, as a part of the Ramanujan Birth Centenary celebrations. The gracious lady came to the Institute, accompanied by the Narayanans, and other friends and relatives. The function was presided over by the Patron of the Institute of Mathematical Sciences, Sri C. Subramaniam, and in the presence of other dignitaries and an audience of several hundred well-wishers and mathematicians, the Hinduja Foundation's Secretary, Mr. A.P. Venkateswaran, presented to her a purse of Rs.20,000/- and announced an honorarium of Rs.1000/- per month, to Janakiammal, from then on.

As stated earlier, from that time onwards, on many an occasion, the author had the pleasure and privilege of calling on the Narayanans, with prior appointment,

[96]The author accompanied each one of them and some others also, when they expressed their desire to meet the gracious lady, as a mark of respect to her husband. When Professor Askey asked her whether she had conjugal relationships with her husband, her answer was in the affirmative and there was a shy smile on her face. The Ramanujans had no children, and it is this fact that, perhaps, prompted Prof. Askey to raise the question.

invariably in the afternoon of a working day, with a visiting Professor of Mathematics to India, who wished to pay his homage to the better half of the one and only mathematical genius of India. The lady also allowed the author to borrow the Bronze Bust of Ramanujan presented to her by Prof. Askey, with contributions by the mathematicians of the world, for display at the Childrens' Congress preceding the 98th Indian Science Congress, at the Engineering College, Guindy, Chennai, as a part of the PIE (πie) Pavilion, whose concept and design was by the present author.

One of her cherished hopes was that a fitting memorial would come up to honour the man who for "natural" genius has been compared to the all-time greats Euler, Gauss and Jacobi. The memorial was also to house the correspondence, research papers, the original Notebooks of Ramanujan, the large-size slate used by him and other such memorabilia. The receptive ears of Professor Askey made him commission the bronze bust of Ramanujan, made by sculptor Paul Granlund. The original was presented to her by Askey, thereby making a part of her dream come true, in the birth centenary year, 1987. The author, when approached by the Registrar, on behalf of the Director of the Society for Electronic Transactions and Security (SETS), in Taramani, Chennai, in the year 2010, for a suggestion to get a bust of Ramanujan made, succeeded in getting a full-length bronze statue of Ramanujan made and installed at the entrance of SETS.

Srimathi Janaki Ramanujan breathed her last on the morning of April 13, 1994, at the home which was by then her own residence, at the age of 94, having lived a trying period of 74 years after the untimely death of her husband, Srinivasa Ramanujan (who died on April 26, 1920). The author was informed on that morning at about 6.30 A.M. which enabled him to place a garland at the feet of the departed soul with a prayer for her soul to rest in peace.

1.10 Human Qualities

In figure he [Ramanujan] *was a little below medium height (5ft. 5in.) and stout until emaciated by disease; he had a big head, with long black hair brushed sideways above a big forehead; his face was square, he was clean-shaven, and his complexion never really dark, grew paler during his life in England; his ears were small, his nose broad, and always his shining eyes were the conspicuous feature that Ramachandra Rao observed in 1910. He walked stiffly, with head erect and toes out-turned; if he was not talking as he walked, his arms were held clear of the body, with hands open and palms downwards. But when he talked, whether he was walking or standing, sitting or lying down, his slender fingers were forever alive, as eloquent as his countenance.*

This is the physical description of Ramanujan as recorded by Professor E.H. Neville.[97] Ramanujan had only one passion in his life—Mathematics. He devoted all his time to this subject and its development. Quoting Professor Neville again,

[97]E.H. Neville, in *Ramanujan: Letters and Reminiscences*, Memorial Number, vol.1, Ed. P.K. Srinivasan, Muthialpet High School (1968) pp. 59–60.

Ramanujan had *an instinctive perfection of manners that made him a delightful guest or companion. Success and fame left his actual simplicity quite untouched. To his friends he was devoted beyond measure, and he devised curiously personal ways of showing his gratitude and expressing his affection. The wonderful mathematician was indeed a lovable man.*

This is in complete accord with the views of his mentor Hardy[98] on Ramanujan: *. . . the picture I want to present to you is that of a man who had his peculiarities like other distinguished men, but a man in whose society one could take pleasure, with whom one could drink tea and discuss politics or mathematics; the picture in short, not of a wonder from the East, or an inspired idiot, or a psychological fraud, but of a rational human being who happened to be a great mathematician.*

The integrity of Ramanujan is transparent from the following statement of Hardy: *All of Ramanujan's manuscripts passed through my hands, and I edited them very carefully for publication. The earlier ones I wrote completely. I had no share of any kind in the results, except of course when I was actually a collaborator, or when explicit acknowledgment was made. Ramanujan was almost absurdly scrupulous in his desire to acknowledge the slightest help.*

In a letter to a friend of Ramanujan, in September 1917, Hardy wrote:[99] *He has been seriously ill but is now a good deal better. It is very difficult to get him to take proper care of himself; if he would only do so, we should have every hope that he would be quite well again before very long.*

In the aforesaid letter, Hardy referred to his discovering that Ramanujan *was not writing to his people nor apparently hearing from them. He was very reserved about it and it appeared to us that there must have been some quarrel.*

Hardy expressed his anxiety regarding the trouble which might have arisen and wanted it to be cleared away.

Ramanujan was shy by temperament and contemplative by nature. He was a man with a great sense of humour: *He had a fund of stories, and such was his enjoyment in telling them that in his great days his irrepressible laughter often swallowed the climax of his narrative*, as quoted by Neville.

Ramanujan was very affectionate towards his brothers and mother, in particular. His wife, Janaki, recollected[100] that he knew astrology and made predictions to some extent and that he knew he would not live beyond 34 years. Sometimes, he is supposed to have made predictions for others also.

Some friends of Ramanujan remembered[101] that Ramanujan could foresee events in visions; that being an ardent devotee of God Narasimha, he saw drops of blood in dreams (which was considered as a sign of the Lord's grace) and that *after seeing*

[98] G.H. Hardy *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*, Chelsea, New York (1940).

[99] G.H. Hardy to Subramanian, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Muthialpet High School, Chennai-1 (1968), pp. 68–75.

[100] 'Notes' of S. Narayana Iyer, CAC, Madras Port Trust, 15-4-1915.

[101] Letter to the Registrar from Sir Francis Spring, undated and handwritten on plain paper.

such drops, scrolls containing the most complicated mathematics used to unfold before him, and these he set down on paper, on waking only a fraction of what was thus shown to him.

Ramanujan's maternal grandmother was a staunch devotee of Goddess Namagiri of Namakkal. Ramanujan himself was known to his friends to be a devotee of the Goddess of Namakkal, and he used to say that the Goddess appeared in his dreams and inspired him to come forth with new formulae.[102]

Professor K. Ananda Rao was at King's College, Cambridge, when Ramanujan was in Trinity College, and he recalled,[103] in 1962, that: *In his nature he was simple, entirely free from affectation, with no trace whatsoever of his being self-conscious of his abilities. He was quite sociable, very polite and considerate to others.*

Ramanujan never forgot that as a first born to his parents, he had to shoulder the responsibility of taking care of them. He was compassionate accepting the University's offer of a scholarship, he wrote to Mr. Francis Dewsbury, the Registrar of the University of Madras, in a letter[104] dated 11 January 1919, from a Nursing home in Putney:

I feel, however, after my return to India, which I expect to happen as soon as arrangements can be made, the total amount of money to which I shall be entitled will be much more than I shall require. I should hope that, after my expenses in England have been paid, £50/- a year would be paid to my parents and that the surplus, after my necessary expenses are met, should be used for some educational purpose, such in particular as the reduction of school-fees for poor boys and orphans and provision of books in schools. No doubt it will be possible to make an arrangement about this after my return. ····· I feel very sorry that, as I have not been well, I have not been able to do so much mathematics during the last two years as before. I hope that I shall soon be able to do more and will certainly do my best to deserve the help that has been given to me.

Ramanujan was by nature—as chronicled in the writings of his mentor, Professor Hardy, and from the letters Ramanujan wrote to his mother, well-wishers and friends—a gentle, soft spoken,[105] dutiful son and a grateful, outstanding mathematician of the twentieth century.

[102]This was probably Ramanujan's way of explaining away his incomparable intuition and success of his discoveries, to those who are unable to comprehend his ability to churn out continuously new theorems, persisted in questioning him as to how he discovered those results!

[103]K. Ananda Rao, in *Ramanujan: Letters and Reminiscences*, Memorial Number, Vol.1, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968) p. 143.

[104]Copy of the letter of Ramanujan to the Registrar, University of Madras, is in the list of Plates of S.R. Ranganathan *Ramanujan: The Man and the Mathematician*, Asia Publishing House (1967).

[105]It is unfortunate that in the movie based on the book of Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Charles Scribner's Sons, New York (1991), Ramanujan is portrayed as an angry youngman, by an actor, who, in one scene, even shouts at Hardy and asks: "Who are you, Hardy?", unthinkable and inexplicable from the writings of any of Ramanujan's contemporaries—our only reliable sources of information available on any aspect of the life of Ramanujan, which inspired the author of this book.

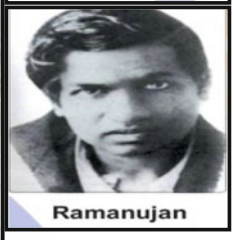
It is fitting to close this chapter with the words of Ramanujan himself, who concluded a letter[106] to Mr. S. Narayana Iyer, dated 11 November 1915, with the following words of gratitude: *I am ever indebted to you and Sir Francis Spring for your zealous interest in my case from the very beginning of acquaintance.*

[106]Ramanujan's letter to S. Narayana Iyer, in *Ramanujan: Letters and Reminiscences*, Memorial Number, Vol.1, Ed. P.K. Srinivasan, Muthialpet High School, Madras (1968) p. 32.

Namagiri thaayar, Namakkal

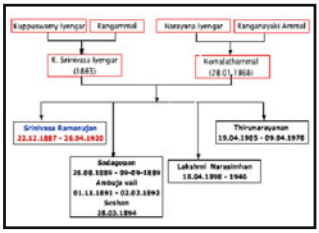
Mandapam, Namakkal

Komalathammal
Mother of Ramanujan
No photo of the father is availlable!



Note: Mother and Son's facial resemblance

Sarangapani Temple, Kumbakonam. One of the 108 Temples of Maha Vishnu, revered in Nalayira Divya Prabhandam by the 12 Saints (or Azhwars) , along the banks of the river Cauvery.



Family Tree of Srinivasa Ramanujan

Horoscope or Birth chart -- shows celestial positions of the planets on Dec.22, 1987, at 9s past 6 p.m.

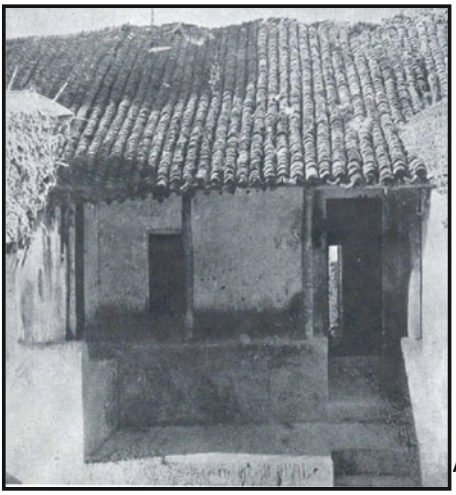
For those curious and interested in Astrology, a pseudoscience, at best!

The horoscope of Ramanujan

Ramanujan (Tengalai Vaishnavite, Bharadwaja gotra) was born at 9 secs. Past 6.00 p.m on Thursday the 9th Margazhi of Sarvajit year corresponding to 22nd December, 1887 at Erode [Lat. 11°20' (N) and Long. 78°00' (E)]. His horoscope cast soon after his birth by his parents, is as follows: Thithi: Ashtami 40-37, Nakshatra (birth star): Uttirattadi 60, Mithuna lagnam, (Gemini ascendent).

<table>
<tr><td>Moon</td><td></td><td></td><td>Lagnam</td></tr>
<tr><td></td><td colspan="2" rowspan="2">Rasi
Chakram</td><td>Rahu
Saturn</td></tr>
<tr><td>Ketu</td><td></td></tr>
<tr><td>Sun</td><td>Mercury
Jupiter</td><td>Venus</td><td>Mars</td></tr>
</table>

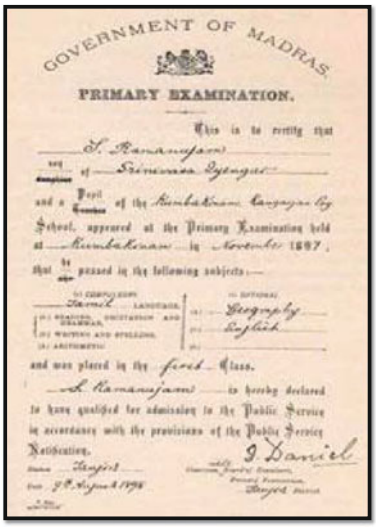
Note: The traditional practice of the orthodox Hindu Brahmins is to follow the Hindu Almanac (Panchangam, meaning the five parameters : "Thithi, vaara, Nakshtra, Yoga, Karana". The planetary positions indicate the longitude and latitude of the planets in the Solar System, at the place of birth. Fixing the Lagnam at the time of birth in the chart and making predictions (by classifying Mercury, Venus, Jupiter as benefics and Mars, Saturn, Sun, Rahu and Ketu as malefic) and assigning attributes to the 12 squares w.r.to the Lagnam, makes the study of the chart, Hindu Astrology.



Ramanujan lived with his parents in Sarangapani Sannidhi Street, Kumbakonam, till 1895, when he left that city in search of benefactors.

Residence of Ramanujan, in Kumbakonam, was acquired by SASTRA University in 2002.

After Renovation, dedicated to the Nation as a Museum by President Dr. A.P.J. Abdul Kalam.



GOVERNMENT OF MADRAS.

PRIMARY EXAMINATION.

This is to certify that

S. Ramanujam

son of Srinivasa Iyengar

and a Pupil of the Kumbakonam Cauvery Sy School, appeared at the Primary Examination held at Kumbakonam in November 1897, that he passed in the following subjects:—

(1) Tamil — LANGUAGE,
(2) READING, DICTATION AND GRAMMAR,
(3) WRITING AND SPELLING,
(4) ARITHMETIC

(1) Geography
(2) English

and was placed in the first Class.

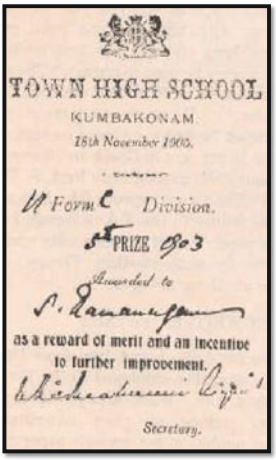
S. Ramanujam is hereby declared to have qualified for admission to the Public Service in accordance with the provisions of the Public Service Notification.

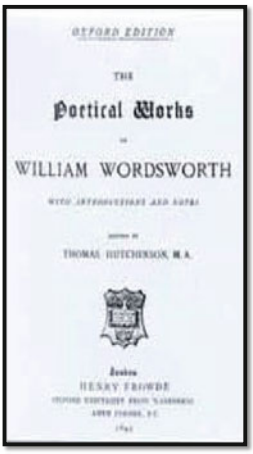
Madras
the 28 August 1898

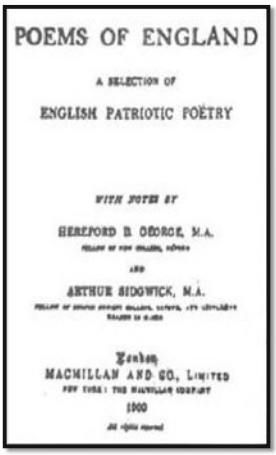
J. Daniel
Director of Public Instruction,
Presidency Commissioner,
Madras.

House in Hanumantharayan Koil Street -- the plaque alone is still there, and the Building replaced by an apartment complex.

Ramanujan passed the Primary Examination, held in November 1897, with a First Class, which entitled him for a half-fee concession, to join the Government Arts College in Kumbakonam.


TOWN HIGH SCHOOL
KUMBAKONAM.
18th November 1903.
Form C Division.
PRIZE 1903
Awarded to
S. Ramanujam
as a reward of merit and an incentive to further improvement.
Secretary.


OXFORD EDITION
THE
Poetical Works
OF
WILLIAM WORDSWORTH
THOMAS HUTCHINSON, M.A.
HENRY FROWDE

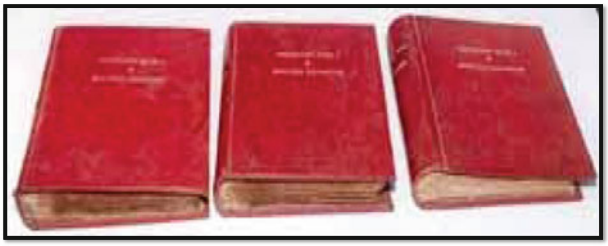

POEMS OF ENGLAND
A SELECTION OF
ENGLISH PATRIOTIC POETRY
WITH NOTES BY
HEREFORD B. GEORGE, M.A.
AND
ARTHUR SIDGWICK, M.A.
London
MACMILLAN AND CO., LIMITED
1900

Prizes "as a reward for merit and an incentive for further improvement", in Form VI, C Division (Form VI corresponds to present day Standard X) awarded to S. Ramanujam by the Town High School, (Secretary), in 1903. Poetical works of William Wordsworth and Poems of England , were awarded to Ramanujam for his proficiency in English. (Note. The name Ramanujam was given to him by his parents and finds a place in all the records. The name S. Ramanujan finds a place in all his publications.

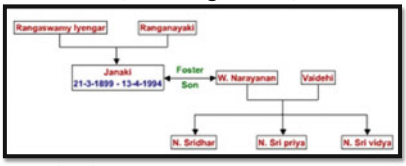
Town High School, Kumbakonam, founded in 1864. Ramanujan studied here from 1898 – 1904. The Calendar year and the academic year were the same in the time of the British, with the vacation being in the month of December.

Government Arts College, Kumbakonam, where Ramanujan Studied for two years after his Schooling. He appeared for the Madras University Examination in 1905 and again as a private candidate in 1907 but failed to pass the First Degree in Arts (F.A.) examinations. His formal Education in India ended with this.

J.A. Yades, the Principal of Pachaiyappa's College, Madras helped Ramanujan by granting him a Scholarship, for proficiency in mathematics.



Ramanujan gave his Notebooks to Mr. R. Radhakrishna Ayyar (Advocate) a classmate in Pachaiyappa's college (1906-1907), before he returned to Kumbakonam due to ill-health – for safe keeping. Ramanujan told him: "If I die, please hand them over to Singaravelu Mudaliar or to the British Professor – Edward B. Ross – of the Madras Christian College". (Ref. "Ramanujan, the Man and the Mathematician", S.R. Ranganathan, Asia Pub.House, 1967).



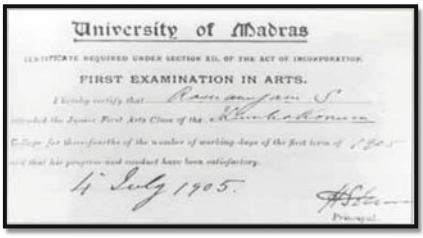
Janaki, wife of Ramanujan and above her Family Tree.

University of Madras, Senate House.

Pachhaiyappa's College, Madras – courtesy, the Hindu.

Subject	Max. Mark	Required (min.)	Marks Obtained
English	200	70	38
Sanskrit	100	35	34
Mathematics	150	45	85
Physiology	75	-	-
History	75	-	-

Ramanujan did not pass the college Examinations and so lost his scholarship. Above the First Examination in Arts 1905 mark sheet – courtesy Madras University, through Mr. C.A. Reddy and Mr. A. Ranganathan, to the author.

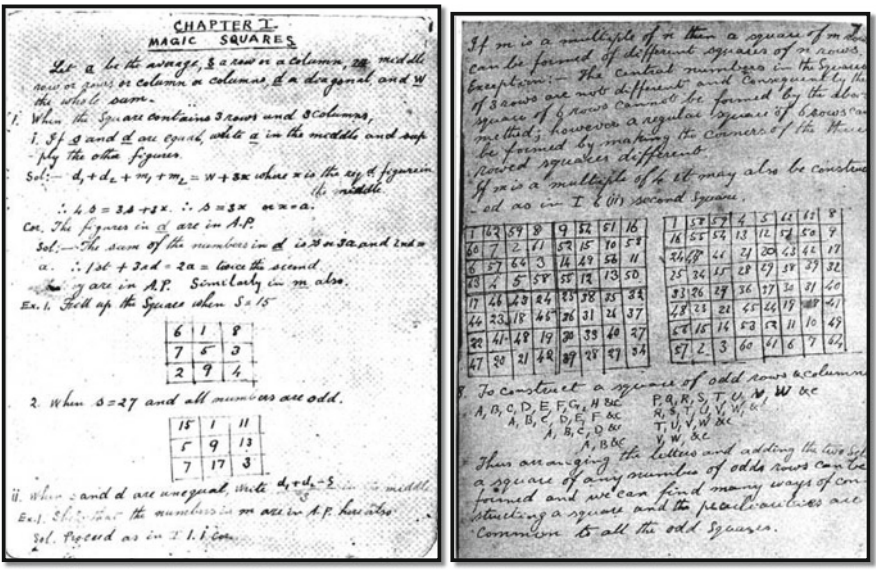

University of Madras

FIRST EXAMINATION IN ARTS.

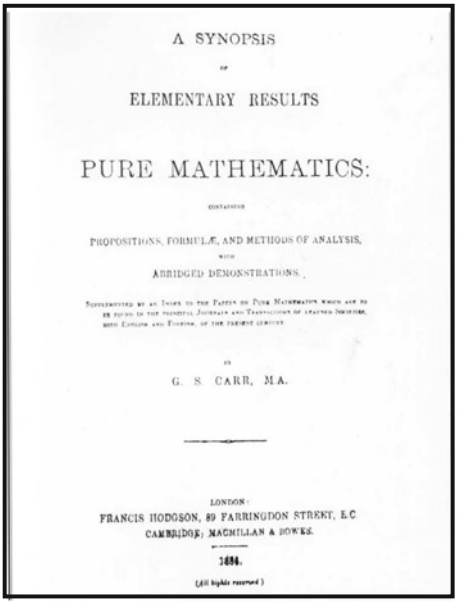
4th July 1905.

Principal

Ramanujan's Attendance Certificate, to appear for the First Examination in Arts (1905).



From the Notebooks of Ramanujan: Page 1 of Notebook 1, with the title "Magic Squares" (the only chapter with a title) and the last page 7 of Notebook 2, also on magic squares. (see: www.imsc.res.in/~rao/ramanujan for the entire contents of the Notebooks)



A SYNOPSIS

OF

ELEMENTARY RESULTS

PURE MATHEMATICS:

CONTAINING

PROPOSITIONS, FORMULÆ, AND METHODS OF ANALYSIS,

WITH

ABRIDGED DEMONSTRATIONS.

SUPPLEMENTED BY AN INDEX TO THE PAPERS ON PURE MATHEMATICS WHICH ARE TO BE FOUND IN THE PRINCIPAL JOURNALS AND TRANSACTIONS OF LEARNED SOCIETIES, BOTH ENGLISH AND FOREIGN, OF THE PRESENT CENTURY.

BY

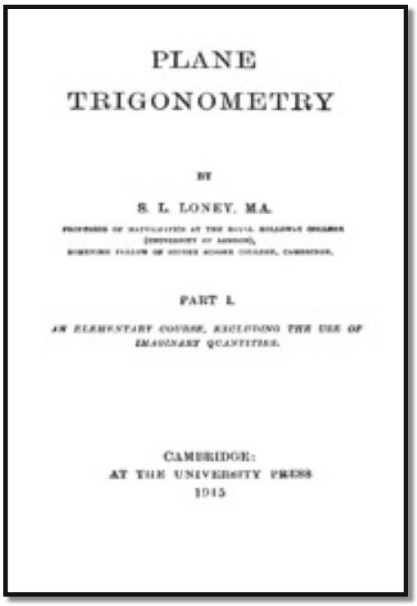
G. S. CARR, M.A.

LONDON:
FRANCIS HODGSON, 89 FARRINGDON STREET, E.C.
CAMBRIDGE: MACMILLAN & BOWES.

1886.

(All rights reserved.)

G.S. Carr's "Synopsis" (1894).



PLANE
TRIGONOMETRY

BY

S. L. LONEY, M.A.

PROFESSOR OF MATHEMATICS AT THE ROYAL HOLLOWAY COLLEGE (UNIVERSITY OF LONDON),
FORMERLY FELLOW OF SIDNEY SUSSEX COLLEGE, CAMBRIDGE.

PART I.

AN ELEMENTARY COURSE, EXCLUDING THE USE OF IMAGINARY QUANTITIES.

CAMBRIDGE:
AT THE UNIVERSITY PRESS
1915

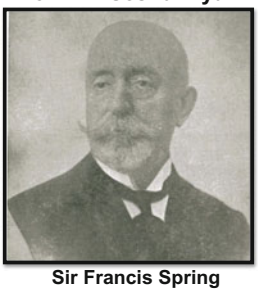
A book mastered by Ramanujan.

Ramanujan's admirers and supporters

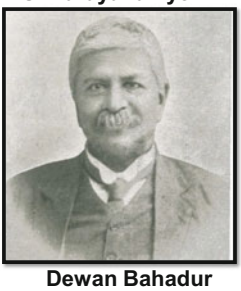
Prof. P.V. Seshu Aiyar

S. Narayana Iyer

V. Ramaswamy Iyer



Sir Francis Spring
Chairman, Madras Port Trust



Dewan Bahadur
Ramachandra Rao

Lord Pentland
Governor of Madras

V. Ramaswamy Iyer (1871 - 1936) was an M.A. from the University of Madras. He joined the Civil Services in 1989 and he became a Deputy Collector in 1901. Founder of the Indian Mathematical Society (IMS) and the Journal of the IMS, in 1909. He was the President of the IMS from 1926 – 1930. He was instrumental in Ramanujan publishing his Research articles and his Questions and Answers to Questions, in the JIMS.

Presidents of the Indian Mathematical Society

B. Hanumanta Rao, (1907 -- 1912) R .N. Apte, (1912 -- 1915) E.W. Middlemast (1915)
R. Ramachandra Rao, (1915 -- 1917) A.C.L. Wilkinson, (1917 -- 1921) H. Balakram (1921 -- 1926)
V. Ramaswami Aiyar, (1926 -- 1930) M.T. Naraniengar, (1930 -- 1932) P.V. Seshu Aiyar (1932 -- 1936)

Ramanujan was living in the 'Summer House', in Sami Pillai Street, Triplicane, Chennai, reluctantly accepting the dole ← from Mr. Ramachandra Rao.

Senate House, University of Madras

← Mr. G.H. Hardy, F.R.S, →
Cayley Lecturer in Mathematics,
Cambridge University

← Mr. G.T. Walker, F.R.S.,
Head of the India
Meteorological
Department, Shimla.

Mr. E.H. Neville, F.R.S.,→
Fellow of Trinity College.

Chapter 2
Ramanujan at Cambridge

Ramanujan wrote a few articles soon after he reached Cambridge, and in June 1914, Hardy presented some of the Entries (Theorems) from the Notebooks of Ramanujan at a meeting of the London Mathematical Society. However, in January 1915, Ramanujan wrote to his friend Subramanian:[1]

"*My notebook is sleeping in a corner for these 4 or 5 months. I am publishing only my present researches as I have not yet proved the results in my notebook rigorously*".

In the quoted statement above Ramanujan mentions about his "results in my notebook". Ramanujan received recognition in the form of Prizes for his proficiency in English at School. Hence, it is worth noting that he refers to "my notebook" and not Notebooks. This makes one wonder whether he was yet to complete the work of "copying" the results from his first Notebook onto the Second Notebook, thereby producing in the end a revised, enlarged version of his first Notebook!

2.1 Research Publications

Ramanujan was attending a lecture by Mr. Berry at the University on elliptic integrals. Mr. Berry was working out some formulae with chalk on the blackboard and a glance at Ramanujan's face, bright with excitement, caused the lecturer to ask Ramanujan whether he was following his lecture and whether he had anything to say. At this opportunity, Ramanujan went to the blackboard and much to everyone's

[1]Letter to S.M. Subramanian, in *Ramanujan: Letters and Reminiscences*, Ed. P.K. Srinivasan, Memorial Number 1, Muthialpet High School, Madras-1 (1968) pp. 20–27.

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
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surprise wrote down some of the results which were yet to be proved. This anecdote was recalled, in later years, by P.C. Mahalanobis.[2]

2.2 On Selected Formulae of Ramanujan

The letter which Ramanujan was induced to write to Professor G.H. Hardy, at the Trinity College, Cambridge University, dated 16 January 1913, is considered as a Historic letter which contributed to the rare Hardy-Ramanujan collaboration in mathematics research. Most probably, this letter was drafted by Narayana Iyer, then Treasurer in the Madras Port Trust.

Hardy's comments on the sheets of paper containing over a hundred theorems culled from the Notebooks of Ramanujan, in 11 pages, are recorded by Hardy in his writings contained in the *Collected Papers of Ramanujan* edited by Hardy with Ramachandra Rao and Seshu Iyer (Cambridge Univ. Press, 1927). The 11 attached sheets contained series formulae, integral formulae and continued fractions.

The series formulae are:

$$1-\frac{3!}{(1!2!)^3}x^2+\frac{6!}{(2!4!)^3}x^4-\cdots$$
$$=\left(1+\frac{x}{(1!)^3}+\frac{x^2}{(2!)^3}+\cdots\right)\left(1-\frac{x}{(1!)^3}-\frac{x^2}{(2!)^3}+\cdots\right). \tag{2.1}$$

$$1-5\left(\frac{1}{2}\right)^3+9\left(\frac{1\cdot 3}{2\cdot 4}\right)^3-13\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^3+\cdots=\frac{2}{\pi}. \tag{2.2}$$

$$1+9\left(\frac{1}{4}\right)^4+17\left(\frac{1\cdot 5}{4\cdot 8}\right)^4+25\left(\frac{1\cdot 5\cdot 9}{4\cdot 8\cdot 12}\right)^4+\cdots=\frac{2^{3/2}}{\pi^{1/2}\{\Gamma(3/4)\}^2}. \tag{2.3}$$

$$1-5\left(\frac{1}{2}\right)^5+9\left(\frac{1\cdot 3}{2\cdot 4}\right)^5-13\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 16}\right)^5+\cdots=\frac{2}{\{\Gamma(3/4)\}^4}. \tag{2.4}$$

"The series formulae I found much more intriguing, and it becomes obvious that Ramanujan must possess much more general theorems. The second is a formula of Bauer well known in the theory of Legendre series, but the others are much harder than they look. The theorems required for proving them can all be found now in Bailey's Cambridge Tract on Hypergeometric Functions (1935)."

[2]"My friendship with Ramanujan in England", P.C. Mahalanobis—who became later the eminent mathematician of India who started the Indian Statistical Institute in Calcutta—in *Ramanujan: Letters and and Reminiscences*, Ed. P.K. Srinivasan, Memorial Number 1, Muthialpet High School, Madras-1 (1968) pp. 145–148.

The integral formulae in the letter of Ramanujan to Hardy (16 January 1913) are:

$$\int_0^\infty \frac{1+\left(\frac{x}{b+1}\right)^2}{1+\left(\frac{x}{a}\right)^2}\frac{1+\left(\frac{x}{b+2}\right)^2}{1+\left(\frac{x}{a+1}\right)^2}\cdots dx = \frac{\pi^{1/2}}{2}\frac{\Gamma(a+1/2)\Gamma(b+1)\Gamma(b-a+1/2)}{\Gamma(a)\Gamma(b+1/2)\Gamma(b-a+1)} \tag{2.5}$$

$$\int_0^\infty \frac{dx}{(1+x^2)(1+r^2x^2)(1+r^4x^2)\cdots} = \frac{\pi}{2(1+r+r^3+r^6+r^{10}+\cdots)}. \tag{2.6}$$

"I thought that, as an expert in definite integrals, I could prove these, and did so, with a good deal more trouble than I had expected. On the whole, the integral formulae seemed least Impressive."

The formulae in the 11 pages attached with Ramanujan's letter which represented theorems "on a different level and obviously both deep and difficult" which even Hardy "had never seen anything in the least like them before" and which Hardy stated[3] "defeated me completely" are:

$$\text{If} \qquad u = \frac{x}{1+}\,\frac{x^5}{1+}\,\frac{x^{10}}{1+}\,\frac{x^{15}}{1+}\cdots \tag{2.7}$$

$$\text{and} \quad v = \frac{x^{1/5}}{1+}\,\frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+}\cdots \tag{2.8}$$

then

$$v^5 = u\,\frac{1-2u+4u^2-3u^3+u^4}{1+3u+4u^2-2u^3+u^4}. \tag{2.9}$$

$$\frac{1}{1+}\,\frac{e^{-2\pi}}{1+}\,\frac{e^{-4\pi}}{1+}\cdots = \left\{\sqrt{\left(\frac{5+\sqrt{5}}{2}\right)}-\frac{\sqrt{5}+1}{2}\right\}e^{2\pi/5}. \tag{2.10}$$

[3] *Ramanujan: Twelve Lectures on Subjects Suggested by His Life Work*, G.H. Hardy, Chelsea, New York (1940) p. 9.

and

$$\frac{1}{1+}\,\frac{e^{-2\pi\sqrt{5}}}{1}+\frac{e^{-4\pi\sqrt{5}}}{1}+\cdots=\left[\frac{\sqrt{5}}{1+\sqrt[5]{\left\{5^{3/4}\left(\frac{\sqrt{5}-1}{2}\right)^{5/2}-1\right\}}}-\frac{\sqrt{5}+1}{2}\right]\times e^{2\pi/\sqrt{5}}. \tag{2.11}$$

Hardy stated that "A single look at them is enough to show that they could be written down by a mathematician of the highest class. They must be true because, if they are not true, no one would have had the imagination to invent them. Finally, (you must remember that I knew nothing whatever about Ramanujan, and I had to think of every possibility), the writer must be completely honest, because great mathematicians are commoner than thieves or humbugs of such incredible skill." Hardy considered it "sufficiently marvelous that he should have even dreamt of problems such as these and Rogers and Watson found the proofs of the extremely difficult theorems", in later years.

2.3 Elementary Mathematics

As an example of Ramanujan's elementary mathematics which possesses some novelty, refer to Question 700 posed by Ramanujan in the JIMS:[4] Sum the series

$$(a+b+1)\left(\frac{a}{b}\right)^2+(a+b+3)\left[\frac{a(a+1)}{b(b+1)}\right]^2+(a+b+5)\left[\frac{a(a+1)(a+2)}{b(b+1)(b+2)}\right]^2+\cdots \tag{2.12}$$

to n terms. The solution provided by K.R. Raman Aiyar and K. Appukuttan Erady[5] required the definition of

$$u_n=\left[\frac{a(a+1)(a+2)\cdots(a+n-1)}{b(b+1)(b+2)\cdots(b+n-1)}\right]^2 \quad \text{and} \quad v_n=(a+b+2n-1)u_n \tag{2.13}$$

[4] J. of the Indian Math. Soc., VII (1915), p. 199.

[5] J. Indian Math. Soc., VIII (1916), p. 152.

so that

$$(b-a-1)v_n = (a+n-1)^2 u_{n-1} - (a+n)^2 u_n. \tag{2.14}$$

Putting $n = 2, 3, \ldots n$ in succession in this equation and adding them results in:

$$v_2 + v_3 + \cdots + v_n = \frac{(a+1)^2 u_1 - (a+n)^2 u_n}{(b-a-1)} \tag{2.15}$$

and finally arrives at the desired sum:

$$v_1 + v_2 + v_3 + \cdots + v_n = \frac{a^2 - (a+n)^2 u_n}{(b-a-1)}, \qquad b \neq a+1. \tag{2.16}$$

Another example of the solution provided by Ramanujan is, in JIMS, vol. IV, p. 226, to the Question 289 posed in JIMS, vol. III, p. 90:

Q.289. Find the values of

$$(i) \qquad \sqrt{1+2\sqrt{(1+3\sqrt{(1+\cdots)\cdots})}} \tag{2.17}$$

$$(ii) \qquad \sqrt{6+2\sqrt{(7+3\sqrt{(8+\cdots)\cdots})}} \tag{2.18}$$

Ramanujan himself gave the following method to solve these two problems:

(i) Notice that

$$n(n+2) = n\sqrt{1+(n+1)(n+3)}. \tag{2.19}$$

Let $f(n) = n(n+2)$, then the first of the above identities becomes:

$$\begin{aligned} f(n) &= n\sqrt{1+f(n+1)} \\ &= n\sqrt{1+(n+1)\sqrt{1+f(n+2)}} \\ &= n\sqrt{1+(n+1)\sqrt{1+(n+2)\sqrt{1+\cdots}}} \end{aligned} \tag{2.20}$$

Putting $n = 1$, we have

$$3 = \sqrt{1+2\sqrt{(1+3\sqrt{(1+\cdots)\cdots})}}. \tag{2.21}$$

(ii) In a similar manner let

$$f(n) = n(n+3) \tag{2.22}$$

then

$$\begin{aligned} f(n) &= n\sqrt{(n+3)^2} \\ &= n\sqrt{(n+5)+(n+1)(n+4)} \\ &= n\sqrt{(n+5)+f(n+1)} \\ &= n\sqrt{(n+5)+(n+1)\sqrt{(n+6)+f(n+2)}} \\ &= n\sqrt{(n+5)+(n+1)\sqrt{(n+6)+(n+2)\sqrt{(n+7)+\cdots}}} \end{aligned} \tag{2.23}$$

Putting $n = 1$, we have

$$4 = \sqrt{6+2\sqrt{(7+3\sqrt{(8+\cdots)\cdots}}}. \tag{2.24}$$

Ramanujan generalized these two results and made an Entry (No. 4, p. 139, of his second Notebook) as:

$$x+n+a = \sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{etc.}}}. \tag{2.25}$$

from which the two results can be obtained by setting, respectively:

$$x = 2, \quad n = 1, \quad a = 0 \qquad \text{for (i)} \tag{2.26}$$

$$x = 2, \quad n = 1, \quad a = 1 \qquad \text{for (ii)} \tag{2.27}$$

This elementary result of Ramanujan was perhaps arrived at by the following simple step-by-step procedure:

$$\begin{aligned} 3 &= \sqrt{9} = \sqrt{1+8} = \sqrt{1+2\cdot 4} \\ &= \sqrt{1+2\sqrt{16}} = \sqrt{1+2\sqrt{1+15}} \\ &= \sqrt{1+2\sqrt{1+3\cdot 5}} = \sqrt{1+2\sqrt{1+3\sqrt{25}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+24}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+4\cdot 6}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{36}}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+35}}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+\cdots}}}}} \end{aligned} \tag{2.28}$$

and

$$\begin{aligned}
4 &= \sqrt{16} = \sqrt{6+10} = \sqrt{6+2\cdot 5} \\
&= \sqrt{6+2\sqrt{25}} = \sqrt{6+2\sqrt{7+18}} \\
&= \sqrt{6+2\sqrt{7+3\cdot 6}} = \sqrt{6+2\sqrt{7+3\sqrt{36}}} \\
&= \sqrt{6+2\sqrt{7+3\sqrt{8+28}}} \\
&= \sqrt{6+2\sqrt{7+3\sqrt{8+4\cdot 7}}} \\
&= \sqrt{6+2\sqrt{7+3\sqrt{8+4\sqrt{49}}}} \\
&= \sqrt{6+2\sqrt{7+3\sqrt{8+4\sqrt{9+40}}}} \\
&= \sqrt{6+2\sqrt{7+3\sqrt{8+4\sqrt{9+5\sqrt{10+\cdots}}}}}
\end{aligned} \tag{2.29}$$

Q[6] **464.** $2^n - 7$ is a perfect square for the values 3, 4, 5, 7, 15 of n. Find other values.

This is another elementary number theoretic problem, for solving the equation:

$$x^2 + 7 = 2^n, \tag{2.30}$$

Ramanujan[7] conjectured that:

$$x = 1, 3, 5, 11, 181 \quad \text{corresponding to} \quad n = 3, 4, 5, 7, 15, \tag{2.31}$$

respectively, are the only five rational integer solutions for x and n. Nagel[8] proved this conjecture decades later, in 1948.

The generalized Diophantine equation

$$x^2 - D = \ell^n, \tag{2.32}$$

where D and $\ell > 0$ are specified integers, has been studied by Helmut Hasse[9] who has surveyed the literature, especially for the case $\ell = 2$, D odd and gave necessary conditions for the existence of the solution. I wish to thank Professor S. Raghavan for these references, not explicitly given in his article (Ref. S. Raghavan, "Impact of Ramanujan's work on modern mathematics", J. Indian Inst. Sci., "Ramanujan Special issue" (1987), pp. 45–53.)

[6] S.Ramanujan, J. Indian Math. Soc. V (1915) p. 120.

[7] *Collected Papers of Srinivasa Ramanujan*, Ed. G.H. Hardy, P.V. Seshu Iyer and B.M. Wilson, Chelsea (1962) p. 327.

[8] T. Nagell, 'The Diophantine equation $x^2 + 7 = 2^n$', Nordisk. Mat. Tidskr 30 (1948) 62–64; Ark. Mat. 4 (1961) 185–187. See also, Th. Skolem, S. Chowla and D.J. Lewis, "The Diophantine equation $2^{n+2} - 7 = x^2$ and related problems", Proc. AMS 10 (1959) 661–669.

[9] Helmut Hasse, "Über eine Diophantische Gleischung von Ramanujan–Nagell and Ihre Verallgemenerung", Nagoya Math. J. 27 (1966) 77–102.

In the words of Bruce C. Berndt[10] "Not only are such results fascinating, but for the most part, Ramanujan's methods remain a mystery."

One of the early papers of Ramanujan was on "Highly Composite Numbers", which Hardy called was "of an elementary but highly ingenious character". Ramanujan had already thought about these numbers since results on these numbers are in his Notebooks. A brief account of these numbers follows, in view of the importance of this work in the career of Ramanujan.

2.4 Highly Composite Numbers

An integer is said to be a prime number[11] if it is divisible only by 1 and by itself. The first few prime numbers are: 2,3,5,7, 11,13,17,19,23, ... In other words, if p is a prime number,[12] it has only two divisors, 1 and p, a fact which is denoted by $d(p) = 2$. A fundamental theorem in number theory, known from the days of Euler, is that there are infinitely many primes. The number of primes below a given integer and their density has been studied extensively by Legendre and Gauss, among others, and this led to the Prime Number Theorem which states that

$$\pi(n) \approx \frac{n}{log\ n}, \tag{2.33}$$

$\pi(n)$ being the number of primes not exceeding n. It is proved using powerful methods of Complex Analysis by Hadamard and de la Vallée Poussin, in 1896.

The Fundamental Theorem of Arithmetic states that every positive integer can be expressed as a product of powers of primes and that this factorization is unique (except for the ordering of the factors).

That is, every number can be written as:

$$N = p_1^{a_1} p_2^{a_2} p_3^{a_3} \cdots p_n^{a_n}, \tag{2.34}$$

where $p_1, p_2, \cdots, p_n$ are distinct prime numbers, and $a_1, a_2, \cdots, a_n$ are non-negative integers. Every school student knows that a number can be either a prime number or a composite number. It is the genius of Ramanujan which made him define[13] what he called as highly composite numbers.

Ramanujan's definition of a highly composite number: *A number N may be said to be a highly composite number, if d(N′) < d(N) for all values of N′ less than N.*

[10] B.C. Berndt, "Ramanujan's Notebooks", Part I (1985)–Part V (1997).

[11] A number which is not a prime number is called a "composite" number. The integer 1 is neither prime nor composite, because it has only one divisor. It is the identity for multiplication.

[12] The number 2 is the only even number which qualifies to be a prime number, since it has two divisors, 1 and 2.

[13] S. Ramanujan, *Highly Composite Numbers*, Proc. London Math. Soc. 2, XIV, 1915, 347–409.

Ramanujan starts his paper on Highly Composite Numbers with the following introductory paragraph: "The number $d(N)$ of divisors of N varies with extreme irregularity as N tends to infinity, tending itself to infinity or remaining small according to the form of N. In this paper I prove a large number of results which add a good deal to our knowledge of the behaviour of $d(N)$."

Table for the divisors of the first 25 natural numbers

N	Divisors of N	d(N)	Remark
$2=2^1$	1,2	2	Prime, highly composite
$3=3^1$	1,3	2	Prime
$4=2^2$	1,2,4	3	Highly composite
$5=5^1$	1,5	2	Prime
$6=2^1 \cdot 3^1$	1,2,3,6	4	Highly composite
$7=7^1$	1,7	2	Prime
$8=2^3$	1,2,4,8	4	Composite
$9=3^2$	1,3,9	3	Composite
$10=2^1 \cdot 5^1$	1,2,5,10	4	Composite
$11=11^1$	1,11	2	Prime
$12=2^2 \cdot 3^1$	1,2,3,4,6,12	6	Highly composite
$13=13^1$	1,13	2	Prime
$14=2^1 \cdot 7^1$	1,2,7,14	4	Composite
$15=3^1 \cdot 5^1$	1,3,5,15	4	Composite
$16=2^4$	1,2,4,8,16	5	Composite
$17=17^1$	1,17	2	Prime
$18=2^1 \cdot 3^2$	1,2,3,6,9,18	6	Composite
$19=19^1$	1,19	2	Prime
$20=2^1 \cdot 5^1$	1,2,4,5,10,20	6	Composite
$21=3^1 \cdot 7^1$	1,3,7,21	4	Composite
$22=2^1 \cdot 11^1$	1,2,11,12	4	Composite
$23=23^1$	1,23	2	Prime
$24=2^3 \cdot 3^1$	1,2,3,4,6,8,12,24	8	Highly composite
$25=5^2$	1,5,25	3	Composite
...	...		...

Hardy has stated that a highly composite number is "as unlike a prime as a number can be". In his second Notebook and in his classic paper on "Highly Composite Numbers", Ramanujan has listed the first 102 of these, the first few of which are: 2, 4, 6, 12, 24, 36, 48, 60, 120. In this table, he gives N, $d(N)$ which is also expressed in terms of powers of primes, and $d(d(N)) \equiv dd(N)$. The last entry in his table is:

$$N = 6746328388800 = 2^6 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \tag{2.35}$$

$$d(N) = 10080 = 2^5 \cdot 3^2 \cdot 5 \cdot 7, \qquad dd(N) = 72. \tag{2.36}$$

The problem thought of solved by Ramanujan "is a very peculiar one, standing somewhat apart from the main channels of mathematical research. But there can be no question as to the extraordinary insight and ingenuity which he has shown in treating it, nor any doubt that his memoir is one of the most remarkable published in England for many years."

Ramanujan concluded the introduction to this paper on "Highly Composite Numbers" with: "I prove also that two successive highly composite numbers are asymptotically equivalent, i.e., that the ratio of two consecutive numbers tends to unity. These are the most striking results. More precise ones will be found in the body of paper. These results give us a fairly accurate idea of the structure of a highly composite number."

"I then select from the general aggregate of highly composite numbers a special set which I call 'superior highly composite numbers' ".[14] I determine completely the general form of all such numbers, and I show how a combination of the idea of a superior highly composite number with the assumption of the truth of the Riemann hypothesis concerning the roots of the ζ-function leads to even more precise results concerning the maximum order of $d(N)$. These results naturally differ from all which precede in that they depend on the truth of a hitherto unproved hypothesis.

As stated earlier, when this paper was submitted to the London Mathematical Society, in 1915, due to the financial difficulties at the time, Ramanujan was requested to reduce the length of the paper. Still, it is his longest paper: 62 pages long with some 260 equations. It was for this work, along with some of his other published papers, that Ramanujan was awarded the B.A. degree by research of the Cambridge University.

An interesting anecdote regarding this paper is reported by George Andrews, in the American Mathematical Society publication Number **66** (Conf. Board of the Mat. Sciences, Regional Conference series in Maths. 1986), entitled: "q-series: their Development and Application in analysis, Number Theory, Combinatorics, Physics and Computer Algorithms": W.N. Bailey, reputed for his Cambridge University Tract and masterpiece on "Generalized Hypergeometric Series", was an undergraduate at Cambridge when Ramanujan arrived at Trinity College. Together they had attended a course of lectures by Hardy. In his inaugural lecture on accepting the Chair of Professor of Mathematics, Bedford College, he included the following tribute to Ramanujan: "...I remember that when Hardy wanted the value of an integral he turned to Ramanujan and asked him for the value, and Ramanujan immediately supplied him with the answer."

"Another occasion I remember was being invited by Ramanujan, along with S. Pollard (who afterwards became a lecturer at Trinity) to go to his rooms in Great Court. There he showed us the manuscript of a long paper he had been writing on 'Highly Composite Numbers', which was afterwards published in the Proceedings

[14]The first few of these superior highly composite numbers are: 2, 6, 12, 60, 120, 360, 2520, 5040, 554400, 720720, $\cdots$.

of the London Mathematical Society. He started at the beginning and quickly turned over the pages as he explained the ideas and the arguments very briefly. Pollard wrestled with the argument and was rewarded by a severe headache. I gave up the struggle earlier and was not troubled by a headache."

"He certainly had a great effect on my own work, and I think that probably, if Ramanujan had not lived, I should not now be the new Professor of Mathematics at Bedford College."

2.5 Partitions

It is in the theory of partitions that Ramanujan and Hardy's collaboration contributed to a remarkable theorem. A partition of an integer n is a division of n into any number of positive integral parts. The number of partitions of n is denoted by $p(n)$. Ramanujan also discovered, for the first time, the congruence properties of $p(n)$.

The first few examples for $p(n)$ are:

$$\begin{aligned}
&p(1) = 1 \text{ and } 1 = 1\\
&p(2) = 2 \text{ and } 2 = 2,\ 1+1\\
&p(3) = 3 \text{ and } 3 = 3,\ 2+1,\ 1+1+1\\
&p(4) = 5 \text{ and } 4 = 4,\ 3+1,\ 2+2,\ 2+1+1,\ 1+1+1+1\\
&p(5) = 7 \text{ and } 5 = 5,\ 4+1,\ 3+2,\ 3+1+1,\ 2+2+1,\\
&\qquad\qquad 2+1+1+1,\ 1+1+1+1+1.
\end{aligned} \tag{2.37}$$

It can be seen that $p(n)$ increases rapidly with n, for $p(6) = 11$, $p(7) = 15, \cdots$, $p(20) = 627, \cdots$, $p(50) = 204226, \cdots$. In the case of $p(5)$, notice that 5, 4+1 and 3+2 are distinct parts of 5, and 5, 3+1+1, 1+1+1+1+1 are odd parts of 5. The observation that the number of these two parts, distinct and odd, is equal led Euler (1707–1783), more than 200 years ago, to a theorem which is today named after him as Euler's theorem.

Euler's Theorem: For any positive integer n, the number of partitions of n with distinct parts equals the number of partitions of n with odd parts.

Basic properties of arithmetic can be used to prove this 200-year-old theorem. Here the lucid exposition of George Andrews in: "Ramanujan: Prodigy and Hero of Mathematics", an article in "Srinivasa Ramanujan, 1887–1920: A tribute", edited by K.R. Nagarajan and T. Soundararajan (Macmillan India Ltd., 1988) is presented. Andrews details the step-by-step procedure to transform all the partitions of 44 with odd parts into partitions of 44 with distinct parts:

$$44 = 7+7+7+7+3+3+3+1+1+1+1+1+1+1. \tag{2.38}$$

Step 1. Collect like terms in the above partition to write:

$$44 = 4 \times 7 + 3 \times 3 + 7 \times 1, \tag{2.39}$$

where 4,3,7 are coefficients.

Step 2. Write the coefficients as sums of distinct powers of 2. Thus,

$$44 = 4 \times 7 + (2 + 1) \times 3 + (4 + 2 + 1) \times 1. \tag{2.40}$$

Step 3. Carry out the indicated multiplications:

$$44 = 28 + 6 + 3 + 4 + 2 + 1. \tag{2.41}$$

This is now a partition of 44 into distinct parts.

In the next example, all the three rules are applied to the partitions of 5 with odd parts:

$$\begin{array}{llll} 5 & = 1 \times 5 & = (1) \times 5 & = 5 \\ 3 + 1 + 1 & = 1 \times 3 + 2 \times 1 & = (1) \times 3 + (2) \times 1 & = 3 + 2 \\ 1 + 1 + 1 + 1 + 1 & = 5 \times 1 & = (4 + 1) \times 1 & = 4 + 1. \end{array} \tag{2.42}$$

Fundamental properties of arithmetic have been used to establish that indeed these three rules always provide a proof of Euler's theorem.

A 150 years after Euler's theorem, Rogers (1894) and Ramanujan (1913) independently found a theorem that is much deeper than Euler's theorem, even though it looks almost the same.

> If Odd numbers are characterized by integers with 1,3,5,7 or 9 as last digit, and we call integers with last digit 1,4,6 or 9 as Strange numbers, then the number of partitions of n into strange parts equals the number of partitions of n into distinct parts no two of which are consecutive integers.

This is the first Rogers-Ramanujan theorem. The following example illustrates this formulation of the Rogers-Ramanujan theorem:

$n = 12$ has		
	$11 + 1$	12
	$9 + 1 + 1 + 1$	$11 + 1$
	$6 + 6$	$10 + 2$
	$6 + 4 + 1 + 1$	$9 + 3$
	$6 + 1 + 1 + 1 + 1 + 1 + 1$	$8 + 4$
	$4 + 4 + 4$	$8 + 3 + 1$
	$4 + 4 + 1 + 1 + 1 + 1$	$7 + 5$
	$4 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$	$7 + 4 + 1$
	$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$	$6 + 4 + 2$
	nine strange parts	nine distinct parts without consecutive integers.

(2.43)

The natural question which arises in one's mind is whether the first Rogers-Ramanujan theorem can also be proved in the same way as Euler's theorem, using only the properties of arithmetic?

This problem remained an open problem for 60 years, until Garsia and Milne[15] found the solution and they published a paper entitled: "A Rogers-Ramanujan Bijection", in the Journal of Combinatorial Theory.

In the words of the discoverer of the "Lost" Notebook of Ramanujan, George E. Andrews, "**Elegance, depth and surprise, are beautifully intertwined**" in this problem conjured by the incomparable Srinivasa Ramanujan.

2.6 Ramanujan's Congruences

Very little was known in Ramanujan's times about the arithmetic properties of $p(n)$—the number of partitions of n. For instance, it was not known whether $p(n)$ is odd or even. Hardy, in "Ramanujan: Twelve Lectures ...", pp. 87–90, states: "*Ramanujan was the first and up to now, the only, mathematician to discover any such properties; and his theorems were discovered in the first instance, by observation. MacMahon had calculated a table of* $p(n)$ *for the first 200 values of* n, *and Ramanujan observed that the table indicated certain simple congruence properties of* $p(n)$".

[15] A. Garsia and S. Milne, J. Combinatorial Theory, Ser.**A.31** (1981) 289–239.

Euler, in 1742, proved the identity:

$$\frac{1}{(1-x)(1-x^2)(1-x^3)\cdots} = 1 + \sum_{n=1}^{\infty} p(n)\, x^n \quad \text{for} \quad |x| < 1. \tag{2.46}$$

Ramanujan used this identity, the following Euler identity:

$$(1-x)(1-x^2)(1-x^3)\cdots = 1 + \sum_{n=-\infty}^{\infty} (-1)^n\, x^{n(3n+1)/2}, \tag{2.47}$$

and the Jacobi formula:

$$\{(1-x)(1-x^2)(1-x^3)\cdots\}^3 = \sum_{n=-\infty}^{\infty} (-1)^n\, (2n+1)\, x^{n(n+1)/2}, \tag{2.48}$$

to prove that $p(5n+4)$ is divisible by 5 for all n. In 1919, Ramanujan proved the following:

$$\begin{aligned} p(5n+4) &= 0 \quad (mod\ 5), \\ p(7n+5) &= 0 \quad (mod\ 7), \\ p(11n+6) &= 0 \quad (mod\ 11). \end{aligned} \tag{2.49}$$

Or, in other words,

$$\begin{aligned} &p(4),\ p(9),\ p(14)\cdots \textit{ are divisible by } 5. \\ &p(5),\ p(12),\ p(19)\cdots \textit{ are divisible by } 7. \\ &p(6),\ p(17),\ p(28)\cdots \textit{ are divisible by } 11. \end{aligned} \tag{2.50}$$

For $n = 0$, these congruences imply:

$$p(4) = 5, \quad p(5) = 7, \quad p(6) = 11. \tag{2.51}$$

Ramanujan also proved congruences with moduli $5^2,\ 7^2,\ 11^2$:

$$\begin{aligned} p(25n+24) &= 0 \quad (mod\ 5^2), \\ p(49n+47) &= 0 \quad (mod\ 7^2), \\ p(121n+116) &= 0 \quad (mod\ 11^2) \end{aligned} \tag{2.52}$$

and went a good deal further to put forward a general conjecture:

$$\text{if} \quad \delta = 5^a\, 7^b\, 11^c \qquad \text{and} \qquad 24\lambda \equiv 1 \ (mod\ \delta), \tag{2.53}$$

$$\text{then} \qquad p(m\delta + \lambda) \equiv 0 \ (mod\ \delta) \quad \text{for every } m. \tag{2.54}$$

This conjecture of Ramanujan led to a considerable amount of work in this area. Hansraj Gupta[16] in "A table of partitions" and "On a conjecture of Ramanujan", while extending the table of MacMahon to values of $p(n)$ up to $n = 300$, found that :

$$p(243) = 133978259344388 \tag{2.55}$$

which is divisible by 7^3. S. Chowla, in his paper entitled "Congruence properties of partitions" (Journal of London Mathematical Society, vol. 9 (1934) 247) observed that, since

$$24 \cdot 243 \equiv 1 \ (mod\ 7^3) \tag{2.56}$$

this contradicts Ramanujan's conjecture! However, Krečmar in "Sur les proprietes de la divisibilite d'une fonction additive" (Bulletin of Academy of Sciences, URSS, Vol. 7 (1933) 763–800) proved the congruence:

$$p(125n + 99) \equiv 0 \ (mod\ 5^3). \tag{2.57}$$

Two of the **remarkable algebraic identities** discovered by Ramanujan are related to the first two of the above congruences. Hardy, in his article in the "Collected Papers" by Srinivasa Ramanujan (first published by the Cambridge University Press, in 1927) stated that: "If I had to select one formula from all of Ramanujan's work, I would agree with Major MacMahon in selecting the formula :

$$p(4) + p(9)x + p(14)x^2 + \cdots = \frac{5\{(1-x^5)(1-x^{10})(1-x^{15})\cdots\}^5}{\{(1-x)(1-x^2)(1-x^3)\cdots\}^6}, \tag{2.58}$$

where $p(n)$ is the number of partitions of n". This formula appears in Ramanujan's paper entitled: "Some properties of $p(n)$, the number of partitions of n", in the Proceedings of Cambridge Philosophical Society, Vol. **19**, 1919, p. 207–210, along with another remarkable identity:

$$p(5) + p(12)x + p(19)x^2 + \cdots = \frac{7\{(1-x^7)(1-x^{14})(1-x^{21})\cdots\}^3}{\{(1-x)(1-x^2)(1-x^3)\cdots\}^4}, \tag{2.59}$$

[16]Proc. London Math. Soc. (2) vol. 39 (1935) p. 142–149.
Proc. Indian Acad. Sci. (A), Vol. 4 (1936) p 625–629.

These identities make the truth of the divisibility of

$$p(5n+4) \quad \text{and} \quad p(7n+5) \quad \text{by 5 and 7, respectively,} \tag{2.60}$$

obvious. Hardy in *Ramanujan's congruences*[17] pointed out that "Ramanujan never published a complete proof" of these two formulae "but proofs have been found by Darling and Mordell".

2.7 Rogers–Ramanujan Identities

In 1917, Hardy,[18] in his book on "Ramanujan" states that "Ramanujan looking through old volumes of the London Mathematical Society, came accidentally across Roger's papers. I can remember very well his surprise and the admiration which he expressed for Rogers' work."

The celebrated Rogers–Ramanujan identities were originally posed by Ramanujan as Q. 584 in the Questions and Answers section of the Journal of the Indian Mathematical Society, Vol. VI (1914) p. 199:

Q[19]**584. (S. Ramanujan):**– "Examine the correctness of the following results:

$$(i) \quad 1+\frac{x}{1-x}+\frac{x^4}{(1-x)(1-x^2)}+\frac{x^9}{(1-x)(1-x^2)(1-x^3)}+\cdots$$

$$= \quad \frac{1}{(1-x)(1-x^6)(1-x^{11})\cdots} \times \frac{1}{(1-x^4)(1-x^9)(1-x^{14})\cdots} \tag{2.61}$$

Here $1, 4, 9, \cdots$ on the left are the squares of natural numbers while, $1, 6, 11, 16, \cdots$ on the right are numbers in A.P. with 5 for common difference.

$$(ii) \quad 1+\frac{x^2}{1-x}+\frac{x^6}{(1-x)(1-x^2)}+\frac{x^{12}}{(1-x)(1-x^2)(1-x^3)}+\cdots$$

$$= \quad \frac{1}{(1-x^2)(1-x^7)(1-x^{12})\cdots} \times \frac{1}{(1-x^3)(1-x^8)(1-x^{13})\cdots} \tag{2.62}$$

Here the nth term of the sequence $2, 6, 12, \cdots$ is $n(n+1)$ and 2,7,12, $\cdots$; 3,8,13, $\cdots$ increase by 5".

[17] G.H. Hardy, *Ramanujan: Twelve lectures on subjects suggested by his life and work*, Camb. Univ. Press (1940) pp. 87–90.

[18] G.H. Hardy, *Ramanujan: Twelve lectures on subjects suggested by his life and work*, AMS Chelsea Publishing (1991) p. 91.

[19] S. Ramanujan, J. Indian Math. Soc. vol. VI (1914) p. 199.

"The formulae have a very curious history. They were formed first in 1894 by Rogers, a mathematician of great talent but comparatively little reputation, now remembered mainly from Ramanujan's rediscovery of his work ⋯ Ramanujan discovered the formulae sometime before 1913. He had then no proof (and knew that he had none). ⋯ They are therefore stated without proof in the second volume of MacMahon's Combinatorial Analysis. The mystery was solved, trebly, in 1917. ⋯ A correspondence followed in the course of which Rogers was led to a considerable simplification of his original proof. About the same time, I. Schur, ⋯, rediscovered the identities again. Schur published two proofs, one of which is combinatorial and quite unlike any other proof known. There are now seven published proofs, the four referred to already, the two much simpler proofs found later by Rogers and Ramanujan and published in the Papers, and a much later proof by Watson based on quite different ideas. None of these proofs can be called both 'simple' and 'straightforward', since the simplest are essentially verifications and no doubt it would be unreasonable to expect a really easy proof" (p. 41 of Hardy's *Ramanujan: Twelve Lectures on subjects suggested by his life and work*).

One of the proofs due to Schur was made into a combinatorial proof by Garsia and Milne, in 1981, who discovered a basic combinatorial fact—their involution principle. These beautiful identities appear in Chapter XVI of Ramanujan second Notebook. This chapter has been thoroughly analysed and edited by C. Adiga, B.C. Berndt and S. Bhargava and by G.N. Watson in "Chapter 16 of Ramanujan's second notebook: Theta-functions and q-series" (Memoir No. 315, American Mathematical Society, Providence 1985).

The Rogers-Ramanujan identities arise in the study of Euclidean Lie algebras, and the search for a proof of these identities in this setting led to a deeper understanding of some of the representation theory of these algebras. These identities arise naturally in R.J. Baxter's solution of the Hard Hexagon model in Statistical Mechanics, 1980.

The distribution of a film of liquid Helium on a graphite plate is a physical example of the Hard Hexagon model. R.J. Baxter (like Rogers and Schur) found and proved the Rogers-Ramanujan identities without knowing them beforehand. R.J. Baxter's solution of the 2-dimensional Hard Hexagon model is detailed in Chapter 14 of "Exactly Solved Models in Statistical Mechanics" (Academic Press, 1982).

Baxter's work on the Hard Hexagon model has led to generalizations, in particular to the work of G.E. Andrews and R.J. Baxter on the "Eight-vertex SOS model and generalized Rogers-Ramanujan-type identities", published in the J. Stat. Phys., Vol. 25 (1984) p. 193. The interested reader is urged to refer to the article of G.E. Andrews entitled: *The Reasonable and Unreasonable Effectiveness of Number Theory in Statistical Mechanics*.[20]

[20]This title should have been inspired by the title of Eugene Paul Wigner's: *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*, Communications on Pure and Applied Mathematics Vol. 13 (1960) 1: pp. 1–14. (http://www.dartmouth.edu/~matc/MathDrama/reading/Wigner.html).

Asked as to what makes Ramanujan a "natural mathematical genius", it is difficult to choose a few theorems from his plethora of thousands of Entries in his Notebooks and from his published papers. However, venturing to answer the query, many experts would draw attention to a one-page abstract[21] in which Ramanujan announced that he had found a number of algebraic identities for two infinite products he defined as $G(x)$ and $H(x)$:

$$G(x) = \prod_{n=1}^{\infty} \frac{1}{(1-x^{5n-4})(1-x^{5n-1})} \quad \text{and}$$
$$H(x) = \prod_{n=1}^{\infty} \frac{1}{(1-x^{5n-3})(1-x^{5n-2})} \tag{2.63}$$

which satisfy

$$H(x)G(x^{11}) - x^2 G(x)H(x^{11}) = 1 + 11xG(x^6)H(x^6) \tag{2.64}$$

and

$$H(x)G(x^{11}) - x^2 G(x)H(x^{11}) = 1. \tag{2.65}$$

Each of these algebraic identities is the simplest of a large class. The Rogers-Ramanujan identities are:

$$G(x) = \sum^{\infty} \frac{x^{n^2}}{(1-x)(1-x^2)\cdots(1-x^n)},$$
$$H(x) = \sum^{\infty} \frac{x^{n(n+1)}}{(1-x)(1-x^2)\cdots(1-x^n)}. \tag{2.66}$$

Ramanujan's "Lost" Notebook contains 40 such identities, such as the one given below:

$$H(x)G(x^6) - xG(x)H(x^6) = \frac{P(x)}{P(x^3)} \quad \text{where} \quad P(x) = \prod_{n=1}^{\infty}(1-x^{2n-1}). \tag{2.67}$$

The three Quarterly Reports submitted by Ramanujan to the University of Madras, as the University's first Research Scholar, contain a large number of formulae for

[21] G.H. Hardy, *Ramanujan: Twelve Lectures inspired by his life and work*, Camb. Univ. Press (1940) paper 29.

definite integrals. Hardy, in 'Ramanujan: Twelve Lectures...', on p.15, refers to a formula Ramanujan was fond of: "There is one particularly interesting formula, viz.

$$\int_0^\infty x^{s-1}\{\phi(0) - x\phi(1) + x^2\phi(2) - \cdots\}\, dx = \frac{\pi\phi(-s)}{\sin\ s\pi} \tag{2.68}$$

of which he was especially fond and made continued use. This is really an 'interpolation formula', which enables us to say, for example, that, under certain conditions, a function which vanishes for all positive integral values of its argument must vanish identically, I have never seen this formula stated explicitly by anyone else, though it is closely connected with the work of Mellin and others."

When

$$\phi(u) = \frac{\lambda(u)}{\Gamma(1+u)}, \tag{2.69}$$

the above formula becomes:

$$\int_0^\infty x^{s-1}\{\lambda(0) - \frac{x}{1!}\lambda(1) + \frac{x^2}{2!}\lambda(2) - \cdots\} = \Gamma(s)\lambda(-s). \tag{2.70}$$

These two integrals are variants of one another and Ramanujan "used them as one of the commonest tools" in his works.

2.8 An 'Astonishing' Theorem

Hardy and Ramanujan showed that $p(n)$, the number of partitions of n, can be calculated exactly to any value of n. E.C. Titchmarsh in his biographical note on Hardy, which appears in the "Collected Papers of Hardy" wrote: "I must now describe Hardy's work on partitions, the 'circle method', in the analytic theory of numbers and his association with Ramanujan. They wrote five papers together, the most famous being that in Vol. 17 of the Proceedings of the London Math. Soc. (1918) 5, a section of which is reproduced on the dust-cover of a Mathematician's Apology.

"In this it is shown that $p(n)$ the number of unrestricted partitions of n can not only be represented approximately by an asymptotic formula, but it can be calculated exactly for any value of n." The "circle method" on which this depends is, no doubt, Hardy's most original creation. It proceeds as follows. The numbers $p(n)$ are the coefficients in the expansion:

$$f(z) = 1 + \sum_{n=1}^{\infty} p(n)z^n = \frac{1}{(1-z)(1-z^2)(1-z^3)\cdots} \tag{2.71}$$

so that

$$p(n) = \frac{1}{2\pi i} \int_{\Gamma} \frac{f(z)}{z^{n+1}} dz \tag{2.72}$$

where Γ is a path enclosing the origin and lying entirely inside the unit circle and is taken to be a concentric circle of radius just < 1. The unit circle is a line of essential singularities of $f(z)$, but certain points are found to have a particularly strong influence on the integral, and it is from these that a dominant term is ultimately derived. In the Dirichlet series method for proving, for example the prime number theorem, the dominant term is easily identified, and almost the whole difficulty lies in showing that it is dominant."

"A whole apparatus, involving the Farey dissection of the circle and the linear transformations of the elliptic modular functions, is needed to produce the result. It is all the more astonishing that the analysis should have been carried out to the point at which the exact value of $p(n)$ could be obtained."

"Similar methods were applied later by Hardy and Littlewood to many problems, particularly to the Waring problem of the expression of a number as a sum of given powers, and to problems involving primes. One such problem which had long defied analysis was Goldbach's theorem, or hypothesis, that any even number can be expressed as the sum of two primes. The success of the circle method in the study of $p(n)$ suggests a similar approach to Goldbach's theorem. ... But the original Goldbach hypothesis still remains unproved."

In an "astonishing theorem" (J.E. Littlewood,[22]) Hardy and Ramanujan have shown that the number $p(n)$ of partitions of n is the integer nearest to

$$\sum_{q=1}^{\nu} \sqrt{q} A_q(n) \Psi(n), \tag{2.73}$$

where

$$A_q(n) = \sum \omega_{p,q} \; exp(-2np\pi/q), \tag{2.74}$$

the sum being over all p prime to q and less than it, $\omega_{p,q}$ is a certain 24 q-th root of unity, ν is of order of $\sqrt{n}$ and

$$\Psi_q(n) = \frac{d}{dn} \left[exp \left\{ \frac{\pi}{q} \sqrt{\frac{2}{3} \left(n - \frac{1}{24} \right)} \right\} \right]. \tag{2.75}$$

[22] P.K. Srinivasan's *Ramanujan: Letters and Reminiscences*, Memorial Number, Vol. 1, Muthialpet High School, Madras, 1968, pp. 154–156; and also in G.E. Andrews, *The Theory of Partitions*, Encyclopedia of Mathematics and its Applications, Vol. 2, Addison-Wesley, Reading, Massachusetts (1976) pp. 68–69.

Taking $\nu = 5$, for $n = 200$, they predicted the value:

$$p(200) = 3972999029388 \tag{2.76}$$

which was computed by Major MacMahon to be correct (without the use of computers!). Littlewood while reviewing Ramanujan's "Collected Papers" has given an exciting account of the collaboration between the two men of "quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him". Later on, D.H. Lehmer showed that the Hardy-Ramanujan series was divergent and Rademacher refined the circle method[23] using what are called Ford Circles, which simplified considerably the estimate and led him to a convergent series for $p(n)$.

In this context, it is worthwhile to record here the comments of the Fields Medalist Atle Selberg, in his "Reflections Around the Ramanujan Centenary", reproduced from "Atle Selberg, Collected Papers, Vol. I" (Springer-Verlag, 1989; reprinted in "Resonance", December 1996.): "The paper by Hardy and Ramanujan contained surely a result that was very remarkable in itself; since $p(n)$ is an integer, it allows exact computation on it. But it was not an exact formula. It was a formula with an error tending to zero as n grows and therefore $p(n)$ being an integer, one could find the exact value."

"If one looks at Ramanujan's first letter to Hardy, there is a statement there which has some relation to his later work on the partition function, namely about the coefficient of the reciprocal of a certain theta series (a power series with square exponents and alternating signs as coefficients). It gives the leading term in what he claims as an approximate expression for the coefficient. If one looks at that expression, one sees that this is the exact analogue of the leading term in the Rademacher formula for $p(n)$ which shows that Ramanujan, in whatever way he had obtained this, had been led to the correct terms of that expression".

"In the work on partition function, studying the paper it seems clear to me that it must have been, in a way, Hardy who did not fully trust Ramanujan's insight and intuition, when he chose the other form of the terms in their expression, for a purely technical reason, which one analyses as not very relevant. I think that if Hardy had trusted Ramanujan more, they should have inevitably ended with the Rademacher series. There is little doubt about that."

[23]For the Hardy-Littlewood-Ramanujan *circle method*, see R. Bellman: *A Brief Introduction to Theta Functions*, Holt, Rinehart and Winston, New York (1961) p. 14.

2.9 Ramanujan on Elliptic and Modular Functions

Euler[24] (1748) gave following integral representation for the hypergeometric function:

$$F(a,b;\ c;\ z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}\, dt, \tag{2.77}$$

where $\Re\, c > \Re\, b > 0$ and $|\arg(1-z)| < \pi$. It is straightforward to show that, for $|k| < 1$,

$$K(k) = \frac{1}{2}\pi\ F(\frac{1}{2}, \frac{1}{2}; 1;\ k^2) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} \tag{2.78}$$

and

$$E(k) = \frac{1}{2}\pi\ F(-\frac{1}{2}, \frac{1}{2}; 1;\ k^2) = \int_0^1 \frac{\sqrt{(1-k^2t^2)}}{\sqrt{(1-t^2)}}\, dt. \tag{2.79}$$

These fundamental integrals are called complete Legendre "elliptic integrals" of the second kind. The complementary integrals E' and K' are integrals with the complementary variables: $f'(k) = f(k')$, and the variable k is called the "modulus" and the variable k' is called the "complementary modulus" and the prime does not denote differentiation. In terms of k', the complementary elliptic integrals E' and K' are:

$$K'(k) = K(k') \qquad \text{and} \qquad E'(k) = E(k'). \tag{2.80}$$

The reason why these are called "elliptic" integrals is because the perimeter A of an ellipse, with semi-major and semi-minor axes being a and b, is given by

$$A = 4aE'\left(\frac{b}{a}\right). \tag{2.81}$$

Elliptic functions can be expressed as inverse functions of elliptic integrals. Jacobi (1829) is the originator of the fascinating theory of theta functions, an important class of functions closely related to the elliptic functions. It is the considered opinion of some[25] that Ramanujan had not seen any standard book on elliptic functions,

[24] *Introduction in analysis Infinitorum* (1748). English translation; *Introduction to Analysis of the Infinite*, by John Blanton (Book I, ISBN 978-0-387-96824-7, Springer-Verlag (1988), Book II, ISBN 978-0-387-97132-2, Springer-Verlag (1989).

[25] "Ramanujan Revisited", edited by George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin, Academic Press Inc. (1988).

before he went to England, since he used his own notation and his work does not reveal his awareness of the fundamental double periodicity property[26] of the elliptic functions.

Hardy, in his "Ramanujan: Twelve Lectures Inspired by His Life and Work" says that it is in Ramanujan's work on elliptic and modular functions that
"*both the profundity and the limitations of Ramanujan's knowledge stand out most sharply ... Ramanujan never professed to have made any major advance in the theory of elliptic function, and it seems that he must have learnt the fundamentals of the theory, so far as he was interested in them, from books. There is a sharp contrast between his attitude here and his attitude about the theory of primes, where he certainly regarded all his results his own. He never writes as if he had 'invented' theta-functions or modular equations, though he sets out a whole theory of them in a language of his own. Cayley's and Greenhill's books were in the Madras University Library, and he could have found a good deal about them there. Indeed Littlewood says that 'Ramanujan somehow acquired an effectively complete knowledge of the formal side of the theory of elliptic functions', and that his ignorance of complex function theory and Cauchy's theorem may seem difficult to reconcile with this'; and adds that 'a sufficient, and I think necessary, explanation would be that Greenhil's very odd and individual 'Elliptic Functions' was his text-book'. In Greenhill's book the complex variable and double periodicity are not mentioned until p. 254, and the double periodicity is deduced somehow from properties of Cartesian ovals. In fact Greenhill knew very little more 'function theory' than Ramanujan"*.

Hardy also stated that: "*And there is some of his work, mostly in the theory of elliptic functions, about which some mystery still remains; it is not possible, after all the work of Watson and Mordell, to draw the line between what he may have picked up somehow and what he must have found for himself.*"

Chapters 16 and 21, of the Second Notebook of Ramanujan dealt with q-series and θ-functions; the fundamental properties of elliptic functions; Jacobi's elliptic functions; modular equations of degrees 3, 5, 7 and associated θ-function identities; modular equations of higher and composite degree and Eisenstein series, respectively. Bruce C. Berndt has studied thoroughly the 834 Entries contained in these six Chapters in Part III of his five volume work: "Ramanujan's Notebooks". G.N. Watson considers that these results "show Ramanujan at his best".

[26] A function of a complex variable $f(z)$ is called a doubly periodic function of z, if $f(z) = f(z + 2\omega_1) = f(z + 2\omega_2)$, where ω_1, ω_2 are two complex numbers for which the ratio ω_1/ω_2 is not a real number and $2\omega_1, 2\omega_2$ are the periods of the function. An elliptic function is a doubly periodic function which is meromorphic in the finite part of the complex plane. (Analytic functions $f(z)$ that have only well separated poles as singularities are called meromorphic).

2.10 Mock Theta Functions

Ramanujan apparently means,[27] by theta-functions, sums, products and quotients of series of the form:

$$\sum_{-\infty}^{\infty}(-)^{\varepsilon n}\, q^{an^2+bn} \qquad \text{where} \quad \varepsilon = 0 \text{ or } 1. \tag{2.82}$$

Examples of theta-functions are:

$$\sum_{-\infty}^{\infty}(-)^{n}\, q^{6n^2+4n} = 1 - q^2 - q^{10} + q^{16} + q^{32} - \cdots \tag{2.83}$$

and

$$\sum_{-\infty}^{\infty}(-)^{n}\, q^{6n^2+4n} = 1 + q^2 + q^{10} + q^{16} + q^{32} + \cdots \tag{2.84}$$

The transformed Eulerian form of the theta-functions are:

$$1 + \frac{q}{(1-q)^2} + \frac{q^4}{(1-q)^2(1-q^2)^2} + \frac{q^9}{(1-q)^2(1-q^2)^2(1-q^3)^2} + \cdots$$

$$= \left(\sum_{n=-\infty}^{\infty} (-1)^n q^{n(3n-1)/2} \right)^{-1} \tag{2.85}$$

which is an identity of Euler, and

$$1 + \frac{q}{(1-q)} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)} + \cdots$$

$$= \frac{\displaystyle\sum_{n=-\infty}^{\infty} (-1)^n q^{n(5n-1)/2}}{\displaystyle\sum_{n=-\infty}^{\infty} (-1)^n q^{n(3n-1)/2}} \tag{2.86}$$

which is the first Rogers-Ramanujan identity.

[27] George E. Andrews: *An Introduction of Ramanujan's 'Lost' Notebook*, Am. Math. Monthly **86** (19169) 89–108.

In his last letter to Hardy, in January 1920, Ramanujan wrote:[28]
"*I am extremely sorry for not writing to you a single letter up to now... I discovered very interesting functions recently which I call Mock theta-functions. Unlike the False theta-functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics beautifully as the ordinary theta-functions. I am sending you with this letter some examples ...*

I have proved that if

$$f(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \cdots \tag{2.87}$$

then

$$f(q) + (1-q)(1-q^3)(1-q^5)\cdots(1-2q+2q^4-2q^9+\cdots) = O(1) \tag{2.88}$$

at all the points $q = -1,\ q^3 = -1,\ q^5 = -1,\ q^7 = -1, \ldots$, *and at the same time*

$$f(q) - (1-q)(1-q^3)(1-q^5)\cdots(1-2q+2q^4) = O(1) \tag{2.89}$$

at all the points $q^2 = -1,\ q^4 = -1,\ q^6 = -1,\ \ldots$ *Also obviously* $f(q) = O(1)$ *at all the points* $q = 1,\ q^3 = 1,\ q^5 = 1, \cdots$. *And so* $f(q)$ *is a Mock* θ *function. When* $q = -e^{-t}$ *and* $t \to 0$

$$f(q) + \sqrt{\frac{\pi}{t}}\ \exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right) \to 4. \tag{2.90}$$

The coefficient of q^n *in* $f(q)$ *is*

$$(-1)^{n-1}\frac{\exp(\pi\sqrt{\frac{n}{6}-\frac{1}{144}})}{2\sqrt{n-\frac{1}{24}}} + O\left(\frac{\exp(\frac{\pi}{2}\sqrt{\frac{n}{6}-\frac{1}{144}})}{\sqrt{n-\frac{1}{24}}}\right) \tag{2.91}$$

It is inconceivable that a single θ *function could be found to cut out the singularities of* $f(q)$.

Mock theta-functions:

$$\phi(q) = 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \cdots \tag{2.92}$$

[28] *Collected Papers of Srinivasa Ramanujan*, Ed. G.H. Hardy, P.V. Seshu Iyer and B.M. Wilson, Chelsea (1962) p. xxxi.

$$\Psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \cdots \tag{2.93}$$

$$\chi(q) = 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \cdots \tag{2.94}$$

These are related to $f(q)$ *as shown below.*

$$2\phi(-q) - f(q) = f(q) + 4\Psi(-q) = \frac{1-2q+2q^4-2q^9+\cdots}{(1+q)(1+q^2)(1+q^3)\cdots} \tag{2.95}$$

$$4\chi(q) - f(q) = \frac{(1-2q^3+2q^{12}-\cdots)^2}{(1-q)(1-q^2)(1-q^3)\cdots} \tag{2.96}$$

These are of the 3rd order".

This was followed by Mock theta-functions of 5th order and of 7th order, which are not related to each other.

By False theta functions are meant simply the theta series with the wrong signs like:

$$\sum_{n=0}^{\infty}(-1)^n q^{6n^2+4n}(1+q^{4n+2}) = 1+q^2-q^{10}-q^{16}+q^{32}+\cdots \tag{2.97}$$

Hardy has commented that "*a mock theta function is a function defined by a q-series convergent when* $|q| < 1$ *for which we can calculate asymptotic formulae, when q tends to a rational point,* $e^{2\pi ir/s}$ *of the unit circle, of the same degree of precision as those furnished for the ordinary theta functions by the theory of linear transformations*".

The classical one-dimensional heat equation and the Kortweg-de Vries equation have been solved in terms of theta functions. In recent times, theta functions have been found to be of great use in the theory of solitons—a field of great interest in modern Physics. Freeman J. Dyson, a renowned theoretical physicist, on the occasion of the Ramanujan birth centenary, in "Ramanujan Revisited" stated:[29] "*My dream is that I will live to see the day when young Physicists, struggling to bring predictions of superstring theory into correspondence with the facts of nature, will be led to enlarge their analytic machinery to include not only theta-functions, but also mock theta-functions*."

[29] 'Ramanujan Revisited', Ed. George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin, Acad. Press Inc. (1988).

2.11 Continued Fractions

It is Ramanujan's results on Continued Fractions, contained in his first historic letter to G.H. Hardy, in 1913, which made the latter state that the formulae "defeated me completely". Ramanujan's major work on Continued Fractions revolves around the generalization of what is called the "Ramanujan continued fraction":

$$(1.1.1) \qquad \frac{1}{1+}\,\frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+\cdots}, \qquad |x|<1. \tag{2.98}$$

Professor K.G. Ramanathan, the renowned mathematician of the Tata Institute of Fundamental Research, Bombay, in his *Homi J. Bhabha medal lecture – 1984* entitled: "Ramanujan's continued fraction", states:[30] "*The continued fraction was, however, first discovered by L.J. Rogers in his beautiful work, (p. 328)*[31] *on expansions of infinite products. It was rediscovered around 1911 by Ramanujan who recognized its importance and applied to it the methods of elliptic and modular functions. Indeed, in his, now famous, first letter to Hardy in 1913, he stated a number of beautiful results about (1.1.1) which led Hardy to believe, and rightly too, that these could have been written down only by a mathematician of the highest class. In the "Lost" note book, he gave a number of generalizations of (1.1.1) and their evaluations at points in an imaginary quadratic field. ...Alte Selberg,*[32] *in 1936, working independently, rediscovered almost all of Ramanujan's generalizations of (1.1.1) and the Rogers-Ramanujan identities. Some of Schur's results on (1.1.1) are also to be found in the 'Lost' note book".*

Ramanujan has done pioneering work on continued fractions. He asserted that

$$1+\frac{aq}{1+}\,\frac{aq^2}{1+}\,\frac{aq^3}{1+\cdots}\,\frac{aq^n}{1}=\frac{A_n}{B_n} \tag{2.99}$$

$$A_n=1+aq\frac{1-q^n}{1-q}+a^2q^4\frac{(1-q^{n-1})(1-q^{n-2})}{(1-q)(1-q^2)}+\cdots \tag{2.100}$$

$$B_n=1+aq^2\frac{1-q^{n-1}}{1-q}+a^2q^6\frac{(1-q^{n-2})(1-q^{n-3})}{(1-q)(1-q^2)}+\cdots \tag{2.101}$$

[30] K.G. Ramanathan, *Ramanujan's continued fraction*, Indian Journal of Pure and Applied Mathematics **16** (1985) 695–724; 'Hypergeometric series and continued fractions', Proc. of the Indian Acad. Sci. (Math. Sci.) **97** (1987) 277–296.

[31] J.J. Rogers, *Second memoir on the expansion of certain infinite products*, Proc. London math. Soc. **25** (1894) 318–343.

[32] *Über einige arithmetische Identitaäten*, Avhand. Utigitt. Norske Vid. Akad. Oslo (1936) No. 8, 23 pages.

for $|q| < 1$. Though this is the only identity which appeared in his published papers, his Notebooks, particularly Chapter XII of the second Notebook, extensively edited in recent times by Bruce C. Berndt, in "Ramanujan's Notebooks", Part II, and the "Lost" Notebook contain a large number of very beautiful results on continued fractions. In a series of papers[33] Professor George E. Andrews and Professor Bruce C. Berndt have undertaken the task of grouping and proving the formulae contained in the "Lost" Notebook and have till date brought out three out of the planned four volumes, which "broadly interpret 'lost notebook' to include all material published *with* Ramanujan's original lost notebook by Narosa in 1988."[34]

[33]G.E. Andrews, "Ramanujan's 'Lost' Notebook": I. "Partial Theta-functions", Advances in Mathematics 41 (1981) 137–172: "Ramanujan's 'Lost' Notebook" II. Theta-function expansions", ibid p. 173–185; "Ramanujan's 'Lost' Notebook III. The Rogers Ramanujan continued fraction", ibid pp. 186–208.

[34]S. Ramanujan, *The Lost Notebook and Other Unpublished Papers*, Narosa, New Delhi, (1988).

Chapter 3
Ramanujan's Mathematics: Further Glimpses

The earliest contributions of Ramanujan to Mathematics were in the form of Questions and/or Answers to questions in the *Journal of the Indian Mathematical Society*. His creativity in posing and answering mathematical problems is best illustrated through some examples mentioned in Chap. 2, Sect. 2.3. In this chapter, we provide further glimpses into his unique mathematical contributions which may be called as Ramanujan's mathematics!

Any set which can be put in a one-to-one correspondence with the natural numbers (or integers) is called a countably infinite, or denumerably infinite, set. Given a countable set S, any other set which can be put into a one-to-one correspondence with S is also countable. Examples of countable sets include the integers, algebraic numbers and rational numbers. Examples of nondenumerable sets include the real, complex, irrational and transcendental numbers.

A transcendental number is defined as a number which is not the solution of an algebraic equation. The decimal digits of a transcendental number are random and unending. The complex constant i is the solution of the simple algebraic equation:

$$x^2 + 1 = 0, \text{ whose solutions are } x = \pm\sqrt{-1}.$$

The universal real mathematical constants π, present[1] in the area, perimeter, volume formulas for the conic sections and regular quadratic surfaces like the sphere, ellipsoid, etc. Napier invented the natural logarithm which led (decades later) to the discovery of the constant e, sometimes called the Euler number. The exponential function is related to growth or decay of many natural phenomena, is unique[2] in that it is the only mathematical function whose derivative is equal to itself:

[1] Peter Beckmann, *History of Pi*, St. Martin's Press (1971), 3rd edition (1976).

[2] Eli Maor, *e: The Story of a Number*, Princeton University Press (1993).

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
https://doi.org/10.1007/978-981-16-0447-8_3

$$\frac{d}{dx}e^x \;=\; e^x \;=\; \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

π and e are transcendental numbers. Their values are:

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971\ \cdots$$

and

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = 2.71828\ 18284\ 59045\ 23536\ 02874\ 71352\ 66249\ 77572\ \cdots,$$

is called the (natural) exponential function. These three constants π, i and e are related through *the most beautiful equation in the whole of mathematics* called the Euler's formula:

$$e^{i\pi} + 1 \;=\; 0,$$

which includes along with the transcendental numbers.[3] π and e, the additive identity 0 and the multiplicative identity 1. The digits of the transcendental number are random and unending.

3.1 Ramanujan Summation and Ramanujan Sum

The infinite series: $1+2+3+4+\cdots\infty$, is a divergent series. For, the partial sums:

$$1, 1+2=3, 1+2+3=6, 1+2+3+4=10, \cdots, \sum_{k=1}^{n} k \;=\frac{n(n+1)}{2},$$

increase without bound as $n \;\rightarrow\; \infty$. Since the sequence of partial sums do not converge to a finite limit, the series does not have a sum. The above numbers are triangular numbers which diverge to $+\infty$, so by definition, the infinite series also diverges to $+\infty$. By the term test, since the terms do not approach zero, the series also diverges as a consequence.

The infinite series

$$1-1+1-1+\cdots=\sum_{n=0}^{\infty}(-1)^n$$

[3] In transcendental number theory, the Lindemann-Weierstrass theorem: if $\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_n$ are algebraic numbers that are linearly independent over the rational numbers $\mathcal{Q}$, then $e^{\alpha_1}, \ldots, e^{\alpha_n}$ are algebraically independent over $\mathcal{Q}$; is a result that is very useful in establishing the transcendence of numbers. The transcendence of π and e was proved by Lindemann in 1882.

is called Grandi's series, after Guido Grandi (1671–1742), an Italian Monk and a Priest, who was a theologian, philosopher, engineer and a mathematician—a polymath, who studied this divergent series in 1703, and observed that if it is treated as a telescoping series and perform subtractions as shown:

$$(1-1)+(1-1)+(1-1)+\cdots = 0+0+0+\cdots = 0.$$

However, by a shifting of the bracketing procedure:

$$1+(-1+1)+(-1+1)+(-1+1)+\cdots = 1+0+0+0+\cdots = 1,$$

which contradicts the earlier summation which gave a result 0, by applying the parentheses differently. Another formal way of manipulation is the following:

$$S = 1-1+1-1+\cdots$$

So, $1-S = 1-(1-1+1-1+\cdots) = 1-1+1-1+\cdots = S$, and therefore,

$$1 = 2S \qquad \text{or} \qquad S = \frac{1}{2}.$$

In modern mathematics, the sum of an infinite series is defined as:

$$\textit{the limit of the sequence of its partial sums, if it exists.}$$

The present day understanding is that it is not valid to perform seemingly innocuous operations on a series, such as grouping or ordering individual terms, unless the series is *absolutely convergent*.

Ramanujan in his Notebook 1, Chap. 8, presents a derivation for

$$1+2+3+4+\cdots = -\frac{1}{12}.$$

"Another way of finding the constant is as follows:

Let us take the series $1+2+3+4+5+\&c$. Let C be its constant.

Then $C = 1+2+3+4+\&c$

$$Therefore,\ 4C = \ 4 \ +8 \ +\&c$$

$$Therefore, -3C = 1-2+3-4+\&c = \frac{1}{(1+1)^2} = \frac{1}{4}.$$

$Therefore, C = -\frac{1}{12}$."

In Number Theory, the sum usually denoted by $c_q(n)$, a function of two positive integer variables, q and n, defined by the formula:

$$c_q(n) = \sum_{p(q)} \exp\{-2np\pi i/q\}$$

which is a sum over p less than and prime to q (or any complete system of residues prime to q) is called a Ramanujan sum. Ramanujan summed a large number of series of the form:

$$\sum_q a_q c_q(n).$$

3.2 Arithmetic Functions

The second letter of Ramanujan to Hardy, dated February 27, 1913, contains the following identity involving hypergeometric functions of unit argument:

If

$$\frac{F(\frac{1}{4},\ \frac{3}{4},\ 1,\ 1-\alpha)}{F(\frac{1}{4},\ \frac{3}{4},\ 1,\ \alpha)} = 5\,\frac{F(\frac{1}{4},\ \frac{3}{4},\ 1,\ 1-\beta)}{F(\frac{1}{4},\ \frac{3}{4},\ 1,\ \beta)}$$

then

$$(\alpha\beta)^{1/2} = [(1-\alpha)(1-\beta)]^{1/2} + [\alpha\beta(1-\alpha)(1-\beta)]^{1/6}[(\alpha\beta)^{1/6} + ((1-\alpha)(1-\beta))^{1/6}].$$

After seeing this identity, Prof. Littlewood remarked that Ramanujan *is at least a Jacobi*, when he was asked by Hardy to comment on the formulae contained in the first two letters of Ramanujan to Hardy, in 1913. Ramanujan developed in a simple and purely algebraic way the basic properties of the classical elliptic and other allied functions in his paper: *On certain elliptic functions*. For instance he gave an a priori proof of the following identity:

$$\left\{\frac{1}{4}\cot(\frac{\theta}{2} + \sum_{n=1}^{\infty} x^n \frac{\sin(n\theta)}{(1-x^n)}\right\}^2 = \left\{\frac{1}{4}\cot(\frac{\theta}{2})\right\}^2 + \sum_{n=1}^{\infty} x^n \frac{\cos(\mathrm{n}\theta)}{(1-x^n)^2}$$

$$+\frac{1}{2}\sum_{n=1}^{\infty} n x^n \frac{1-\cos(n\theta)}{(1-x^n)},$$

which forms the foundation of his theory. For, from this identity he derives many interesting results. Venkatachaliengar[4] has remarked that this identity is contained in Jordan's *Cours de Analyze II* in a somewhat different form. Ramanujan himself remarks in his paper that the elementary proofs of these formulae contained in his paper "seem to be of some interest in itself". Ramanujan has derived the basic properties of elliptic functions also in a purely algebraic way.

Ramanujan defined the symmetric function:

$$\phi_{r,s} = \sum_m \sum_n m^r n^s q^{mn} = \phi_{s,r}$$

for natural numbers r, s, such that $rs \neq 0$ and $(r + s)$ odd. Bernoulli numbers B_r occur in

$$\phi_{2r-1,0} = \sum_n n^{2r-1} \frac{q^n}{1-q^n} - (-1)^r \frac{B_r}{4r} = \phi_{0,2r-1}.$$

Ramanujan proved that $\phi_{r,s}$ is a polynomial over the field of rational numbers in P, Q, R, which are the following functions introduced by him:

$$\begin{aligned} P &= 1 - 24\sum_{n=1}^{\infty} \frac{nq^n}{1-q^n} \\ Q &= 1 + 240\sum_{n=1}^{\infty} \frac{n^3q^n}{1-q^n} \\ R &= 1 - 504\sum_{n=1}^{\infty} \frac{n^5q^n}{1-q^n}, \qquad \text{for} \qquad |q| < 1. \end{aligned}$$

Ramanujan used these functions to not only derive the classical identity named after Jacobi :

$$Q^3 - R^2 = 1728q \prod_{n=1}^{\infty} (1-q^n)^{24},$$

in the simplest algebraic way, but also discovered many new identities, for which we refer the interested reader to the book of Venkatachaliengar.

[4]K. Venkatachaliengar, *Development of Elliptic Functions according to Ramanujan*, Madurai Kamaraj University, Technical Report 2 (1987).

3.3 Ramanujan's τ-Function

The Ramanujan τ-function is defined by

$$g(x) = x\{(1-x)(1-x^2)(1-x^3)\cdots\}^{24}$$

$$= x\prod_{n=1}^{\infty}(1-x^n)^{24} = \sum_{n=1}^{\infty}\tau(n)x^n.$$

Ramanujan tabulated $\tau(n)$ up to $n = 30$ and showed that

$$\tau(p) \equiv 0 \ (\text{mod } p), \text{for} \quad p = 2, 3, 5, 7, 23$$

and for these primes

$$\tau(pn) \equiv 0 \ (\text{mod} \quad p), \text{for every} \quad n.$$

Ramanujan was the first to observe interesting congruence properties of this τ-function, such as:

$$\tau(n) = \sigma_r(n) \qquad (\text{mod } 691), \qquad \sigma_r(n) = \sum d^r$$

where the sum is over all d dividing n. The prime number 691 which occurs here (and in the Bernoulli number $B_{12} = 691/2730$) is called the Ramanujan Prime Number.

The Riemann ζ-function is defined as:

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}, \qquad \text{for } s \geq 1,$$

and the Dirichlet series is defined as:

$$F(s) = \sum_{n=1}^{\infty}\frac{f(n)}{n^s}.$$

Ramanujan discovered the factorization of the Dirichlet series for his τ-function:

$$\sum_{n=1}^{\infty}\frac{\tau(n)}{n^s} = \prod_{all\ primes}\{1-\tau(p)p^s + p^{11-2s}\}^{-1}.$$

Ramanujan also conjectured

$$|\tau(p)| \leq 2p^{11/2} \qquad \text{for every prime } p.$$

Deligne proved this famous conjecture of Ramanujan, in 1974, using advanced methods of Algebraic Geometry, and this was considered as a significant contribution of his along with others made by him and won for him a Fields medal. Another Fields Medalist, Professor Atle Selberg, in 1988, at an extempore talk at the Tata Institute of Fundamental Research, Colaba, Bombay, observed in his "Reflections Around the Ramanujan Centenary": "*that a felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem. ... Ramanujan's recognition of the multiplicative properties of the coefficients of modular forms that we now refer to as cusp forms and his conjectures formulated in this connection, and their later generalization, have come to play a more central role in the mathematics of today, serving as a kind of focus for the attentions of quite a large group of the best mathematicians of our time. Other discoveries like the mock-theta functions are only in very early stages of being understood and no one can yet assess their real importance. So the final verdict is certainly not in, and it may not be in for a long time, but the estimates of Ramanujan's stature in mathematics certainly have been growing over the years. There is no doubt about that.*"

Commenting on the quality of the theorems in the "Lost" Notebook, Richard Askey says: "*Try to imagine the quality of Ramanujan's mind, one which drove him to work unceasingly while deathly ill, and one great enough to grow deeper while his body became weaker. I stand in awe of his accomplishments, understanding is beyond me. We would admire any mathematician whose life's work was half of what Ramanujan found in the last year of his life while he was dying.*"

In this context one may observe that only very few are destined to leave "footprints in the sands" of mathematics and Ramanujan has left behind his indelible Entries in his Notebooks for generations of mathematicians to work on and provide a proof, wondering whether Ramanujan would have provided a simpler proof!

Bruce Berndt in Part I of his five part work on *Ramanujan's Notebooks* (p. 14) has drawn our attention to the following assessments of Ramanujan and his work: "*In notes left by B.M. Wilson, he tells us how George Polya was captivated by Ramanujan's formulas. One day in 1951 while Polya was visiting Oxford, he borrowed from Hardy his copy of Ramanujan's notebooks. A couple of days later, Polya returned them in almost a state of panic explaining that however long he kept them, he would have to keep attempting to verify the formulae therein and never again would have time to establish another original result of his own.*"

E.H. Neville, who on his visit to India to lecture on Complex Analysis to the students of the University of Madras, was commissioned by Hardy to call on Ramanujan and look at his Notebooks, began a broadcast (in Hindustani) in 1941, with the declaration: "*Srinivasa Ramanujan was a mathematician so great that his name transcends jealousies, the one superlatively great mathematician whom India has produced in the last thousand years.*"

Paul Erdös who visited India many times has passed on to us Hardy's personal ratings of mathematicians: Suppose that we rate mathematicians on the basis of pure talent on a scale from 0 to 100. Hardy gave himself a score of 25, Littlewood 30, Hilbert 80 and Ramanujan 100.

This section in which the statements of several great men have been strung together is concluded with the apt observation of Dr. S. Chandrasekhar, the Nobel Laureate and renowned astrophysicist: "*As long as people do mathematics, the work of Ramanujan will continue to be appreciated*."

The author hopes that with the glimpses provided here, the interested mathematics student/reader will begin his own appreciation of the work of Ramanujan.

3.4 Ramanujan's Notebooks

The history of the Notebooks, in brief, is the following: Ramanujan had noted down the results of his researches, without proofs, as in *A Synopsis of Elementary Results: A Book on Pure Mathematics*, by George Shoobridge Carr, in three Notebooks, most probably between the years 1903 and 1914, till he left for Trinity College, Cambridge University, Cambridge, England. These were the Notebooks which he showed to every prospective benefactor to convince them about his abilities as a mathematician and to get financial support for himself and his family. The results in his Notebooks were organized by him.

The first Notebook of Ramanujan has 16 Chapters in 134 pages. The second, contains 21 Chapters in 252 Pages. The fact that his second Notebook contained also results that were in his first Notebook, it has been the conclusion of several that the second Notebook is a revised, enlarged version of the first Notebook and that the Second Notebook is a fair "copy" of the first Notebook. Being a creative genius in Mathematics, in the attempt of rewriting the Entries from the first into the second Notebook, he added new Entries. For example, Chap. 1 of the first Notebook—which is incidentally the only Chapter for which there is a title: "Magic Squares"—is in three pages but in the second Notebook there are eight pages on Magic Squares.

There is also a Third Notebook, which contains 33 pages of unorganized entries, which are included at the end of the second Notebook, after it is turned through 180°. Though Ramanujan took these Notebooks with him to Cambridge, he wrote in one of his letters to a friend that he had no time to look into them and most probably he did not put them to use during his five-year stay abroad. This is probably due to the fact he was discovering more results on his own and wanted to perhaps impress Hardy about his new discoveries made during his stay there.

The Preface to the *Collected Papers of Srinivasa Ramanujan* contains the following appraisal of the work in the Notebooks: *There is still a large mass of unpublished material. None of the contents of Ramanujan's notebooks has been printed, unless incorporated in later papers, except that one chapter, on generalized hypergeometric series, was analyzed by Hardy.*[5] *in the Proceedings of the Cambridge Philosophical Society. This chapter is sufficient to show that, while the notebooks are naturally unequal in quality, they contain much which should*

[5]G.H. Hardy, A chapter from Ramanujan's notebook, *Proc. Camb. Phi. Soc.* ***21*** *(1923) 492–503.*

certainly be published. It would be a very formidable task to work through them systematically, select particular passages, and edit these with adequate comment, and it is impossible to print the notebooks as they stand without further monetary assistance. The singular quality of Ramanujan's work and the romance which surrounds his career encourage us to hope that this volume may enjoy sufficient success to make possible the publication of another.

Hardy tried to persuade the University of Madras to undertake such a task, and in turn, Professor G.N. Watson was requested by the Madras University, in 1931, to edit the Notebooks in a suitable form for publication. This was a formidable task since the Notebooks contained about 4000 theorems.

Ramanujan was aware of the fact that his mother Komalathammal could recite 1000 of the 4000 verses in *Naalaayira Divya Prabhandam (four thousand divine verses).* Carr's *Synopsis* is another instance of a compendium of more than 4000 theorems and it is not out of place to conjecture that these influenced him to think that a compilation of such a magnitude would result in recognition and a place in the history of mathematics. This would have made him set his goal to write down the 3254 Entries in his Notebooks. Of course, the results which were coming to him from his subconscious mind, as an incessant torrent, might not have afforded him the time to idly think of "how many have I discovered today"; or, to reckon and keep a count of the same. Such activities are for the mundane and for some (not all) of the worldly wise scientists of today, who are constantly in search of promotions and recognitions on the basis of their publications!

Professor G.N. Watson gave a lecture on "Ramanujan's Notebooks", published later in the Journal of London Mathematical Society, Volume **6**, (pages 137–153), in 1931, and published several papers also between 1928 and 1936, motivated and inspired by Ramanujan's mathematical works. Watson's Presidential Address to the London Mathematical Society, in 1935, was on *mock-theta functions*, discovered by Ramanujan during the period when he was terminally ill—after his return to India in March 1919 and his death on April 26, 1920.

Watson, entrusted with the job by Hardy, undertook the task of editing the Notebooks with Professor B.M. Wilson and *they focused their attention on the Second Notebook. Chapters 2–13 were to be edited by Wilson, and Watson was to examine Chapters 14–21. Unfortunately, Wilson passed away prematurely in 1935 at the age of 38*. This is pointed out to us by Bruce Berndt in *Ramanujan's Notebooks*, Part I, 1985, Page 5. The untimely death of Wilson possibly put an end to the joint effort of Watson with Wilson to edit the Notebooks of Ramanujan and had to await the work to be taken up in right earnest by Bruce Berndt in early 1970s.

In 1957, with monetary assistance from Sir Dadabai Naoroji Trust, at the instance of Professor Homi Jehangir Bhabha and Professor K. Chandrasekaran of the Tata Institute of Fundamental Research, the Tata Institute published a facsimile edition of the Notebooks of Ramanujan in two large (A4 sized) volumes. The reproduction is thoroughly faithful to the original. About a thousand copies were made then and in the Ramanujan Birth Centenary Year, 1987, Springer-Verlag and Narosa Publishing House reprinted the same.

The formidable task of truly editing the Notebooks—viz. to either prove each of the results or provide references to the literature where the proofs may be found—which was started by G.N. Watson and B.M. Wilson was but never completed, until Bruce Berndt took it up. Professor Bruce C. Berndt has been publishing a series of papers, since 1981, by himself or in collaboration with his students and others including Indian Mathematicians S. Bhargava, C. Adiga and Padmini T. Joshi. He has also utilized the earlier work of G.N. Watson and B.M. Wilson on the Notebooks with the Trinity College, in Cambridge. This dedicated work of Berndt, published by Springer- Verlag, as *Ramanujan's Notebooks*, is now available in five parts, the first of which appeared in 1985 and the fifth in 1997.

At the beginning of his work, in Part I, Berndt points out: *The notebooks were originally intended primarily for Ramanujan's own personal use and not for publication … Some of Ramanujan's incorrect 'theorems' in number theory found in his letters to Hardy have been publicised. Thus, perhaps, some think that Ramanujan was prone to making errors. However, such thinking is erroneous. The notebooks contain scattered errors. Especially if one takes into account the roughly hewn nature of the material and his frequently formal arguments, Ramanujan's accuracy is amazing*.

3.5 Resurgence of Interest

After Ramanujan returned to India, though he was seriously ill and bedridden for most of the time, he was working continuously. His lifelong habit had been to work on a slate and transfer the final results to the notebook or sheets of paper. His brother-in-law (as pointed out by R. Srinivasa Iyengar, in Volume 1, of *Ramanujan: Letters and Reminiscences*, Edited by P.K. Srinivasan) states that *a vast quantity of papers containing his notes was handed over to the University after his death*. Three days after Ramanujan died, his bother S. Lakshminarasimhan, in a letter to Professor Hardy, dated 29 April 1920, informing Hardy about his demise, wrote: *All his MSS that were in his trunk were handed over on the day of his death to Mr. Ramachandra Rao's son in law, since the former is at Nilgiris. Not only those but also the journals, magazines – all he possessed except the books which he had, were taken away by them*.

In a letter to Hardy, dated 3 December 1920, Mr. R. Ramachandra Rao wrote: *Ramanujan's M.S.S. whatever they are are with me and will be handed over intact to the University of Madras who I understand is already in correspondence with you regarding the methods of publication*. These papers reached Professor Hardy, who should have passed them on to Professor G.N. Watson. Hence their final destination—viz. the estate of Professor Watson, in the underground cellar of Trinity College, Cambridge. After he wrote several papers inspired by Ramanujan's results, in 1976, Professor George E. Andrews of the Pennsylvania State University, State College, Pennsylvania, in USA, discovered the 'Lost' Notebook.

Professor Watson died in February 1965. A decade later, in the Spring of 1976, when Professor George Andrews was going through the estate of Watson, he discovered a box of paper and he wrote: *The most interesting item in this box was a manuscript of more than one hundred pages in Ramanujan's distinctive handwriting which contains over six hundred mathematical formulae listed one after the other without proof. It is my contention that this manuscript, or notebook, was written during the last year of Ramanujan's life after his return to India from England. My evidence for this assertion is all indirect; in the words of Stephen Leacock, 'It is what we call circumstantial evidence – the same that people are hanged for.'*

These are now referred to as the contents of the "Lost" Notebook of Ramanujan. Andrews assessed—in *Srinivasa Ramanujan: The Lost Notebook and Other Unpublished Papers*, with an introduction by George E. Andrews (published by Narosa Publishing House, New Delhi (1987)): "*While it is impossible to categorize the various formulas completely, a rough approximation of its contents is the following:*

q-series and related topics including mock θ-functions: 60%
Modular equations and relations, singular moduli: 30%

Integrals, Dirichlet series, congruences, asymptotics, misc.: 10% I give only this rough break down because of the chaotic nature of this manuscript. On many pages there are fragments of numerical computations and infinite series running off in all directions. It may be possible eventually to make sense out of some of these pages; if so, the above percentage breakdown may change. . . . In any event it is clear that q-series investigations make up the bulk of the work in the Lost Notebook. Indeed I count about 380 formulae that I consider to belong more to the q-series than to modular relations or other topics.

Fifty-six years after the death of Ramanujan, due to the discovery of the "lost" Notebook by George Andrews, in 1976, and the editing of the three Notebooks of Ramanujan by Professor Bruce Berndt, there has been a resurgence of interest in the work of Ramanujan. These Notebooks of Ramanujan have formed the basis of numerous papers by many mathematicians, who gave proofs of the theorems and conjectures of Ramanujan obtained by him through his intuition and sheer brilliance. This is a singular, unique and unparalleled phenomenon in the annals of mathematics.

In the centenary year of his birth, 1987, several conferences were held in many countries to focus the attention on his mathematical work and the recent developments there from. Mystery still surrounds *some of his work mostly in the theory of elliptic functions*, wherein *it is not possible, after all the work of Watson and Mordell, to draw the line between what he may have picked up and what he must have found for himself*, points out Hardy on p. 10 of his *Ramanujan: Twelve Lectures inspired by his life and work* book.

Hardy regretted that he could easily have but did not find out all the details from Ramanujan himself since Ramanujan *was quite able and willing to give a straight answer to a straight question, and not in the least disposed to make a mystery of his achievements.*. However, Hardy argues (p. 11 of his *Ramanujan: Twelve Lectures inspired by his life and work*) that Ramanujan *was not particularly interested in*

his own history or psychology; he was a mathematician anxious to get on with the job. And after all I too was a mathematician, and a mathematician meeting Ramanujan had more interesting things to think about than historical research. It seemed ridiculous to worry him about how he found this or that known theorem, when he was showing me half a dozen new ones almost everyday.

3.6 A Page from the Second Notebook of Ramanujan

To provide a feeling for Ramanujan's Notebooks, reproduced here is a page from Chapter XVIII of his second Notebook—actually for the sake of continuity and relevance, the last three lines of pp. 224 and 225 are put together in this page. There is no particular reason for choosing this page except that it contains an entry which has become a "folklore" which illustrates the attachment Ramanujan had to numbers which prompted Littlewood to state that to Ramanujan *every number is a personal friend*.

There are two entirely distinct parts of Ramanujan's interests in mathematics juxtaposed and noted down by him on this page. The first part concerns the geometrical construction of a "square whose area is equal to that of a given circle". Circle squaring has been of interest from early times and dates back to ancient times. It is the Greeks who set up the circle-squaring problem with the following two conditions:[6] the solution should use only a straight edge and compass, so that the proofs can be reduced to Euclid's theorems and the solution must use only a finite number of steps. If, in fact, $\pi r^2 = a^2$, where r is the radius of the circle and a is the length of the side of the square, then it follows that $\pi = (a/r)^2$, or that π is rational. In fact, π has been shown to be not only an *irrational number* but also proved by Ferdinand von Lindemann, in 1882, to be a *transcendental number*. Furthermore, the phrase "squaring the circle" in common usage suggests a project doomed to failure! Hardy commented that he found in Ramanujan's collection of books in Cambridge, some books by "quacks" on this theme.

Ramanujan published a paper entitled, "*Squaring the circle*", in the Journal of the Indian Mathematical Society, in 1913, in which he gives a geometrical construction for finding the length of the side of a square whose area equals that of the circle. He also reproduced this and another geometrical construction for π in his latter paper on *Modular equations and approximations to* π. In this paper, he deduced several formulae like:

$$\pi = \frac{63}{25}\frac{17 + 15\sqrt{5}}{7 + 15\sqrt{5}} = 3.14159265380\ldots$$

[6]See, *The Joy of* π, David Blattner, Penguin Press, 1997 - a delightful book, the first edition costing 6.99 Pounds, in 1997.

$$\pi = \frac{24}{\sqrt{142}} \log \left\{ \sqrt{\left(\frac{10 + 11\sqrt{2}}{4} \right)} + \sqrt{\left(\frac{10 + 7\sqrt{2}}{4} \right)} \right\}$$

formulae correct to 9, 16, 18 decimal places; and

$$\frac{1}{\pi} = 2\sqrt{2} \left\{ \frac{1103}{99^2} + \frac{27493}{99^6} \cdot \frac{1}{2} \cdot \frac{1 \cdot 3}{4^2} + \frac{53883}{99^{10}} \cdot \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1 \cdot 3 \cdot 5 \cdot 7}{4.8^2} + \cdots \right\}$$

$$= 2\sqrt{2} \sum_{n=0}^{\infty} \frac{(\frac{1}{4})_n (\frac{1}{2})_n (\frac{3}{4})_n}{(1)_n (1)_n (1)_n} (1103 + 26390\, n) \left(\frac{1}{99} \right)^{4n+2}$$

which he asserted would be "*rapidly convergent*", for the first term itself gives the sum to 8 decimal places. In 1986, two computer scientists, Borwein and Borwein, in their book "Pi and the AGM" (Canadian Mathematical Society, John Wiley (1987)) used a version of Ramanujan's formula to calculate π to 17 million places and found that the formula of Ramanujan converges on the exact value with far greater efficiency than any previous method.[7] This success of Ramanujan's formula being found correct with the help of a fast digital computer by the Borwein brothers of Canada proved that Ramanujan's insight which made him state that a particular infinite series formula, one of 19 he had discovered, for $1/\pi$ would be *rapidly convergent* was vindicated.

The other Entries on this page 225, in Chapter XVIII of Ramanujan's second Notebook are solutions of the Diophantine equations:

$$X^3 + Y^3 = U^2$$
$$X^3 + Y^3 + Z^3 = U^3 \quad (Euler's\ equation)$$

followed by numerical examples for these equations—6 solutions for the first and 12 solutions for the Euler's equation and one which looks like but which is not a solution of the Euler's equation. Characteristically, Ramanujan does not give the values of the parameters for the examples. The interested reader may however refer to Part III of Bruce Berndt's *Ramanujan's Notebooks* p. 199 for the examples with the values of the parameters appended.

[7] David and Gregory Chudnovsky are two mathematicians who hold several world records for calculating the highest number of the digits of π: first up to 450 million digits, then up to 1 billion digits and then to 2 billion digits. The current record is over 51 billion digits, stated in David Blatner's classic small book entitled, *The Joy of* π, The Penguin Press (1997).

Ramanujan's solution of Euler's equation in rational numbers given here is:

$$\begin{aligned} X &= m^7 - 3m^4(1+p) + m(2+6p+3p^2) \\ Y &= 2m^6 - 3m^3(1+2p) + 1 + 3p + 3p^2) \\ Z &= m^6 - 1 - 3p - 3p^2 \\ U &= m^7 - 3m^4(1+p) + m(3p^2 - 1) \end{aligned}$$

where m and p denote arbitrary numbers. He has also given another solution to this equation elsewhere (see Question 441 which follows), but neither of these two parameter solutions is the general solution[8] which contains three parameters, prompting Hardy (in p. 11 of *Ramanujan: Twelve Lectures on subjects suggested by his life and work*) to comment that *Diophantine equations should have suited him, but he did comparatively little with them, and what he did was not his best.*

The second of the 12 examples given for the Euler equation on this page

$$1^3 + 12^3 = 9^3 + 10^3 \qquad (1729)$$

corresponds to ($m = 2, p = 3$). This has been made famous by the following **taxi cab number episode** associated with it and made known to us by Hardy:[9] *He could remember the idiosyncracies of numbers in an almost uncanny way. It was Littlewood who said that every positive integer was one of Ramanujan's personal friends. I remember going to see him once when he was lying ill in Putney. I had ridden in taxi-cab number 1729, and remarked that the number seemed to me rather a dull one, and that I hoped that it was not an unfortunate one, and that I hoped that it was not an unfortunate omen. 'No', he replied, 'it is a very interesting number, it is the smallest number expressible as a sum of two cubes in two different ways'. I asked him, naturally, whether he could tell me the solution of the corresponding problem for fourth powers; and he replied, after a moment's thought, that he knew no obvious example, and supposed that he knew no obvious example, and supposed that the first such number must be very large.*

The simplest known solution of the Diophantine equation:

$$x^4 + y^4 = z^4 + t^4$$

is Euler's: $59^4 + 158^4 = 133^4 + 134^4 = 635318657$. Euler gave a solution involving two parameters, but no "general" solution is known.

The anecdote reveals Ramanujan's remarkable feeling for numbers and his sharp memory which made him recall one Entry out of several thousands he made for himself in his Notebooks just like that and the fact that he had not recorded in his

[8]The last of the examples $1^3 + 6^3 + 8^3 = 9^3$ does not belong to this class of solutions, since no values of m and p yield this solution to the Euler's equation. So, its presence reveals that perhaps, Ramanujan was aware that his solution was not the most general for Euler's equation.

[9]G.H. Hardy: *Ramanujan: Twelve Lectures on subjects suggested by his life and work.*

Notebooks the observation made about the Taxi cab number 1729 which came from him spontaneously when Professor Hardy made the innocuous observation about the number.

Though the number 1729 itself finds no explicit mention in the Notebooks, the Euler equation

$$x^3 + y^3 + z^3 = u^3$$

and its solution:

$$1^3 + 12^3 = 9^3 + 10^3,$$

finds mention in three Questions.[10] These are detailed below:

- **Question 441**, in the J. Indian Math. Soc. Vol. V, p. 39: Shew that

  $$(6a^2-4ab+4b^2)^3 = (3a^2+5ab-5b^2)^3+(4a^2-4ab+6b^2)^3+(5a^2-5ab-3b^2)^3$$

  and find other quadratic expressions satisfying similar relations. [The *Solution* given by S. Narayanan, J. Indian Math. Soc. vol. VI, p. 226.]
- **Question 661**, in the J. Indian Math. Soc. Vol. VII, p. 119: Solve in integers

  $$x^3 + y^3 + z^3 = u^6,$$

  and deduce the following:

  $$6^3 - 5^3 - 3^3 = 2^6, \qquad 8^3 + 6^3 + 1^3 = 3^6,$$
  $$12^3 - 10^3 + 1^3 = 3^6, \qquad 46^3 - 37^3 - 3^3 = 6^6,$$
  $$174^3 + 133^3 - 45^3 = 14^6, \quad 1188^3 - 509^3 - 3^3 = 34^6.$$

  [*Solutions* are by N.B. Mitra, in the J. Indian Math. Soc. Vol. XIII, p. 15–17. Additional solution and remarks by N.B. Mitra, XIV, 73–77.]
- **Question 681**, in the J. Indian Math. Soc. Vol. VII, p. 160: Solve in integers

  $$x^3 + y^3 + z^3 = 1,$$

  and deduce the following:

[10] I am thankful to (late) P.K. Srinivasan for drawing my attention to these and his publications: *An Introduction to Creativity of Ramanujan*, an instructional guide to mathematics teachers in Primary Schools, Middle Schools and, with T. Dharmarajan, for High Schools, published by the Association of Mathematics Teachers of India (AMTI), Madras, in collaboration with Triveni Academy, Madras, in the Ramanujan Birth Centenary Year 1987.

$$6^3 + 8^3 = 9^3 - 1, \qquad 9^3 + 10^3 = 12^3 + 1,$$
$$135^3 + 138^3 = 172^3 - 1, \qquad 791^3 + 812^3 = 1010^3 - 1,$$
$$11161^3 + 11468^3 = 14258^3, +1 \; 65601^3 + 67402^3 = 83802^3 + 1.$$

[*Partial Solution* by N.B. Mitra, in the Journal of the Indian Mathematical Society, Vol. XIII, p. 17. See also N.B. Mitra, XIV, 73–77 (76–77).]

- If p, q, r are quantities so taken that

$$p + 3a^2 = q + 3ab = r + 3b^2 = (a + b)^2$$

and m, n are any two quantities, then

$$n(mp + nq)^3 + m(mq + nr)^3 = m(np + mq)^3 + n(nq + mr)^3.$$

A particular case of the above theorem is:

$$(3a^2+5ab-5b^2)^3+(4a^2-4ab+6b^2)^3+(5a^2-5ab-3b^2)^3 = (6a^2-4ab+4b^2)^3.$$

(Bruce C. Berndt, *Ramanujan's Notebooks*, Part II, p. 266.)

- If $\alpha^2 + \alpha\beta + \beta^2 = 3\lambda\gamma^2$, then

$$(\alpha + \lambda^2\gamma)^3 + (\lambda\beta + \gamma)^3 = (\lambda\alpha + \gamma)^3 + (\beta + \lambda^2\gamma)^3.$$

(Bruce C. Berndt, *Ramanujan's Notebooks*, Part II, p. 387.)

Note that in the above 4-parameter solution for the Euler's equation, it has been observed[11] that for $\alpha = 3$, $\beta = 0$, $\lambda = 3$, and $\gamma = 1$, we get

$$12^3 + 1^3 = 10^3 + 9^3 = 1729.$$

The fact that Ramanujan could come out spontaneously with the statement that 1729 is **the smallest number which could be expressed as the sum of two cubes in two different ways**, a statement which he had not jotted down in his Notebooks explicitly, is possibly an evidence for the confirmation of the observation of Professor Littlewood that to Ramanujan every integer is a "personal friend".

[11] P.K. Srinivasan, *An Introduction to Creativity of Ramanujan*—An Instructional Guide to Mathematics Teachers in (a) Primary Schools, (b) Middle Schools, (c) High Schools (with T. Dharmarajan), a Publication of the Association of Mathematics Teachers of India (1987).

3.7 Bruce Berndt's Work on the Notebooks

Hardy's statement that two thirds of Srinivasa Ramanujan's work contained in the Notebooks consisted of rediscoveries of known results is refuted by Bruce C. Berndt, who states, after his extensive studies, that Hardy's estimate is too high. Berndt's main goal as stated by him was: "To prove each of Ramanujan's theorems" and for known results refer to literature where proofs may be found. Tables of the contents of Berndt's *Ramanujan's Notebooks*, Part I to Part V, are given below. The titles of the Chapters, provided by Berndt, give the reader an idea about the range and nature of the problems and the number of results Ramanujan had recorded in each Chapter of his Notebooks.

In the Introduction to Part V of "*Ramanujan's Notebooks*", Berndt states: *This volume, however, should be regarded as the closing chapter on Ramanujan's notebooks. Instead, it is just the first milestone on our journey to understanding Ramanujan's ideas. . . . It is our fervent wish that these volumes will serve as springboards for further investigations by mathematicians intrigued by Ramanujan's remarkable ideas. . . . Ramanujan remarked that several of his series arose from alternative theories of elliptic functions . . . The first of three alternative theories is the most interesting and the most important, and we feel that a large body of work remains to be discovered here.*

Ramanujan's work on hypergeometric series, contained in Chapters X and XI of his second Notebook, was edited by Hardy, as stated earlier. Later we will take a closer look at some of the beautiful work of Ramanujan in this area of hypergeometric series, where he discovered for himself all that was known in Europe at that time about the hypergeometric series, including the most general summation theorem known due to Dougall, called the ${}_7F_6(1)$ summation theorem, which Hardy has rechristened as Dougall-Ramanujan summation theorem.

Ramanujan's Notebooks, Part I, Bruce C. Berndt (1985)

Ch.	Subject	# of results
1.	Magic Squares	43
2.	Sums related to the Harmonic Series or the Inverse Trigonometric Function	68
3.	Combinatorial Analysis and Series Inversions	86
4.	Iterates of the Exponential Function and an Ingenious Formal Technique	50
5.	Eulerian Polynomials and Numbers, Bernoulli Numbers and the Riemann Zeta-Function	94
6.	Ramanujan's Theory of Divergent Series	61
7.	Sums of Powers, Bernoulli Numbers and the Γ Function	110
8.	Analogues of the Gamma Function	108
9.	Infinite Series Identities, Transformations, and Evaluations Ramanujan's Quarterly Reports	139

Ramanujan's Notebooks, Part II, Bruce C. Berndt (1989)

Ch.	Subject	# of results
10.	Hypergeometric Series I	116
11.	Hypergeometric Series II	103
12.	Continued Fractions	113
13.	Integrals and Asymptotic Expansions	92
14.	Infinite Series	87
15.	Asymptotic Expansions Modular Forms	94

Ramanujan's Notebooks, Part III, Bruce C. Berndt (1991)

Ch.	Subject	# of results
16.	q- Series and Theta Functions	134
17.	Fundamental Properties of Elliptic Function	162
18.	The Jacobi Elliptic Functions	135
19.	Modular Equations of Degree 3, 5 and 7 and Associated Theta Function Identities	185
20.	Modular Equations of Higher and Composite Degrees	173
21.	Eisenstein Series	45

The 100 pages of unorganized material after Chapter 21 of Srinivasa Ramanujan's second Notebook and the 33 pages of his third Notebook were organized into Chapters 22 to 31 by Berndt.

In the concluding Part V, Berndt examines the unorganized pages in all the three Notebooks of Ramanujan on continued fractions, alternative theories of elliptic functions, class invariants and singular moduli, explicit values of theta functions, modular equations, infinite series, approximations and asymptotic expansions, etc. Berndt points out that *very few claims in this volume are pertaining to Ramanujan's published papers and problems.*

Ramanujan's Notebooks, Part IV, Bruce C. Berndt (1994)

Ch.	Subject	# of Results
22.	Elementary Results	47
23.	Number Theory	108
24.	Theory of Prime Numbers	24
25.	Theta Function and Modular Equations	86
26.	Inversion Formulas for Lemniscate and other functions	10
27.	q Series	9
28.	Integrals	63
29.	Special Functions	139
30.	Partial Fraction Expansions	15
31.	Elementary and miscellaneous analysis	36
	16 Chapters of the First Notebook	54

Ramanujan's Notebooks, Part V, Bruce C. Berndt (1997)

Ch.	Subject	# of Results
32.	Continued Fractions	73
33.	Ramanujan's Theories of Elliptic Functions to Alternative Bases	62
34.	Class Invariants and Singular Moduli	196
35.	Values of Theta-Functions	24
36.	Modular Equations and Theta-Function Identities in Notebook 1	87
37.	Infinite Series	53
38.	Approximations and Asymptotic Expansions	46
39.	Miscellaneous Results in the First Notebook	24

Many theorems communicated by Ramanujan to Hardy in his January 16, 1913, and February 27, 1913, letters are found in Chapters 22–31. In all, $759 + 605 + 834 + 491 + 565 = 3254$ theorems have been studied in the Parts I to V by Professor Berndt. Hardy had estimated that the Notebooks of Ramanujan contained approximately 3000–4000 statements of theorems, and besides editing one Chapter of Ramanujan's second Notebook, in 1923, he tried to get them edited by other mathematicians. His intentions have at long last been fulfilled by the tireless efforts of the indefatigable Bruce C. Berndt[12] of the University of Illinois at Champagne-Urbana and his students.

It is the author's opinion that as long people do mathematics, the work of Ramanujan and the stupendous effort of Bruce Berndt in editing the Notebooks will be appreciated. (The interested reader may refer to the article *Ramanujan Papers*, by K. Srinivasa Rao, Maths Student, (1997).)

3.8 Ratan P. Agarwal's Work on the Notebooks

Professor Ratan P. Agarwal is the founder of a school of ordinary and basic hypergeometric series in India. It is he who inspired the author to switch his field of interest from Theoretical Nuclear Physics to Special Functions. In recent times, Professor R.P. Agarwal consolidated a part of his work in a three volume work entitled: *Resonance of Ramanujan's Mathematics*. The present author has provided a review of the same in the Book Reviews section of Current Science journal, Volume **73**, No. 12, (1997) pages 1124–1125. This work concerns some of the Entries in the Notebooks of Srinivasa Ramanujan.

[12]The author wishes to acknowledge with thanks Professor Bruce Berndt for readily communicating the Preface and the Contents of Part V, by e-mail, on request.

Volume I of R.P. Agarwal's work has five chapters. Chapter 1 concerns certain summation and asymptotic formulae for generalized, ordinary and basic hypergeometric series. Chapter 2 is on Rogers-Ramanujan identities. Chapters 3–5 deal with certain definite Integrals of interest to Ramanujan.

Professor R.P. Agarwal opines, like Hardy, that *a major part of Ramanujan's systematic work on ordinary hypergeometric series has been in the nature of a rediscovery of the work already done by previous authors*—the ${}_7F_6(1)$ summation formula of Dougall (1907) and particular cases of this sum, such as, the Pfaff (1797)-Saalschütz (1890) theorem for the ${}_3F_2(1)$ and the Gauss and Kummer summation theorems for the ${}_2F_1(1)$ and ${}_2F_1(-1)$, respectively, and Dixon's summation theorem for a well-poised ${}_3F_2(1)$. These and Ramanujan's Entries in his Notebooks concerning the deep results on the hypergeometric series, which centre around the behaviour of the logarithmic solutions of the hypergeometric differential equation, are presented in Chap. 1.

Chapter 2 deals with the derivations of the celebrated Rogers-Ramanujan identities, which we have already referred to while providing glimpses into Ramanujan's work, their combinatorial interpretations, their generalizations and their applications in Statistical Mechanics for the solution of the two-dimensional Hard Hexagon Model, by Professor R.J.Baxter. Several proofs of these identities and the interesting mathematical developments have been described very well in R.P. Agarwal's cogently written chapter.

The next three chapters of Volume I deal with some of the integrals studied by Ramanujan. Professor Agarwal presents definite integrals associated with Fourier transforms in the Second Notebook and self-reciprocal functions given in Ramanujan's "lost" Notebook in Chap. 3. He has shown how some of Ramanujan's results on integrals in his Notebook can be established with the help of other Entries of his and *that every entry given in the notebooks by him has a purpose, although sometimes a particular entry may look out of context at the first instance. . . . We have advocated elsewhere also that it may be interesting many a time to try to prove 'unproven' entry of Ramanujan with the help of some other entry given by him earlier or later on.*

Chapter 4 is focused on Ramanujan's Master Theorem for the evaluation of definite integrals, viz.:

$$\int_0^\infty x^{n-1}\, F(x)\, dx = \Gamma(n)\phi(-n), \qquad \text{where} F(x) = \sum_{k=0}^{\infty} \frac{\phi(k)(-x)^k}{k!},$$

in some neighbourhood of $x = 0$ and for n not necessarily a positive integer. The three Quarterly Reports submitted by Ramanujan to the Madras University, as a requirement stipulated by the University for awarding him the *First Research Scholarship in Mathematics*, contain various applications of this theorem to the *evaluation of a huge variety of integrals* and expansion formulae. The appropriate conditions of convergence for its validity have been provided by Hardy in his lectures contained in Hardy's *Ramanujan: Twelve Lectures on subjects suggested*

by his life and work. The q- extensions of these works by several mathematicians including Thomae (1879), F.H. Jackson (1904), R. A. Askey (1978), G.E. Andrews and R. Askey (1981), Arun Verma and V.K. Jain (1992) are also included in this Chapter l.

Chapter 5 is devoted to the study of integrals of the type:

$$\int_{-\infty}^{\infty} \frac{e^{at^2+bt}}{e^{ct}+d}\,dt$$

for particular values of a, b, c, d evaluated by Ramanujan. Mordell (1933) classified these according to the values of the parameters and evaluated them in terms of Jacobi's theta and other related functions, using the methods of complex contour integration. Professor R.P. Agarwal presents the general method of evaluation of different standard forms of this integral and relates them to those studied by Ramanujan, who used transform calculus to evaluate his integrals. Integrals associated with Riemann's zeta functions and elliptic modular relations are not discussed, as pointed out by the author himself. The Chapter concludes with the mention of the example of a definite integral with fractional derivatives in the integrand stated in the third Quarterly Report of Ramanujan. Ramanujan's formal deduction is based on his Master Theorem, and it agrees with the classical Liouville's definition of a fractional derivative of a function.

In Volume II, Professor R.P. Agarwal presents a critical appraisal of Ramanujan's extensive and intriguing work on elliptic functions and results pertaining to theta functions, partial theta functions, "mock" theta functions, discovered by Ramanujan during the last year of his life after his return to India, as well as Lambert series and their relationship with elliptic functions. While theta functions and partial theta functions are covered in Chap. 1 of this volume and Lambert series and related functions are dealt with in Chap. 5, the major part of this volume is concerned with the "mock" theta function results of Ramanujan, presented in three extensive Chaps. 2–4, of the "Lost" Notebook, which was made available to the world in a facsimile edition, by Narosa Publishing House, for the first time only in the centenary year of Ramanujan's birth, 1987, though it was discovered in the estate of G.N. Watson by Professor George E. Andrews of the Pennsylvania State University, in the Spring of 1976.

After giving the definitions of the four Jacobi functions and the "false" Theta functions (viz. theta functions with the "wrong" signs for the terms) defined by L.J. Rogers, Professor R.P. Agarwal introduced the notation for the theta function in terms of Ramanujan's symmetric function:

$$f(a,b)=\sum_{k=-\infty}^{\infty}(ab)^{k(k-1)/2}(a^k+b^k), \qquad |ab|<1.$$

In terms of this symmetric function, Ramanujan defines:

$$f(qe^{2iz}, qe^{-2iz}) = \theta_3(z,q), \quad \text{where} \quad q=e^{i\pi\tau} \text{ and } \quad |q|<1.$$

Several general properties of the function $f(a, b)$ are then presented. Jacobi's triple product identity and other such identities due to D. Hickerson (1988) and G.E. Andrews and D. Hickerson (1991) are derived explicitly, due to their importance to simplify calculations in later chapters on "mock" theta functions. A number of theta function expansions in the "lost" Notebook pertain to "partial" theta functions. Some of these are stated, and G.E. Andrews (1981) derived a general basic hypergeometric identity and showed the results of Ramanujan as special cases of it.

Ramanujan defined four third-order mock theta functions and to this set three more were added by G.N. Watson. Watson used the transformations of the basic hypergeometric series for obtaining new definitions for all the seven functions. Agarwal himself has contributed significantly to the theory of "mock" theta functions by showing that most of the general identities of Andrews (referred above) belong to a very general class of basic hypergeometric transformations. His results are presented and those of Andrews deduced from them. Furthermore, Agarwal defines the third-order mock theta functions in terms of a basic ${}_2\Phi_1$ basic hypergeometric series, and deduces many of the mock theta function properties from the well-known properties of the ${}_2\Phi_1$. Ramanujan gave ten mock theta functions of order five, in two groups of five each, and three functions of order seven. He asserted that the members of each of the two groups of mock theta functions of order five are related among those belonging to the same group only, while the three mock theta functions of order seven are not related to each other. The works of G.E. Andrews, R.P. Agarwal and M. Gupta on the mock theta functions of order five are presented in detail. A. Gupta and R.P. Agarwal showed that mock theta functions of order five and seven are defined through basic hypergeometric series of the type ${}_3\Phi_2$ and ${}_4\Phi_3$. They have also made attempts to develop the theory of the above types of basic hypergeometric series to obtain new transformations and definitions for certain mock theta functions of orders five and seven. It is pointed out that the absence of a general transformation theory for the ${}_3\Phi_2(a, b, c; e, f; z)$ series [except when the argument z is itself q the base parameter] or ef/abc] is the main reason why there exists no general transformation theory for mock theta functions of order five.

Observations of R.P. Agarwal on the relationship between the "mock" theta functions and the basic series lead him to define: *the order of a mock theta function as* $(2r + 1)$ *if it is expressible in terms of a* ${}_{r+1}\Phi_r$ *Series, on a single base* q^k, $q < r + 1$. *There may be in the definition of the Mock theta function an additive term with* ${}_{r+q}\Phi_r$ *consisting of* θ*-products, which do not affect the order.*

While this is an interesting observation by itself, it is simplistic, to say the least. For, mystery still surrounds the incomplete work of Ramanujan in this area, contributing to the belief that Ramanujan was working on a general comprehensive theory of mock theta functions, whose preliminary results are the ones in the "Lost" Notebook of Ramanujan, and that he was snatched away by fate before he could reveal his grand scheme!

This two volume work of Professor R.P. Agarwal is a deep study of some of the work of Ramanujan on ordinary and basic hypergeometric series, on definite integrals and theta, partial theta and mock theta functions. It is very original in parts

and has a minimum overlap with the comprehensive five part work on the Notebooks of Ramanujan by Bruce C. Berndt.[13] This work presents, in as coherent a way as is possible, the work of Agarwal and his school of students and co-workers. It is a very valuable supplement for the research scholar and for mathematicians and presents the author's understanding of the "how" and "why" of what Ramanujan did.

3.9 Dyson on Ramanujan's Notebooks

Freeman J. Dyson, the renowned theoretical physicist, in *Ramanujan Revisited*[14] concludes his article entitled: *A walk through Ramanujan's Garden*, with the following sagacious advice:
In conclusion, I would like to urge all of you who are working in the many fields of mathematics which have been enriched by Ramanujan's ideas to go back to the source: the Collected Papers and the Notebooks. ⋯ The Notebooks ⋯ are now appearing in a splendidly annotated version edited by Bruce C. Berndt. The 'Lost' Notebook is now accessible to us through the devoted labours of George Andrews. When I started my walk through Ramanujan's Garden 47 years ago, only the Collected Papers were available. A year after I chose Hardy and Wright's 'Theory of Numbers' (Oxford, Clarendon Press, 1938) as a school prize, I won another prize. For the second prize I chose Ramanujan's Collected Papers. The Collected Papers have travelled with me from England to America and are still as fresh today as they were in 1940. Whenever I am angry or depressed, I pull down the Collected Papers from the shelf and take a quiet stroll in Ramanujan's garden. I recommend this therapy to all of you who suffer from headaches or jangled nerves. And Ramanujan's papers are not only a good therapy for headaches. They also are full of beautiful ideas which may help you to do more interesting mathematics.

Certainly several non-mathematician readers will not find this therapy useful. Freeman Dyson, Paul Erdös and Atle Selberg being the very best in their chosen fields, they were greatly benefited by an early introduction to the *Collected Papers of Srinivasa Ramanujan*. Most mathematicians can derive benefit of the extensive study of Ramanujan's work by Hardy in his books: *Ramanujan: Twelve Lectures on subjects suggested by his life and work* and Bruce Berndt's *Ramanujan's Notebooks*, Part I to Part V, to begin appreciating the prolific, original work of the "natural" mathematical genius and see also the original Notebooks of Ramanujan, written in his inimitable beautiful, neat handwriting, a facsimile edition of which was published by the Tata Institute of Fundamental Research, Mumbai, in 1957.

[13] R.P. Agarwal's reference to these Parts I to V as the Berndt Notebooks is unfortunate, especially in view of the significance of the *Ramanujan Notebooks* and the "*Lost*" *Notebook*, which are the source materials for both Bruce C. Berndt and R.P. Agarwal.

[14] Ed. by George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin, Academic Press, Inc. (1988) p.25.

3.10 CD ROMs on the Life and Work of Ramanujan

Two CD ROMs on "The Life and Work of Srinivasa Ramanujan" have been brought out with the present author providing the design, the script and the photographs and visuals. They were produced by the National Multimedia Resource Center (NMRC), Pune. This was a two and a half year Project funded by the Department of Science and Technology, Government of India, when Dr. V. S. Ramamurthy was the Secretary, DST, between December 2002 and June 2005. The same was released by Dr. Ramamurthy on December 22, 2005, at New Delhi.[15]

A Pilot CD ROM Project, executed with the help of the K-12 group[16] of the National Institute of Information Technology, New Delhi, was also exhibited at the Indian Science Congress Exhibition, ISCE '99. The enthusiastic and whole-hearted cooperation of Mr. P.K. Srinivasan and Mr. A.T.B. Bose (Secretary) of the Avvai Kalai Kazhagam (Avvai Academy) in Royapuram, as resource persons and volunteers to the National Children's Science Congress (NCSE 98), and the Indian Science Congress Exhibition (ISCE 99) contributed to the success of the whole project. Visitors to the "πie Pavilion", Pilot CD on Ramanujan and the Ramanujan Museum (Replica) which were juxtaposed to each other, in one Hall, included: Dr. R. Chidambaram, Dr. A.P.J. Abdul Kalam, Dr. V.S. Ramamurthy, heads of Institute of Mathematical Sciences and the Ramanujan Institute of the University of Madras and the Vice Chancellors of the Madras University and Anna University. The DAE Chairman, Dr. R. Chidambaram suggested to the author that the project may be replicated at the Nehru Museum of the Tata Institute of Fundamental research, Mumbai. Dr. A.P.J. Abdul Kalam, of the Central Ministry wanted the exhibition to be on wheels, as a Rath Yatra, and taken to all parts of the country, especially rural areas.

The "Pie Pavilion and the Ramanujan Museum (Replica)" were fused into a Ramanujan Gallery, by Dr. K. Srinivasa Rao, after the ICSE 99 and the Gallery was inaugurated by Ms. C.K. Gariyali, I.A.S., Vice Chairperson, Science City, and it was dedicated to the public at the Periyar Science and Technology Center, in Kotturpuram, Chennai, on Science Day, February 28, 1999. The Department of Science and Technology further provided the required funds to create a Ramanujan Photo Gallery by the PSTC. Work on this Gallery, which consists of 110 laminated photographs pertaining to the life and work of Ramanujan, was completed by the author with the help of Mr. A.T.B. Bose and opened to the public from the end of June 1999.

Thanks to the encouragement from the authorities of DST, Dr. R. Chidambaram and Dr. V.S. Ramamurthy, in particular, and the enthusiastic support of the Executive Director of the Periyar Science and Technology Center (PSTC) in Kotturpuram

[15] In a meeting arranged by the Jamia Millia Islamia University, for which the author wishes to express his thanks again here.

[16] The help of Dr. Gaurav Bhatnagar, then with the NIIT is gratefully acknowledged for this pilot CD Rom Project on Ramanujan.

which houses the Birla Planetarium and the offices of the Science City also in a spacious campus, the Pie Pavilion and the Ramanujan Photo Gallery are being expanded into a full-fledged Ramanujan Maths Museum for which a 2000 sq. ft. hall has been ear marked. It also houses some of the mathematical hands-on equipment to enable the students/children to interact with the mathematical models on display and derive greater enjoyment. The PSTC attracts the attention of students from schools and visitors and is open on all days of the year, except on January 26, India's Republic Day; August 15, the Independence Day, and October 2, the Father of the Nation, Mahatma Gandhi's Birth Day.

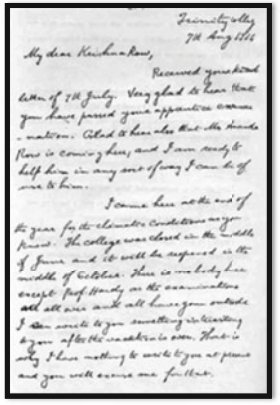
Dr. V. S. Ramamurthy, the Secretary Department of Science and Technology, Government of India, supported the idea of a CD ROM project proposed by the author, as the principal Investigator, and provided adequate funds, Rs. 20 lakhs in all, for a two-and-a-half-year Project at the Institute of Mathematical Sciences. With the help of a software engineer and a postgraduate student in Mathematics, as assistants, the contents provided by the author, with all the figures and visuals prepared for the purpose, the Project was executed successfully, in 2005. The Project was presented to a committee including Dr. Ramamurthy, and Dr. B.D. Acharya, Head of the Mathematical Sciences Division of DST, and the final approved copies (2000) were provided to the DST, by the author, for dissemination by the Vigyan Prasar, New Delhi. The twin-set CD ROMs was released on the occasion of the 118th Birth Anniversary of Ramanujan, December 22, 2005, by Dr. V.S. Ramamurthy, at the Jamia Millia Islamia University.

One copy of the CD ROM set was presented to the President of India Dr. A.P.J. Abdul Kalam, at the Rashtrapathi Bhavan, by the author, along with Dr. Ramamurthy and Dr. Acharya, the Director of Vigyan Prasar, the Head of the National Multimedia Resource Center, Pune—a part the Center for Development of Advanced Computing (C-DAC), New Delhi, whose team produced the CDs.

The author has presented the CD ROMs after a presentation of the life and work of Srinivasa Ramanujan in lectures at several leading centres for research in India and abroad, including the Flemish Royal Academy of Belgium, at Brussels; Max Planck Institut für Mathematik in Frankfurt; the National Institute for Advanced Study in Bangalore; the Indian Institute of Science, Bangalore; National Institute for Advanced Study, Bangalore; Indian Institute of Information Technology (IIIT), Sri City, Andhra Pradesh; Indian Institute of Science Education and Research, Tirupathi, Andhra Pradesh; IOP, Bhubaneswar; IIT Madras; Indian Institute of Astronomy and Astrophysics, Pune; Indian Institute of Science Education Research (IISER), Pune; and several universities in India (University of Madras, Annamalai University, University of Rajasthan, Pune University, North Eastern Hill University, Benaras Hindu University, Mysore University, Calicut University, Cochin University of Science and Technology, Delhi University, Madurai Kamaraj University, Tezpur University).

Letter to R. Krishna Rao

P.C. Mahalanobis



Letter to R. Krishna Rao

A long Letter of Ramanujan to his mother Komalathammal (Sep. 11, 1914) in Tamizh.

First Published by Cambridge University Press, in 1927; Chelsea, New York, in 1962; American Mathematical Society in 1999.

Ramanujan with Fellows of Trinity College R is in the Center, Hardy is at the Right end

Ramanujan in Convocation robes (B.A. 1916)

Certificate for a Candidate for Election

Excerpt from Records of the Royal Society & F.R.S.
Certificate for a Candidate for Election.

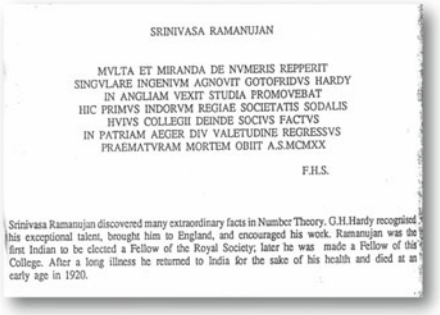
Name and Title or Designation *Srinivasa Ramanujan*
Profession *Research Student in Mathematics*
Usual Place of Residence *Trinity College, Cambridge*
Qualifications(Not to exceed 250 words):
Distinguished as pure mathematician, particularly for his investigations in elliptic functions and the theory of numbers.
Author of the following papers, amongst others: 'Modular equations and approximations to ', Quarterly Journal, vol. 45; 'New expressions of Riemann's function and , ibid, vol. 46; 'Highly composite numbers', Proc. London. Math. Soc., vol. 14; 'On certain arithmetical functions', Trans. Camb. Phil. Soc., vol. 22; 'On the expression of a number in the form $ax^2 + by^2 + cz^2 + dt^2$,' Proc. Camb. Phil. Soc., vol. 19.
Joint author with G.H. Hardy, F.R.S., of the following papers: 'Une formulae asymptotique pour le nombre des partitions de n', Comptes Rendus, 2 Jan. 1917; 'Asymptotic Formulae for the distribution of numbers of various types', Proc. London Math. Soc., vol. 16; 'The normal number of prime factors of a number n', Quarterly Journal, vol. 47; 'Asymptotic Formulae in Combinatory Analysis', Proc. London Math. Soc., (awaiting publication).
being desirous of admission into the ROYAL SOCIETY OF LONDON, we the undersigned propose and recommend him as deserving that honour, and as likely to become a useful and valuable Member.

From General Knowledge.	**From Personal knowledge.**	
(sd.) E.T. Whittaker	**(sd.)G.H. Hardy**	***Proposer.***
(sd.) A.R. Forsyth	**(sd.)P.A.MacMahon**	***Seconder.***
(sd.) A.N. Whitehead	**(sd.)J.H. Grace**	
	(sd.)Joseph Larmor	
	(sd.)T.J.I'A.Bromwich	
	(sd.)E.W. Hobson	
	(sd.)H.F. Baker	
	(sd.)J.E. Littlewood	
	(sd.) J.W. Nicholson	

Suspended for 1918. **Delivered at the Apartments of the Society on the 18th day of Dec.1918 Read to the Society on the 24th day January 1918.**

Trinity College, Cambridge

Ramanujan's F.R.S. citation read:
"Srinivasa Ramanujan, Trinity College, Cambridge. Research student in Mathematics Distinguished as a pure mathematician particularly for his investigations in elliptic functions and the theory of numbers."



SRINIVASA RAMANUJAN

MVLTA ET MIRANDA DE NVMERIS REPPERIT
SINGVLARE INGENIVM AGNOVIT GOTOFRIDVS HARDY
IN ANGLIAM VEXIT STUDIA PROMOVEBAT
HIC PRIMVS INDORVM REGIAE SOCIETATIS SODALIS
HVIVS COLLEGII DEINDE SOCIVS FACTVS
IN PATRIAM AEGER DIV VALETUDINE REGRESSVS
PRAEMATVRAM MORTEM OBIIT A.S.MCMXX

F.H.S.

Srinivasa Ramanujan discovered many extraordinary facts in Number Theory. G.H.Hardy recognized his exceptional talent, brought him to England, and encouraged his work. Ramanujan was the first Indian to be elected a Fellow of the Royal Society; later he was made a Fellow of this College. After a long illness he returned to India for the sake of his health and died at an early age in 1920.

The plaque at the Chapel of Cambridge University, Cambridge,

Translated by Jonathan Smith, Archivist, April 1, 1997.

(Courtesy Dr. T. D. Krishnamachari)

The Chapel

The Great Court, Trinity College, Cambridge.

S. Ramanujan to G.H. Hardy 12 January 1920

University of Madras

I am extremely sorry for not writing to you a single letter upto now. I discovered very interesting functions recently which I call 'Mock' ϑ-functions. Unlike the 'False' ϑ-functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary ϑ-functions. I can write you with this letter some examples.

If we consider q as a function in the transformed Eulerian form e.g.

$$(A) \quad 1 + \frac{q}{(1-q^2)} + \frac{q^4}{(1-q^2)(1-q^4)} + \frac{q^9}{(1-q^2)(1-q^4)(1-q^6)} + \cdots$$

$$(B) \quad 1 + \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^2)(1-q^5)} + \cdots$$

and if we examine the nature of the singularities at the points $q = 1, q^6 = 1, q^{10} = 1, q^{14} = 1, \ldots$ we have how beautifully the asymptotic form of the function can be expressed in a very neat and closed exponential form. For instance when $q \to 1$ and $t \to 0$

$$(A) \quad \sim \sqrt{\frac{\pi}{2t}} \exp\left(\frac{\pi^2}{6t} - \frac{t}{24}\right) + \cdots (1)$$

$$(B) \quad \sim \frac{\exp\left(\frac{\pi^2}{6t} - \frac{5t}{24}\right)}{\sqrt{5t\sqrt{5}}} + \cdots (2)$$

and similar results at other singularities. * It is not necessary that there should be only one term like this. There may be many terms but the number of terms must be finite. I Also (1) may turn out to be $O(1)$. That is all. For instance when $q \to 1$ the function

$$\frac{1}{(1-q)(1-q^2)(1-q^3)\ldots)^{10}}$$

is equivalent to the sum of five terms like (*) together with $O(1)$ instead of (1).

Ramanujan's last letter to Hardy

'Gometra', Chetpet, Madras, Ramanujan's last residence

CORPORATION OF MADRAS

HEALTH DEPARTMENT

B & D No. 5956/20

Certificate of Death — Issued under Section 12 — of the Registration of Births and Deaths Act, 1969.

This is to certify that the following information has been taken from the original record of death which is in the register for Division ... of the Corporation of Madras of the State of Tamil Nadu, S. India.

1. Name S. Ramanujan.
2. Sex Male.
3. Date of death 26th April 1920.
4. Place of death Gometra. Hanington Rd.
5. Registration No. 2825
6. Date of registration 27th April 1920
7. Name of Father / Mother / Husband —
8. Nationality Indian
9. Permanent Address Gometra. Hanington Rd.

Signature of the Issuing Authority
Health Officer
Corporation of Madras

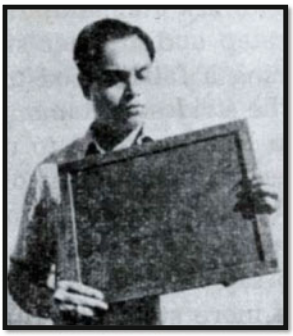
Date 1/1/96 Seal

Note:— In the case of death, No disclosure shall be made of particulars regarding the cause of death as entered in the register.

CHECKED

Death Certificate

s.s. Nevasa by which Ramanujan returned to India (27 Feb. 1919 – 27 March 1919)



Slate of Ramanujan held by Subbanarayanan son of Narayana Iyer (in the 1960s)

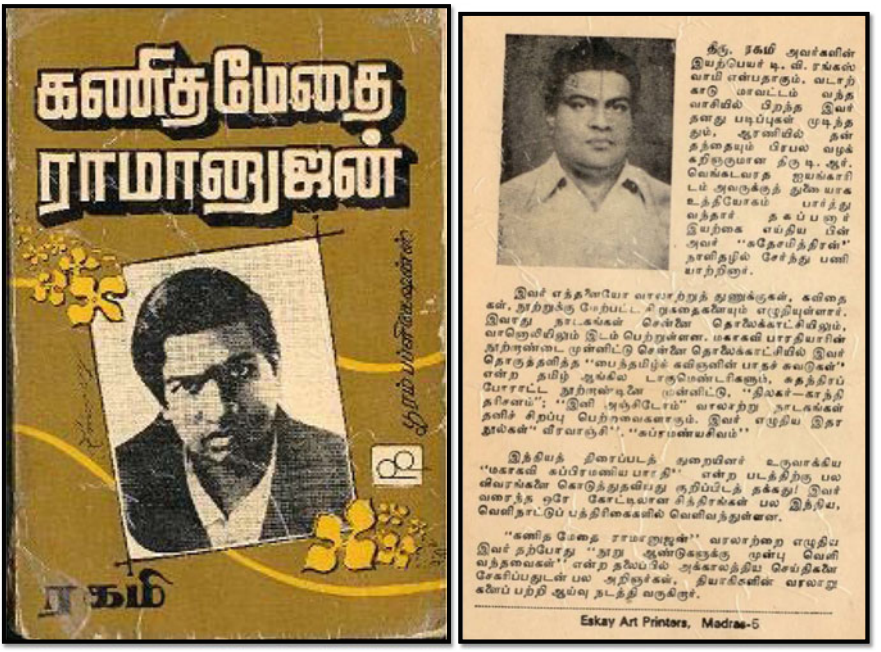
Slate of Ramanujan with Prof. Bruce Berndt, Univ. of Illinois (1987).

Ramanujan 1987 Birth Centenary Celebrations. The `Lost' Notebook of Ramanujan released by Prof. George E. Andrews. The 1st Copy of the book was presented to Janaki Ammal. She is seated at the Center. Robert Rankin and his wife are behind Andrews. The foster son and his wife are standing at the right and left ends, with their relatives.

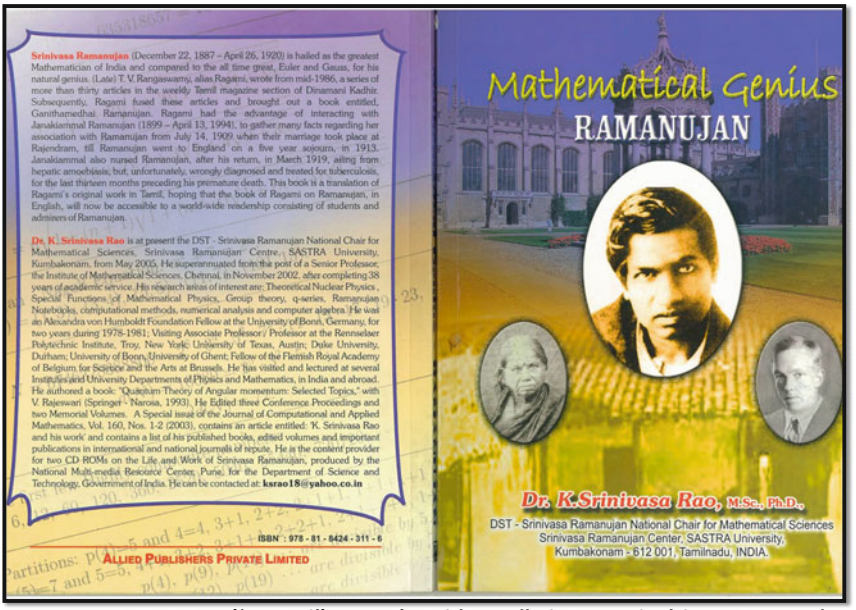
Janaki Ammal honoured by the Hinduja Foundation during the Birth Centenary year with a purse of Rs.20,000/- and a lifelong Pension by the Foundation announced by Professor E.C.G. Sudarshan, Director, IMSc. L to R: Vaidehi Narayanan, Janaki, Hon'ble C. Subramaniam, V.C. Kulanadiswami, E.C.G. Sudarshan and A.P. Venkateswaran (Secretary, Hinduja Foundation).

This function was organized at the IMSc. (by the author, at the instance of Prof. Sudarshan, Director IMSc.)

Janaki Ammal after she was honoured at a function (holding a garland of flowers, in 1962). She is seen with the Port Trust Chairman, Mr. A. Selvaraj, after a function in which the lady was Honoured, with Mr. W. Narayanan in the center and other relatives of Janakiammal.



Gnitha Medhai Ramanujan by 'Ragami', Pooram Publications (1987)


Srinivasa Ramanujan (December 22, 1887 – April 26, 1920) is hailed as the greatest Mathematician of India and compared to the all time great, Euler and Gauss, for his natural genius. (Late) T.V. Rangaswamy, alias Ragami, wrote from mid-1986, a series of more than thirty articles in the weekly Tamil magazine section of Dinamani Kadhir. Subsequently, Ragami fused these articles and brought out a book entitled, Ganithamedhai Ramanujan. Ragami had the advantage of interacting with Janakiammal Ramanujan (1899 – April 13, 1994), to gather many facts regarding her association with Ramanujan from July 14, 1909 when their marriage took place at Rajendram, till Ramanujan went to England on a five year sojourn, in 1913. Janakiammal also nursed Ramanujan, after his return, in March 1919, ailing from hepatic amoebiasis, but, unfortunately, wrongly diagnosed and treated for tuberculosis, for the last thirteen months preceding his premature death. This book is a translation of Ragami's original work in Tamil, hoping that the book of Ragami on Ramanujan, in English, will now be accessible to a world-wide readership consisting of students and admirers of Ramanujan.
Dr. K. Srinivasa Rao is at present the DST - Srinivasa Ramanujan National Chair for Mathematical Sciences, Srinivasa Ramanujan Centre, SASTRA University, Kumbakonam, from May 2005. He superannuated from the post of a Senior Professor, the Institute of Mathematical Sciences, Chennai, in November 2002, after completing 38 years of academic service. His research areas of interest are: Theoretical Nuclear Physics, Special Functions of Mathematical Physics, Group theory, q-series, Ramanujan Notebooks, computational methods, numerical analysis and computer algebra. He was an Alexandra von Humboldt Foundation Fellow at the University of Bonn, Germany, for two years during 1978-1981; Visiting Associate Professor / Professor at the Rensselaer Polytechnic Institute, Troy, New York; University of Texas, Austin; Duke University, Durham; University of Bonn, University of Ghent; Fellow of the Flemish Royal Academy of Belgium for Science and the Arts at Brussels. He has visited and lectured at several Institutes and University Departments of Physics and Mathematics, in India and abroad. He authored a book: "Quantum Theory of Angular momentum: Selected Topics," with V. Rajeswari (Springer - Narosa, 1993). He Edited three Conference Proceedings and two Memorial Volumes. A Special issue of the Journal of Computational and Applied Mathematics, Vol. 160, Nos. 1-2 (2003), contains an article entitled: K. Srinivasa Rao and his work and contains a list of his published books, edited volumes and important publications in international and national journals of repute. He is the content provider for two CD ROMs on the Life and Work of Srinivasa Ramanujan, produced by the National Multi-media Resource Center, Pune, for the Department of Science and Technology, Government of India. He can be contacted at: ksrao18@yahoo.co.in
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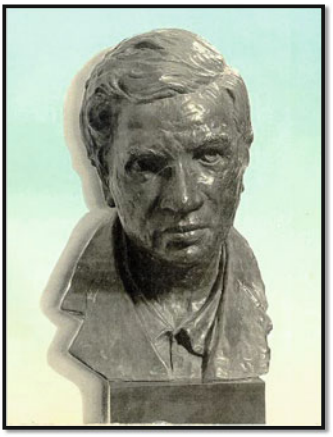
Mathematical Genius
RAMANUJAN
Dr. K.Srinivasa Rao, M.Sc., Ph.D.,
DST - Srinivasa Ramanujan National Chair for Mathematical Sciences
Srinivasa Ramanujan Center, SASTRA University,
Kumbakonam - 612 001, Tamilnadu, INDIA.

T.V. Rangaswamy Iyengar ('Ragami') wrote `Ganithamedhai Ramanujan', in 1987. Translated by the author with a Foreword by Governor Dr. Sardar Surjit Singh Barnala, March 2008.

The Passport Photograph ➔ of Ramanujan taken in 1919 required to get a Passport for him to return to India. Note: He did not have one when he left for UK in 1914!

Bust of Ramanujan at the Port Trust, Madras.



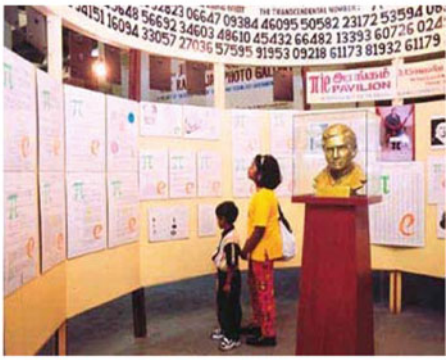
Bust of Ramanujan at the Ramanujan Institute for Advanced Study of the University of Madras, Chepauk. Sculptor: Masilamani

Paul Granlund's Bust of Ramanujan, made with Prof. Richard Askey's initiative. Ten copies are at: RRI, IASc., Bangalore; TIFR, Bombay; IUCAA, Pune; Library of Cambridge Univ.; Royal Soc., London; Gustavus Adolphus College; the Vaughn Foundation, and two are in Private Collections in USA.

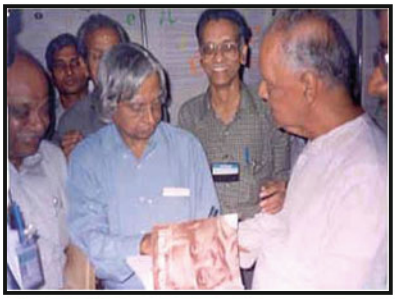
Janaki Ammal Ramanujan seated. Standing behind the lady are (L to R): author, W. Narayanan, Vaidehi Narayanan, Geetha Srinivasa Rao, at the IMSc Seminar Hall (1986).

Author with George E. Andrews, the discoverer of the `Lost' Notebook of Ramanujan inSpring 1976, at the ICM, (Hyderabad), 2010.

Dr. Kapil Sibal at 98th Indian Science Congress Exhibition, at the SRM University, Chennai (Jan. 3-7, 2011)



The "πie Pavilion" and "Ramanujan Gallery", renamed as the RAMANUJAN MATHEMATICS MUSEUM, with a grant-in-aid of 70 Lakhs, proposed by Dr. K. Srinivasa Rao, Senior Professor (Retd.), IMSc., open to the public since 2014, at the Periyar Science and Technology Center, Kottupuram, Chennai . (Punch line of this museum : NEVER SAY π = 22/7 NEVER AGAIN !)



Dr. A.P.J. Abdul Kalam signing a Copy of his Book "Wings of Fire" for Mr. P.K. Srinivasan

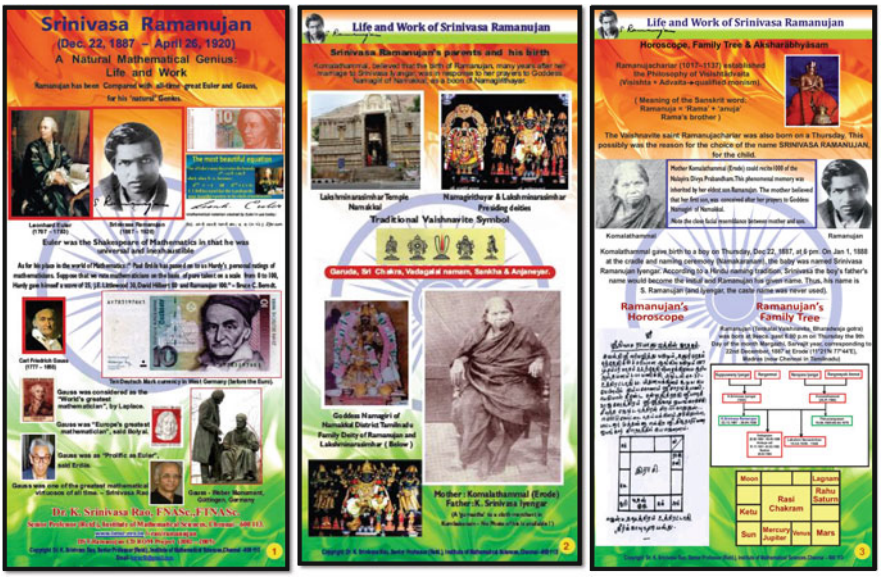
Dr. R. Chidambaram and Dr. R. Ramachandran is to his left at the PSTC.

Dr. V. S. Ramamurthy at the Exhibition

Pie Pavilion and Ramanujan Gallery, Periyar Science and Technology Center (PSTC), Chennai, on the inaugural day of the ISC Exhibition, Anna University, Chennai, Jan. 3 – 9, 1999.

Dr. Bruce C. Berndt with the author on a visit to IMSc. (1986 - 1987)

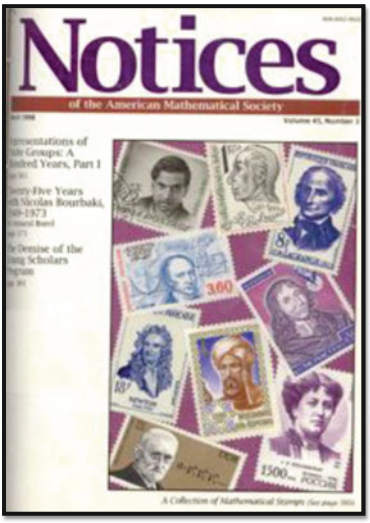
A poster at the 125th Birth Anniversary of Ramanujan, Science City, Chennai (2012)


Srinivasa Ramanujan
(Dec. 22, 1887 – April 26, 1920)
A Natural Mathematical Genius:
Life and Work
Life and Work of Srinivasa Ramanujan
Srinivasa Ramanujan's parents and his birth
Traditional Vaishnavite Symbol
Life and Work of Srinivasa Ramanujan
Horoscope, Family Tree & Aksharabhyasam
Ramanujan's Horoscope
Ramanujan's Family Tree

The author created 32 posters, like the ones above depicting the Life and work of Ramanujan for Exhibitions at several centers in Tamilnadu , Andhra Pradesh, Karnataka and Kerala States.

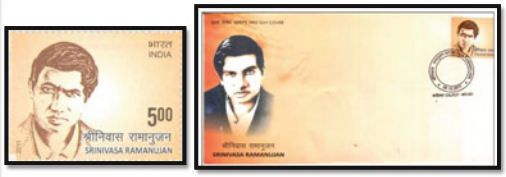
The Indian Institute of Science Education and Research (IISER) Tirupathi observed the 98th Remembrance Day of Ramanujan (April 26, 2008) with a 'Ramanujan Mathematics Museum', consisting of 32 (6' x 3') standee posters, designed by the author for the occasion.

From left above: Prof . K.N. Ganesh, Director, IISER, Prof. K . Srinivasa Rao, and Prof. P.V. Arunachalam, at the inaugural function. (April 26, 2018)



Cover of Notices of the American Mathematical Society, March 1998 (Volume 45, issue number 3)

Stamp and First Day Cover, 1962
75th Birth Anniversary of Ramanujan



Stamp and First Day Cover, 2012
125th Birth Anniversary of Ramanujan
Note that the Stamp shows a mirror image of Ramanujan's original Passport photo!
(why this mirror image of the Passport photo?)

Chapter 4
Hardy on Ramanujan

In Mathematics, George Harold Hardy (G.H. Hardy) on Ramanujan is akin to James Boswell on Samuel Johnson, in literature. The role of Hardy, Professor of Mathematics at the Trinity College, of Cambridge University, in Cambridge city, in the life and uplift of the career of Ramanujan is beyond praise. Hardy considered Ramanujan was in a way his discovery, though he has stated that "like all other great men, he invented himself". Ramanujan was considered a treasure and in Hardy's words:

"I still know more about Ramanujan than any one else, am still the first authority on this particular subject. . . . I owe more to him than to anyone else in the world with one exception, and my own association with him is the one romantic incident in my life".

4.1 Ramanujan Discovers Hardy

This association started with the discovery of Hardy's book on "Orders of Infinity", by Ramanujan. A copy of the book—due to the British ruling over India at that time, 1909—appeared almost immediately in the Library of Presidency College, where Ramanujan chanced to browse through it when he went to meet the Professor of Mathematics, P.V. Seshu Iyer, who was transferred from the Government Arts College, Kumbakonam, to the prestigious Presidency College's Department of Mathematics.

Hardy was thus in a unique position to observe and understand Ramanujan at close quarters. He has also been very generous in doing his very bets to propagate the mathematics of Ramanujan through lectures and articles, after Ramanujan died in April 1920, which actually came as a surprise and shock to Hardy, when he first heard about it from a letter written to him by Ramanujan's brother, Subbanarayanan. Here, we wish to draw heavily on Hardy's views of Ramanujan and quotations

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
https://doi.org/10.1007/978-981-16-0447-8_4

of Hardy[1] are from either his NOTICE in the "Collected Papers of Srinivasa Ramanujan" or from his book.[2]

Hardy was neither the first nor the only eminent British mathematician to whom Ramanujan wrote or appealed to. In Chap. 1, the response of Prof. M.J.M. Hill to Ramanujan's work on divergent series has been pointed out.

In the words of the eminent writer C.P. Snow, "Hardy was not the first eminent mathematician to be sent the Ramanujan manuscripts. There had been two before him, both English, both of the highest professional standard. They had each returned the manuscripts without comment. I don't think history related what they said, if anything, when Ramanujan became famous".

As for their identity, Snow adds that "out of chivalry Hardy concealed this in all that he said or wrote about Ramanujan". However, the names are given by Ashish Nandy,[3] who claims the two to be H.F. Baker and W.W. Hobson.

It is hard to imagine what would have happened if Hardy, like Hobson and Baker, also returned the manuscript of Ramanujan without comment. In view of the pivotal role Hardy played in the career of Ramanujan, it is necessary to give a biographical account of Hardy's life and work.

4.2 Hardy: A Brief Biography

Godfrey Harold Hardy was born on February 7, 1877, at Cranleigh, Surrey, in a gifted and mathematically inclined family. His father, Isaac Hardy, was an Art Master, Bursar and House Master of the preparatory branch of Cranleigh School, and his mother, Sophie Hardy, was a senior mistress at the Lincoln Training College for teachers. It is said that he was writing down numbers up to millions—his earliest sign of mathematical precociousness—when he was just 2 years old. He and his younger sister were sent to a Victorian nursery and later to the Cranleigh School. At School, hardy had private coaching in Mathematics, and not in a class, because of his precocity. He was first in his class in all subjects, but extremely shy to receive the prizes due to his self-consciousness. In Hardy's words, "I thought of mathematics in terms of examinations and scholarships: I wanted to beat other boys and this seemed to be the way in which I could do so most decisively."

Though frail and good looking, he did not like to see himself in a mirror or be photographed. At the age of 12, Hardy won a scholarship at Winchester public school, the best mathematical school in England then. He left the school

[1] G.H. Hardy, *Srinivasa Ramanujan (1887–1920)*, in "Collected Papers of Srinivasa Ramanujan", Ed. by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, AMS Chelsea Publishing, Am. Math. Soc., Providence, Rhode Island (2000) xxi.

[2] G.H. Hardy, *Ramanujan: Twelve lectures on subjects suggested by his life and work*, Chelsea, New York (1940).

[3] Ashish Nandy, *Alternative Sciences*, Allied Publishers, New Delhi (1980).

with honours and was offered an open scholarship to Trinity College, Cambridge University, Cambridge. He was not only good at studies but was also a natural Tennis player and a batsman in Cricket—a game he could enjoy playing or watching till almost the end of his life.

By the time he finished his schooling, he was an atheist though he came from an orthodox religious family and he stopped going into any chapel. This created problems for him at Trinity College, since attending the chapel was compulsory. He referred to God as his personal enemy all through his life. His strong feelings about the church and orthodoxy are clearly reflected in his statement: "It's rather unfortunate that some of the happiest hours of my life should have been spent within sound of a Roman Catholic Church". The church referred to by Hardy is the one close to Fenner's, the cricket ground of Cambridge university.

Hardy, who graduated in 1899, managed to come as fourth Wrangler in Part I of the Mathematical Tripos examination of the Cambridge University, in 1898, by any measure a superb performance, which determined the Wrangler standing. The top candidates were ranked as Wranglers meant in practical terms a First Class. An excellent account of the Tripos and the Wrangler ranking can be found in Kanigel's,[4] 'The Man Who Knew Infinity: A Life of the Genius Ramanujan'. Hardy appeared for Part II of the Tripos, the more provocative and more challenging part, in 1900, and secured the first division of the first class and was elected 1900 to a Prize Fellowship at Trinity, at the age of 22.

Hardy won the Smith's Prize in 1901. It was during his coaching classes for competitive Mathematical Tripos examination, that his coach, Professor A.E.H. Love, advised him to read Jordan's famous *Cours d' Analyse* (the first volume of which was published in 1882 and the third and last volume was published in 1887) about which Hardy said: "I shall never forget the astonishment with which I read that remarkable work, the first inspiration for so many mathematicians of my generation, and learned for the first time as I read it what mathematics really meant. From that time onward I was in a way a real mathematician, with sound mathematical ambitions and a genuine passion for mathematics."

Hardy is the second hero in Kanigel's The Man Who Knew Infinity: a Life of the Genius Ramanujan. Kanigel portrays Hardy as scrupulously honest, fastidious about giving others their due; ever so kind to the weak ones; and one in whom "coexisted a stern, demanding streak with an indulgent liberal-mindedness, a formidable and forbidding exterior with a soft and fragile core".

Though Hardy was the "best and truest" friend Ramanujan ever had, Kanigel expresses his view that Hardy never really knew Ramanujan. The aloof, formal nature of their friendship is best illustrated by the fact that in one note written by Ramanujan to Hardy from a sanatorium, nearly three years after working together, Ramanujan explains that Srinivasa in his name is not his first name but a part of

[4]Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Charles Scribner's Sons, New York (1991); Rupa & Co. (1994).

his father's name. The richness and extent of Ramanujan's inner spiritual life was, perhaps, something he never revealed to Hardy.

In what could be called as a larger than life portrayal, in Kanigel's biography of Ramanujan, Hardy emerges as an aristocrat of the intellect; busy being brilliant; effectively inculcating in Ramanujan the Gospel of proof and rigour in mathematics; the embodiment of all that was highest and best in the Western mathematical tradition; the most stern taskmaster; the most generous man with not even a wisp of envy tainting his relationship with Ramanujan; who championed and hailed the gifts of Ramanujan, recognizing Ramanujan's genius and pushing it towards its limits.

4.3 Hardy's Collaborations

Collaboration is rare in the area of mathematics research. Hardy had two fruitful collaborations in mathematics in his life. He always considered J.E. Littlewood as a better mathematician than himself—and as quoted earlier on, in his self-assessment, he gave himself 25, Littlewood 30, Hilbert 80 and Ramanujan 100 out of 100—and Ramanujan as a natural mathematical genius. Unfortunately, the historic Hardy-Ramanujan collaboration overlapped with the First World War period, but continued uninterrupted for five years from 1914, until Ramanujan's return to India, in March 1919. During those five years, they wrote five papers of the highest class together, which revealed that "Ramanujan intuition incarnate, had run smack into Hardy the Apostle of Proof" (observation of Kanigel). In the words of Littlewood, the collaboration was between two mathematicians of "quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him".

The Hardy-Littlewood collaboration started in 1911 and barring the interference due to the war years when Littlewood was away (for four years), doing ballistics as a Second Lieutenant in the Royal Artillery, this partnership lasted 35 years. Together they published nearly a hundred papers and in the words of C.P. Snow: "a good many of them 'in the Bradman class' and it is "the most famous collaboration in the history of mathematics. . . The Hardy - Littlewood researches dominated English pure mathematics, and much of word pure mathematics, for a generation. . . Of its enduring value there is no question . . . But no one knows how they did it . . . No one has any evidence how they set about it," since during their most productive period, they were not even at the same University.

Hardy with Littlewood and G. Polya published a book on "Inequalities", in 1934. Littlewood and Hardy conducted a seminar or "conversational class" at Cambridge which attracted the mathematical community due to its informal nature and openness in discussions.

Hardy was a confirmed bachelor, in whose life no woman other than his mother and sister played the slightest substantive role. Hardy was a member of a secret intellectual society called the Apostles along with some of the most brilliant Cambridge men of that period—Bertrand Russell, Whitehead and James Clerk Maxwell, to name a few. A homosexual current ran through the Society.

Kanigel in his book, The Man Who Knew Infinity asserts that the Hardy-Ramanujan relationship was marked by distance, not comradely intimacy and that Mathematics was the common ground of their relationship. A fact which indirectly corroborates this view is that after his return to India, in April 1919, during the entire period of almost 13 months before he died, Ramanujan wrote only one letter to Hardy, in January 1920. In that letter he did not mention about his deteriorating health. However, he wrote about the new results he had on what he created and named as "mock theta functions". This led Hardy to feel, as he stated later on, that Ramanujan was recovering with treatment for his illness diagnosed as Tuberculosis by the British doctors and confirmed by their Indian counterparts. So, the announcement of Ramanujan's death in a letter by Ramanujan's brother to Hardy, was a shock to the latter.

We have already discussed the Hardy-Ramanujan collaboration which led to five papers together and the "astonishing theorem" on the number of partitions below a given number n, denoted by $p(n)$. The best in Ramanujan was brought out by Hardy, "The great bulk of Ramanujan's published work was done in England. His mind was hardened to some extent, and he never became at all an 'orthodox' mathematician, but he could still learn to do new things, and do them extremely well. It was impossible to teach him systematically, but he gradually absorbed new points of view. In particular, he learnt what was meant by proof, and his later papers, while in some ways as odd and individual as ever, read like the works of a well-informed mathematician. His methods and weapons, however, remained essentially the same."

Hardy did perhaps everything which one human being could have ever done to another to keep Ramanujan in the best of spirits—the Bachelors Degree in Arts, the Fellowship of the Royal Society, the Fellowship of the Trinity College, conferred on Ramanujan were mainly due to the untiring efforts of Hardy. Together Hardy and Ramanujan were a formidable mathematical pair who left indelible footprints in the sands of pure mathematics—a unique pair in the realm of Mathematics.

4.4 The Hardy-Ramanujan Collaboration: Pros and Cons

Hardy has been eloquent to state that Ramanujan "was by far the greatest formalist of his time"; that Ramanujan's work in the fields of identities and continued fractions is a masterpiece, and he wrote that his "extremely strong language", in the following paragraph, "is not extravagant":

"It was his insight into algebraical formulae, transformations of infinite series, and so forth, that was most amazing. On this side most certainly I have never met his equal, and I can compare him with only Euler or Jacobi. He worked, far more than the majority of modern mathematicians, by induction from numerical examples; all his congruence properties of partitions, for example, were discovered in this way. But with his memory his patience, and his power of calculation he combined a power of generalization, a feeling for form, and a capacity for rapid modification of

his hypothesis, that were often really starling, and made him, in his own peculiar field, without a rival in his day."

On Ramanujan's work on the Prime Number Theorem, Hardy stated: "All that is plain is that Ramanujan found the form of the Prime Number Theorem by himself. This was a considerable achievement; for the men who had found the form of the theorem before him, like Legendre, Gauss, and Dirichlet, had all been very great mathematicians, and Ramanujan found other formulae which lie still further below the surface."

On the importance of Ramanujan's work, Hardy wrote: "Opinions may differ about the importance of Ramanujan's work, the kind of standard by which it should be judged, and the influence which it is likely to have on the mathematics of the future. It has not the simplicity and the inevitableness of the very greatest work; it would be greater if it were less strange. One gift it shows which no one can deny, profound and invincible originality. He would probably have been a greater mathematician if he could have been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt, of greater importance."

In a report submitted to the University of Madras on Ramanujan's mathematical work in England, Hardy wrote: "In one respect, Mr. Ramanujan has been most unfortunate. The war has naturally distracted three-quarters of the interest that would otherwise have been taken in his work, and has made it almost impossible to bring his results to the notice of the continental mathematicians most certain to appreciate it. It has moreover deprived him of the teaching of Mr. Littlewood, one of the great benefits which his visit to England was intended to secure. All this will pass; and, in spite of it, it is already safe to say that Mr. Ramanujan has justified abundantly all the hopes that were based upon his work in India and has shown that he possesses powers as remarkable in their way as those of any living mathematician. His work is only the more valuable because his abilities and methods are of so unusual a kind, and so unlike those of a European mathematician trained in the orthodox school.

Hardy concludes this report to the University with the following comments: "My account of Ramanujan's work has been necessarily fragmentary and incomplete. I have said enough, I hope, to give some idea of its astonishing individuality and power. India has produced many talented mathematicians in recent years, a number of whom have come to Cambridge and attained high academical distinction. They will be the first to recognize that Mr. Ramanujan's work is of a different category. In him India now possesses a pure mathematician of the first order, whose achievements suggest the brightest hopes for its scientific future.

The above statements of Hardy are all positive and strong and bordering on adulation. To show that Hardy was objective in his assessment of Ramanujan, a few of the drawbacks of Ramanujan pointed out by Hardy are reproduced here:

"One would have thought that such a formalist as Ramanujan would have reveled in Cauchy's theorem, but he practically never used it, and the most astonishing testimony to his formal genius is that he never seemed to feel the want of it in the least…"

"... in the analytic theory of numbers, he had, in a sense, discovered a great deal, but he was a very long way from understanding the real difficulties of the subject ... in analytic theory of numbers, even Ramanujan's imagination led him seriously astray ... he showed, as always, astonishing imaginative power, but he proved next to nothing, and a great deal even of what he imagined was false."

Ramanujan "had no knowledge at all, at any time, of the general theory of arithmetical forms."

"In analysis proper, Ramanujan's work is inevitably less impressive, since he knew no theory of functions, and you cannot do real analysis without it ... Still, Ramanujan rediscovered an astonishing number of the most beautiful analytic identities. Thus the functional equation for the Riemann Zeta-function ... stand (in an almost unrecognizable notation) in the notebooks. So does Poisson's summation formula ... and so also does Abel's functional equation..."

"Ramanujan knew nothing at all about the theory of analytic functions... Ramanujan's theory of primes was vitiated by his ignorance of functions of a complex variable. It was (so to say) what the theory might be if the Zeta function had no complex zeros. His method depends upon a wholesale use of divergent series... That his proofs should have been invalid was only to be expected. But the mistakes went deeper than that, and many of the actual results were false. He had obtained the dominant terms of the classical formulae, although by invalid methods; but none of them are such close approximations as he supposed... This may be said to have been Ramanujan's one great failure."

Hardy was thus not only lavish in his appreciation of the originality and the mathematical genius of Ramanujan, but also harsh and severe in his criticism of the shortcomings of Ramanujan. While he himself edited only one chapter of Ramanujan's second Notebook on hypergeometric series, he estimated that "about two thirds of Ramanujan's best Indian work was rediscovery, and comparatively little of it was published in his lifetime, though Watson, who has worked systematically through his notebooks, has since disinterested a good deal more."

This estimate of Hardy's has been refuted to be too high by Bruce C. Berndt, who studied each and every one of the 3254 Entries of Ramanujan in his Notebooks, over a period of more than two decades (1974–1997) and to understand this one has to study the five Part magnum opus of Berndt.

Hardy in his A Mathematician's Apology has stated that all his best work, after he began his lifelong collaboration with Littlewood, in 1911, and after he "discovered Ramanujan", in 1913, "has been bound up with theirs, and it is obvious that my association with them was the decisive event in my life. I still say to myself when forced to listen to pompous and tiresome people. Well, I have done one thing you could never have done, and that is to have collaborated with both Littlewood and Ramanujan on something like equal terms."

4.5 Hardy's Lectures on Ramanujan's Work

Hardy who devoted several years of his lifetime to the study of the work of Ramanujan gave many lectures on Ramanujan's work at a number of Universities and Societies in America and England as well as courses of lectures at Princeton and Cambridge. The notes of his lectures at Princeton were written by Marshall Hall and these were printed as Lecture Notes. A copy of the same is in the Library of the University of Wisconsin, Madison, Wisconsin—a fact I learnt from a private communication with Professor Richard A. Askey, renowned for the Askey-Wilson polynomial scheme.

Hardy, in his A Mathematician's Apology categorically states: "I hate 'teaching' and have had to do very little, such teaching as I have done having been almost entirely supervision of research; I love lecturing, and lectured a great deal to extremely able classes; and I have always had plenty of leisure for the researches which have been the one great permanent happiness of my life." The matter, delivery and hand-writing (a specimen of which appears on the dust cover of A Mathematician's Apology) were alike fascinating. This passage is also quoted by Titchmarsh in the Collected Papers of G.H. Hardy.

Hardy has done yeoman service to Mathematics by publishing these lecture courses of his as a book[5] entitled: *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work*. Hardy was a Visiting Professor at Princeton and at the California Institute of Technology during 1928–1929. He delivered two lectures at the Hardy Tercentenary Conference of Arts and Sciences in the fall of 1936—an occasion which "brought to Harvard more than twenty-five hundred scholars for lectures under broad rubrics of knowledge". On September 16, before an august audience consisting of "no fewer than eleven Nobel Prize winners" Hardy gave his lectures on Ramanujan ... "the most romantic figure in the recent history of mathematics; a man whose career seems full of paradoxes and contradictions, who defies almost all the canons by which we are accustomed to judge one another, and about whom all of us will probably agree in one judgment only, that he was in some sense a very great mathematician."

This Harvard lecture is reprinted from the American Mathematical Monthly, where it was first published, in the above said book on "Ramanujan". As pointed out earlier, Hardy had lectured at a "number of Universities and Societies in America and England and connected courses in Princeton and Cambridge" on Ramanujan's work, and the remaining 11 chapters in "Ramanujan" are based on these lectures of Hardy and "Ramanujan is the thread which holds the whole together." Hardy's aim in these lectures is stated clearly in his own words, in the Harvard lecture: "... the reputation of a mathematician cannot be made by failures or by rediscoveries; it must rest primarily, and rightly, on actual and original achievement. I have to justify Ramanujan on this ground, and I hope to do so in my later lectures."

[5] G.H. Hardy, *Ramanujan: Twelve Lectures on subjects suggested by his life and work*, Chelsea, New York (1940).

The chapter titles given below indicate the range of problems on which Ramanujan had worked during his short productive period, as a professional researcher, after he came into contact with Hardy:

I. The Indian mathematician Ramanujan
II. Ramanujan and the theory of numbers
III. Round numbers
IV. Some more problems of the analytic theory of numbers
V. A lattice-point problem
VI. Ramanujan's work on partitions
VII. Hypergeometric series
VIII. Asymptotic theory of partitions
IX. The representation of numbers as sums of squares
X. Ramanujan's function $\tau(n)$
XI. Definite integrals
XII. Elliptic and modular functions

Apart from these lectures, Hardy has submitted a report on Ramanujan's work in England, for publication, at the request of the University of Madras. This report is with the Wren Library of Trinity College and has the Card Catalogue number Add.Ms.a.94[12(79)] (see Appendix B for details about the catalogued articles). The report gives summaries of twelve (out of 21) papers published by Ramanujan from England, including one (of the seven) he wrote with Hardy. Ten of these are in the *Collected papers*, enumerated as 11, 12, 6, 14, 16, 12, 15, 17, 18 and 20 (see Appendix A for the titles of these papers). One of the 12 corresponds to 19 (of Appendix A) though with a slightly different title: *Some series for Euler's constant* and another (with Hardy) has the title: *A problem in the analytic theory of numbers*, which was unpublished at that time but communicated to the London Mathematical Society. Obviously, since this title is not one of the seven joint papers listed as 31–37 in the *Collected papers*, it was written subsequently under a different title.

4.6 Ramanujan's Work on Hypergeometric Series

Hardy remarks that any student of the notebooks can see that Ramanujan's ideal of presentation had been copied from Carr's *Synopsis*. Three years after Ramanujan's death, in 1923, Hardy spent about four months on Chapter XII—which is Chapter X in G.N. Watson's copy of the 'second edition', as stated in a Note at the end of his lecture VII in his book "Ramanujan: Twelve Lectures on subjects suggested by his life and work'—of the second Notebook, on hypergeometric series, and in the introduction to the paper he published on this topic, Hardy wrote that "a systematic verification of the results would be a very heavy undertaking." A task taken up by G.N. Watson and B.M. Wilson, which ended abruptly with the premature death of

B.M. Wilson, and awaited the devoted continuing efforts of Bruce C. Berndt, from the mid-1970s.

This section is intended for the mathematically inclined readers. This is also the area of research interest to this author from the 1970s. Preceding the birth centenary of Ramanujan, the Director of the Institute of Mathematical Sciences, Professor E.C.G. Sudarshan, suggested my going to Mt. Abu to participate in a two-week workshop on Special Functions, convened and conducted by the then Vice Chancellor of Rajasthan University, Professor Ratan P. Agarwal, who became my guru and for whose lectures I volunteered to take lecture notes and the same were brought out as a Report. This was my formal introduction to the work of the group in India, led by Ratan P. Agarwal, on hypergeometric series and I am thankful to the friendships formed at that time, especially with Professor R. P. Agarwal himself and with Arun Verma, in particular.

The British mathematician John Wallis, in his book, Arithmetica Infinitorum (London, 1655), studied the series

$$1 + a + a(a+1) + a(a+1)(a+2) + \cdots \tag{4.1}$$

as a generalization of the ordinary geometric series:

$$1 + a + a^2 + a^3 + a^4 + \cdots + a^n + \cdots \;=\; (1-a)^{-1} \tag{4.2}$$

and called (4.1) as the "hypergeometric series" (Note: the Greek work "hyper" means above or beyond). Historically, the term "hypergeometric", from the Greek $\nu\pi\epsilon\rho$, was first used by John Wallis to denote any series which is beyond the ordinary binomial series $(1-a)^{-1} = 1 + a + a^2 + a^3 + \cdots$, is called a geometric series.

The infinite series

$$1 + \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!} + \cdots, \tag{4.3}$$

usually represented by the later day symbol, ${}_2F_1(a,b;c;z)$ was defined and introduced by Carl Friedrich Gauss, in 1812, in his famous thesis " *Disquisitiones generales circa seriem infinitam*", before the Royal Society in Göttingen, Germany. Gauss stated explicitly that the ${}_2F_1(a,b;c;z)$, a solution of the second-order Ordinary Differential Equation (ODE), should be considered as a function of four variables and not as a function of the variable z, with a, b as the numerator and c as the denominator parameters of the series. On January 20, 1812, Gauss also announced the discovery of the series at the Paris International Congress of Mathematicians (ICM 1812). He proved the famous summation theorem, since named after him as the Gauss summation theorem:

$${}_2F_1(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \tag{4.4}$$

where $\Gamma(n)$ is the ordinary gamma function

$$\Gamma(n) = (n-1)\Gamma(n-1) = (n-1)(n-2)\cdots 2\cdot 1 = (n-1)! \text{ and } \Gamma(0) = 1. \tag{4.5}$$

Gauss gave a remarkably comprehensive discussion on the convergence properties of the series:

$$\text{(i) the series is convergent for } |z| < 1, \tag{4.6a}$$

$$\text{(ii) divergent for } |z| > 1 \text{ and for} \tag{4.6b}$$

$$\text{(iii) } z = 1, \text{ it is convergent if } R\ell(c-a-b) > 0$$
$$\text{and divergent if } R\ell(c-a-b) < 0. \tag{4.6c}$$

If a numerator parameter is zero or a negative integer, $-n$, say, then the series (4.3) has only $n+1$ terms and it becomes a polynomial of degree n in z. If the denominator parameter c is a negative integer, or, if the zero due to the denominator parameter occurs before the zero due to the numerator parameter, then, the hypergeometric series is not defined. This is the accepted convention.

Gauss established 15 relations between six contiguous functions:

$$_2F_1(a \pm 1,\ b;\ c;\ z) \equiv F(a \pm 1), \quad F(b \pm 1), \quad F(c \pm 1) \tag{4.7}$$

and showed that there exists between any two of these contiguous functions linear relations with coefficients, at most linear in z.

Gauss proved the famous (Gauss) summation theorem (4.4) and gave many recurrence relations between two or more of these series. The summation theorem is easily proved from the Euler integral representation for the Gauss function:

$$_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}\,dt,\ c > b > 0, \tag{4.8}$$

for $c \neq 0, -1, -2, \ldots$ and $R\ell(c-a-b) > 0$.

When $z = 1$ in the Euler integral representation becomes:

$$_2F_1(a,b;c;1) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}\,(1-t)^{c-a-b-1}\,dt \tag{4.9}$$

where the integral can be recognized as the beta function which has the property:

$$B(m,n) = \int_0^1 t^{m-1}\,(1-t)^{n-1}\,dt = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},\ R\ell(m) > 0,\ R\ell(n) > 0, \tag{4.10}$$

so that

$$ {}_2F_1(a, b; c; 1) \;=\; \frac{B(b, c-a-b)}{B(b, c-b)} \;=\; \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \tag{4.11} $$

for $c - a - b > 0$ for all a, b, c complex (even without $R\ell(c - a - b) > 0$).

Ramanujan came across the Gauss summation theorem (4.4) in Carr's *Synopsis*, and he was able to provide an alternate proof of this theorem without recourse to the Euler integral representation for the beta function. Entry 8, in Chapter X of Ramanujan's Notebook 2, contains this remarkable theorem and its proof:

Theorem *If* $R\ell(x + y + n + 1) > 0$, *then*

$$ {}_2F_1(-x, -y;\; n+1;\; 1) \;=\; \frac{\Gamma(n+1)\Gamma(x+y+n+1)}{\Gamma(x+n+1)\Gamma(y+n+1)} \tag{4.12} $$

Proof Assume n, x are integers, $n \geq 0,\ x \geq 0$, and let

$$ (1+u)^{y+n} \;=\; \sum_k {}^{y+n}C_k \qquad \text{and} \qquad \left(1 + \frac{1}{u}\right)^x \;=\; \sum_\ell {}^{x}C_\ell \left(\frac{1}{u}\right)^\ell, \tag{4.13} $$

then

(i) $(1+u)^{y+n}\left(1+\frac{1}{u}\right)^x \;=\; \sum_{k,\ell} {}^{y+n}C_k\, {}^{x}C_\ell \;=\; \sum_n a_n u^n$ (say), where

$$ a_n \;=\; \sum_\ell {}^{y+n}C_{\ell+n}\, {}^{x}C_\ell \;=\; \frac{\Gamma(y+n+1)}{\Gamma(n+1)\Gamma(y+1)}\, {}_2F_1(-x, -y;\; n+1;\; 1). \tag{4.14} $$

(ii)

$$ \frac{(1+u)^{x+y+n}}{u^x} \;=\; \sum_\ell {}^{x+y+n}C_\ell\, u^{\ell-x} \;=\; \sum_n a_n u^n \;(say), \tag{4.15} $$

then

$$ a_n \;=\; {}^{x+y+n}C_{x+n} \;=\; \frac{\Gamma(x+y+n+1)}{\Gamma(x+n+1)\Gamma(y+1)} \tag{4.16} $$

and hence,

$$ {}_2F_1(-x, -y;\; n+1;\; 1) \;=\; \frac{\Gamma(n+1)\Gamma(x+y+n+1)}{\Gamma(x+n+1)\Gamma(y+n+1)}, \tag{4.17} $$

which for

$$x \to -a, \quad y \to -b, \quad n \to c-1, \tag{4.18}$$

yields the Gauss summation theorem for ${}_2F_1(a, b;\ c;\ 1)$ given in (4) above.

Barnes,[6] in 1907, developed the modern day notation and introduced the integral representations for the hypergeometric functions, first introduced by Euler, in his '*Introduction in Analysis Infinitorum*', Lusanne (1748), vol. I. Conventionally, in the notation due to Barnes, the Gauss hypergeometric function is written as:

$${}_2F_1(a, b; c; z) \ = \ \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \ \frac{z^n}{n!}, \tag{4.19}$$

where the symbol

$$(a)_n \ = \ \frac{\Gamma(a+n)}{\Gamma(a)} \ = \ a(a+1)(a+2)\cdots(a+n-1), \quad (a)_0 = 1, \tag{4.20}$$

is called the Pochammer symbol,[7] or the shifted/raising factorial, or a generalized power.

Kummer,[8] in 1836, showed that there exists in all 24 solutions for the Gauss hypergeometric Second-Order Ordinary Differential Equation:

$$\frac{d^2y}{dz^2} + \left[\frac{c}{z(1-z)} - \frac{1+a+b}{1-z}\right]\frac{dy}{dz} - \frac{ab}{z(1-z)}y \ = \ 0, \tag{4.21}$$

from which 0 and 1 are seen to be regular singular points of the equation. If $1/z$ replaces z, then ∞ is also a regular singular point of the Gauss equation.[9] Kummer's 24 solutions to the Gauss equation cover the entire complex z-plane, provided that a, b and c are not integers nor zero.[10] In a self-contained, short communication[11]Lievens, Srinivasa Rao and Van der Jeugt showed that the set of 24 Kummer solutions of the classical second-order ordinary differential equation of Gauss has an *elegant, simple group theoretic structure associated with the*

[6]E.W. Barnes, Proc. London Math. Soc. (2) **5** (1907) 59–116; **6** (1907) 141–177.

[7]L. Pochammer, *Über die hypergeometrische Funktionen n-ter Ordnung*, J. für Math. (Crelle), **71** (1870) 316–340. $F(\alpha, \beta, x)$, J. für Math. **15** (1836) 39–83 and 127–172.

[8]E.E. Kummer, *Über hypergeometrische Reihe* $F(\alpha, \beta, x)$, J. für Math. **15** (1836) 39–83 and 127–172.

[9]E.T. Whittaker and G.N. Watson, *A course of Modern Analysis*, Cambridge University Press, Cambridge (1902, Fourth Edn. 1927), §10.3.

[10]Lucy J. Slater, *Generalized Hypergeometric Functions*, Cambridge University Press, Cambridge (1966).

[11]S. Lievens, K. Srinivasa Rao and J. Van der Jeugt, Int. Trans. Spl. Func. **16** (2005) 153–158.

symmetries of a cube, or, in other words, that the underlying symmetry group is the Symmetric group S_4.

The hypergeometric function has retained its significance in modern mathematics because of its powerful unifying influence, since an intimate relationship exists between the ${}_2F_1(z)$ and the special functions of mathematical physics—ref. Theorem 9.1 in W.W. Bell.[12]

Clausen (1828) introduced a series which is a generalization of (3), by increasing the number of numerator and denominator parameters to 3 and 2, respectively, ${}_3F_2(a, b, c; d, e; z)$. This led to further generalizations in this direction, and summation theorems, like the Gauss summation theorem for the ${}_2F_1(1)$, have been obtained by Saalschütz, Dixon (1903) and Dougall (1907) for generalized hypergeometric series. The first monograph of W.N. Bailey on "Generalized Hypergeometric Series" provides an excellent summary of the whole theory, from its origins, as it existed then. Some of the subsequent books on generalized hypergeometric functions of interest are due to[13] Lucy J. Slater, G. Gasper and M. Rahman, William J. Thompson.

What is remarkable about Ramanujan's "natural" genius is that with only the hint of the Gauss summation theorem, which he found in Carr's *Synopsis*, Ramanujan starts the Chapter XII of Notebook 1, and Chapter X of his Notebook 2, with the most general summation theorem, as Entry 1:

$$\begin{aligned}
&\frac{n\,\Gamma(x+n+a,\,y+n+1,\,z+n+1,\,u+n+1)}{n!\,\Gamma(x+y+z+u+n+1,\,x+y+n+1,\,x+z+n+1,\,x+u+n+1)}\\
&\quad\times\frac{\Gamma(x+y+z+n,\,y+z+u+n,\,z+u+x+n,\,x+y+u+n)}{\Gamma(y+z+n+1,\,y+u+n+1,\,z+u+n+1)}\\
&\quad=\;n\;-\;(n+2)\frac{x}{1!}\frac{x(x-1)}{(x+n+1)(x+n+2)}\cdot\frac{y(y-1)}{(y+n+1)(y+n+2)}\\
&\qquad\times\frac{z(z-1)}{(z+n+1)(z+n+2)}\cdot\frac{u(u-1)}{(u+n+1)(u+n+2)}\\
&\qquad\times\frac{(x+y+z+u+2n+1)(x+y+z+u+2n+2)}{(x+y+z+u+n)(x+y+z+u+n+1)}\\
&\qquad\qquad-\;\&c.
\end{aligned}\tag{4.22}$$

[12] W.W. Bell, *Special Functions for Scientists and Engineers*, von Nostrand, New York (1968).

[13] Lucy J. Slater, *Generalized Hypergeometric Functions*, Cambridge University Press, Cambridge (1966); G. Gasper and M. Rahman, *Encyclopedia of Mathematics and its Applications* 35, Cambridge University Press, Cambridge (1990). See also William J. Thompson, *Angular Momentum - An Illustrated Guide to Rotational Symmetries for Physical Systems*, John Wiley & Sons, Inc. (1994) and Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim (2004).

This Entry is followed by Ramanujan's note:

NB. The above result is not true for all values of x, y, z, u and n. For example it is not true when $z + y + z + u + n = x, y, z$ *or* u owing to the extraneous factor containing all the quantities x, y, z and u in each term, unless we get rid of this factor, identities deduced from the above won't be true for all values. The only way to get of this is to make u infinitely great. The solution for the theorem is evident from the result.

Chapter 5
Ramanujan and Hypergeometric Series

Ramanujan saw the Gauss summation theorem in Carr's *Synopsis*, and it remains a mystery till date as to how in one sweep of intuitive imagination he was able to arrive at the most general summation theorem with only a hint of the Gauss summation theorem (Eqs. 4.4 and 4.11 in Chap. 4) for a terminating hypergeometric series, viz. the ${}_7F_6(1)$ summation theorem (22) stated at the end of Chap. 4. This summation theorem of Dougall[1] (1907) was independently found by Ramanujan during 1910–1912, and finds a place as Entry 1 in Chapter XII of Ramanujan's Notebook 1 (and the corresponding Entry 1 in Chapter X of Notebook 2). It is not possible to assert an exact date or year of Ramanujan's discovery, since there are no dates anywhere in the Notebooks of Ramanujan. In any case, all the experts in the field agree that Ramanujan discovered not only all that was known in Europe on hypergeometric series at that time, but he also discovered several new theorems. In particular, theorems on products of hypergeometric series[2] and different types of asymptotic expansions.

It is also known from Hardy's articles in his book (1944) *Ramanujan*[3]that Ramanujan did not publish any of his Theorems/Entries from his Chapters on hypergeometric series. In particular, Chapters X and XI, in his second Notebook, are considered as *revised, enlarged* versions of the chapters in his first Notebook. However, example 7, after Entry 43, in Chapter XII of the first Notebook does not find a place in the "corresponding" examples after Entry 10 in Chapter X of

[1]Dougall, *On Vandermonde's theorem and some more general expansions*, Proc. Edinburgh Math. Soc. **25** (1907), 114–132.

[2]Like the Appell and Kampe de Feriet series. See for example, L.J. Slater, *Generalized hypergeometric functions*, Cambridge Univ. Press, Cambridge (1966).

[3]G.H. Hardy, *Ramanujan: Twelve lectures on subjects suggested by his life and work*, Cambridge University Press, Cambridge (1944).

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
https://doi.org/10.1007/978-981-16-0447-8_5

his second Notebook, and is therefore not discussed by Bruce C. Berndt,[4] in his *Ramanujan Notebooks*, Part II.

In this chapter we continue to provide a few glimpses into the work on series by Ramanujan.

5.1 A Few Illustrative Examples

In the historic letter of Ramanujan to Hardy, in January 1913, the first four formulae (given in Chap. 1, Sect. 1.4) were "theorems on divergent series to calculate the convergent values corresponding to the divergent series.... Theorems to calculate such values for any given series (say $1 - 1^1 + 2^2 - 3^3 + 4^4 - 5^5 + \cdots$,) and the meaning of such values... I have also dealt with such questions, 'When to use, where to use, and how to use such values, where do they fail and where do they not?'"

Ramanujan considered the hypergeometric series (in Chapters XII, XIII of his first Notebook, corresponding to Chapters X, XI of his second Notebook):

$$ {}_3F_2\left[\begin{matrix}\alpha_1, \alpha_2, \alpha_3 \ ; \ 1 \\ \beta_1, \beta_2\end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n (\alpha_2)_n (\alpha_3)_n}{(1)_n (\beta_1)_n (\beta_2)_n}, \tag{5.1} $$

where

$$ (\alpha)_n = \alpha(\alpha+1)(\alpha+2)\cdots(\alpha+n-1), \text{ with } (\alpha)_0 = 1. \tag{5.2} $$

His Chapter XII "contains in a condensed form practically everything that is known about the summation and transformation of this series... It contains 47 main theorems, many of them followed by a separate statement of a number of corollaries and particular cases. Thus, formula 40 is followed by 'examples' 1–18."

S.R. Ranganathan recalls in his book, Ramanujan: The Man and the Mathematician, a conversation he had, in March 1925, with Hardy, in which talking about Hardy's work on a Chapter of Ramanujan on hypergeometric series, Hardy said that "This work has taken me several weeks. If I am to edit the entire notebook, it will take the whole of my life-time. I cannot do my own work: This would not be proper."

This author (K. Srinivasa Rao) and K. Venkatesh used the Horner scheme for polynomial evaluation, developed by Horner (see for instance, John A.N. Lee, "Numerical Analysis for Computers", Reinhold Pub. Corpn, 1966) to write the polynomial in the folded form:

$$ \begin{aligned} f(x) &= a_n x^n + a_{n-1}x^{n-1} + \cdots + a_x + a_0 \\ &= ((\cdots(((a_n x + a_{n-1})x + a_{n-2})x + a_{n-3})x + \cdots + a_1)x + a_0), \end{aligned} \tag{5.3} $$

[4] Bruce C. Berndt, *Ramanujan Notebooks*, Part II, Springer-Verlag, New York (1987).

so that the number of multiplications is also n same as the number of additions, and thus the number of multiplications is reduced from $O(n^2)$ to $O(n)$ for large n. This is a perceptible reduction in computer time (ref. K. Srinivasa Rao and K. Venkatesh, Computer Phys. Commun. **15** (1978) 227.) and the hypergeometric series has this folded form built into it. This is exhibited by the following formula:

$$ {}_pF_q\left(\begin{matrix}\alpha_1,\alpha_2,\cdots,\alpha_p;\\ \beta_1,\beta_2,\cdots,\beta_q\end{matrix} z\right) = \left[1+\frac{x_0}{y_0}\left(z+\frac{x_1}{y_1}\left(z+\frac{x_2}{y_2}\left(x+\cdots\right)\right)\right)\right] \tag{5.4} $$

where

$$ x_i=\prod_{j=1}^{p}(\alpha_j+i) \qquad \text{and} \qquad y_i=(i+1)\prod_{k=1}^{q}(\beta_k+i). \tag{5.5} $$

This folded form of the hypergeometric series is also useful to recognize or identify the variables in the hypergeometric function.

To illustrate the work of Ramanujan on this topic, we look at a few of "curious formulae" of his, in the standard notation of Barnes for generalized hypergeometric series:

$$ 1+3\frac{x-1}{x+1}+5\frac{x-1}{x+1}\frac{x-2}{x+2}+\cdots=x \tag{5.6} $$

It can be shown that (5.6) corresponds to:

$$ {}_3F_2\left(\begin{matrix}1,\frac{3}{2},-x+1;\\ \frac{1}{2},x+1\end{matrix}-1\right)=x, \tag{5.7} $$

which is a special case of

$$ {}_3F_2\left(\begin{matrix}a,\frac{a}{2}+1,b;\\ \frac{a}{2},1+a-b\end{matrix}-1\right)=\frac{\Gamma(\frac{a}{2}+\frac{1}{2},1+a-b)}{\Gamma(a+1,\frac{a}{2}+\frac{1}{2}-b)} \tag{5.8} $$

for $a=1, b=-x+1$. Another series formula of Ramanujan is the alternating sum:

$$ 1-3\frac{x-1}{x+1}+5\frac{x-1}{x+1}\frac{x-2}{x+2}-\cdots=0 \tag{5.9} $$

a special case of Dixon's theorem specialized to terminating series:

$$ {}_3F_2\left(\begin{matrix}a, & b, & -n;\\ 1+a-b, & 1+a+n\end{matrix}1\right)=\frac{(1+a)_n(1+\frac{a}{2}-b)_n}{(1+\frac{a}{2})_n(1+a-b)_n}, \tag{5.10} $$

for $a = 1, b = \frac{3}{2}, n = x - 1$. A third formula of Ramanujan reads:

$$1 + 3\left(\frac{x-1}{x+1}\right)^2 + 5\left(\frac{x-1}{x+1}\frac{x-2}{x+2}\right)^2 + \cdots = \frac{x^2}{2x-1} \tag{5.11}$$

which corresponds to

$${}_4F_3\left(\begin{matrix}1, \frac{3}{2}, -x+1, -x+1; 1\\ \frac{1}{2}, x+1, x+1\end{matrix}\right) = \frac{x^2}{2x-1}, \tag{5.12}$$

which is a special case of the summation theorem:

$$\begin{aligned}&{}_4F_3\left(\begin{matrix}a, & \frac{a}{2}+1, & b, & c\,;\ 1\\ \frac{a}{2}, & 1+a-b, & 1+a-c\end{matrix}\right)\\ &\quad = \frac{\Gamma(\frac{a}{2}+\frac{1}{2}, 1+a-b, 1+a-c, \frac{1}{2}+\frac{a}{2}-b-c)}{\Gamma(a+1, \frac{1}{2}+\frac{a}{2}-b, \frac{1}{2}+\frac{a}{2}-c, 1+a-b-c)}\end{aligned} \tag{5.13}$$

for $a = 1, b = c = -x + 1$. Another formula reads:

$$1 - 3\left(\frac{x-1}{x+1}\right)^2 + 5\left(\frac{x-1}{x+1}\frac{x-2}{x+2}\right)^2 + \cdots = \frac{\Gamma(x+1)^2}{\Gamma(2x)} \tag{5.14}$$

which corresponds to

$${}_4F_3\left(\begin{matrix}1, & \frac{3}{2}, & -x+1, & -x+1\,;\ -1\\ \frac{1}{2}, & x+1, & x+1\end{matrix}\right) = \frac{\Gamma(x+1)^2}{\Gamma(2x)}, \tag{5.15}$$

a special case of

$${}_4F_3\left(\begin{matrix}a, & \frac{a}{2}+1, & b, & c\,;\ -1\\ \frac{a}{2}, & 1+a-b, & 1+a-c\end{matrix}\right) = \frac{\Gamma(1+a-b, 1+a-c)}{\Gamma(1+a, 1+a-b-c)}, \tag{5.16}$$

for $a = 1, b = c = -x + 1$. A fifth formula reads:

$$1 + \frac{x-1}{x+1} + \frac{x-1}{x+1}\frac{x-2}{x+2} + \cdots = 2^{2x-1}\frac{\Gamma(x+1)^2}{\Gamma(2x+1)}, \tag{5.17}$$

which corresponds to:

$${}_2F_1\left(\begin{matrix}1, -x+1; -1\\ x+1\end{matrix}\right) = 2^{2x-1}\frac{\Gamma(x+1)^2}{\Gamma(2x+1)}, \tag{5.18}$$

a special case of Kummer's theorem:

$$ {}_2F_1\left(\begin{matrix} a, & b; -1 \\ 1+a-b & \end{matrix}\right) = \frac{\Gamma(1+\frac{a}{2}, 1+a-b)}{\Gamma(1+a, 1+\frac{a}{2}-b)}, \tag{5.19} $$

for $a = 1, b = 1 - x$, obtained after using the duplication formula:

$$ 2^{2x}\Gamma\left(x+\frac{1}{2}\right)\Gamma(x+1) = \sqrt{\pi}\Gamma(2x+1). \tag{5.20} $$

Instead of (5.17), if we have an alternating sum

$$ 1 - \frac{x-1}{x+1} + \frac{x-1}{x+1}\frac{x-2}{x+2} - \cdots = \frac{x}{2x-1}, \tag{5.21} $$

it corresponds to a special case of the Gauss summation theorem:

$$ {}_2F_1(a, b; c; 1) = \frac{\Gamma(c, c-a-b)}{\Gamma(c-a, c-b)}, \tag{5.22} $$

for $a = 1, b = 1 - x, c = 1 + x$. A variant of the above alternating sum:

$$ 1 - \frac{1}{3}\frac{x-1}{x+1} + \frac{1}{5}\frac{x-1}{x+1}\frac{x-2}{x+2} - \cdots = \frac{2^{4x}(\Gamma(x+1)^4}{4x\Gamma(2x+1))^2}, \tag{5.23} $$

corresponds to

$$ {}_3F_2\left(\begin{matrix} 1, & \frac{1}{2}, & -x+1; 1 \\ \frac{3}{2}, & x+1 & \end{matrix}\right) = \frac{2^{4x}(\Gamma(x+1))^4}{4(\Gamma(2x+1))^2}, \tag{5.24} $$

a special case of Dixon's theorem:

$$ {}_3F_2\left(\begin{matrix} a, & b, & -n; 1 \\ 1+a-b, & 1+a+n & \end{matrix}\right) = \frac{(1+a)_n(1+\frac{a}{2}-b)_n}{(1+\frac{a}{2})_n(1+a-b)_n}, \tag{5.25} $$

for $a = 1, b = \frac{1}{2}$ and $n = x-1$, obtained after using the duplication formula (5.20). Yet another example of this type is the result:

$$ 1 + \left(\frac{x-1}{x+1}\right)^2 + \left(\frac{x-1}{x+1}\frac{x-2}{x+2}\right)^2 + \cdots = \frac{2x}{4x-1}\left(\frac{\Gamma(x+1)}{\Gamma(2x+1)}\right)^4 \Gamma(4x+1) \tag{5.26} $$

which corresponds to:

$$ {}_3F_2\left(\begin{matrix} 1, & 1-x, & 1+x; 1 \\ 1-x, & 1+x & \end{matrix}\right) = \frac{2x}{4x-1}\left(\frac{\Gamma(x+1)}{\Gamma(2x+1)}\right)^4 \Gamma(4x+1) \tag{5.27} $$

This is a special case of Dixon's theorem:

$$
\begin{aligned}
&{}_3F_2\left(\begin{matrix} a, & b, & c \\ 1+a-b, & 1+a-c & \end{matrix}; 1\right) \\
&\quad = \frac{\Gamma(\frac{a}{2}+1, 1+a-b, 1+a-c, 1+\frac{a}{2}-b-c)}{\Gamma(a+1, 1+\frac{a}{2}-b, 1+\frac{a}{2}-c, 1+a-b-c)},
\end{aligned} \tag{5.28}
$$

for $a = 1, b = c = 1 - x$, obtained after using the duplication formula (5.13). The formulae (5.3), (5.9), (5.23) and (5.26) are given in the examples to be proved (by the reader) in Bailey's monograph.[5]

However, the key formula in Ramanujan's Chapter XII of the first Notebook (and Chapter X of the second Notebook) in the modern day notation is:

$$
\begin{aligned}
&{}_7F_6\left(\begin{matrix} s, 1+\frac{1}{2}s, -x, -y-, -z, -u, x+y+z+u+2s+1 \\ \frac{1}{2}s, x+s+1, y+s+1, z+s+1, u+s+1, v+s+1 \end{matrix}; 1\right) = \\
&\frac{1}{\Gamma(s+1, x+y+z+u+s+1)} \prod_{x,y,z,u} \frac{\Gamma(x+s+1, y+z+u+s+1)}{\Gamma(z+u+s+1)} \\
&\times \prod_{x,y,z,u} \frac{\Gamma(x+s+1, y+z+u+s+1)}{\Gamma(z+u+s+1)}
\end{aligned} \tag{5.29}
$$

where we have used the notation

$$
\Gamma(x, y, \cdots) = \Gamma(x)\Gamma(y)\cdots \tag{5.30}
$$

Hardy states: "In algebra, Ramanujan's main work was concerned with hypergeometric series and continued fractions (I use the word algebra, of course, in its old-fashioned sense). These subjects suited him exactly, and here he was unquestionably one of the greatest masters. There are now three famous identities, the 'Dougall-Ramanujan identity'... and the 'Rogers-Ramanujan identities', in which he had been anticipated by British mathematicians... As regards hypergeometric series one may say, roughly, that he discovered the formal theory, set out in Bailey's tract, as it was known up to 1920. There is something about it in Carr, and more in Chyrstal's 'Algebra', and no doubt he got his start from that.... Ramanujan seems to have found the formula about 1910 or 1911, but he had been anticipated by Dougall. The formula looks formidable, but Dougall's proof is very simple. I imagine that Ramanujan argues similarly, but there is nothing in the notebooks to show" (ref. p.12 and p.102 of Hardy's *Ramanujan: Twelve Lectures on subjects suggested by his life and work*).

[5] W.N. Bailey, *Generalized Hypergeometric Series*, Cambridge Tracts in Mathematics and Mathematical Physics, Hafner Publishing Company, New York, 1964.

Saalschütz's theorem (1890) for a terminating ${}_3F_2(1)$ and Dixon's theorem (1903) for any convergent well-poised series:[6] ${}_3F_2(a, b, c; 1+a-b, 1+a-c; 1)$, are two of the important results of Ramanujan obtained as limiting cases of this key formula, christened by Hardy as the Dougall-Ramanujan summation theorem for a well-poised ${}_7F_6(1)$.

Ramanujan seems to have never published this result and it is found only in his Notebook. Most of the results of Ramanujan have been derived from this key result.

Basic hypergeometric series were first introduced and studied by Heine, soon after Gauss introduced the (ordinary) hypergeometric series. He replaced the parameters a, b, c in the ${}_2F_1(a, b; c; z)$) by "basic" numbers

$$\frac{1-q^a}{1-q} \equiv [a]_q, \tag{5.31}$$

similarly for b and c, where $0 < q < 1$ and the resultant series is called the basic hypergeometric series denoted by

$${}_2\phi_1(q^a, q^b; q^c; z) \equiv {}_2\phi_1(a, b; c; z) \tag{5.32}$$

which in the limit $q \to 1$ becomes the Gauss series, ${}_2F_1(a, b; c; z)$. q-extensions of almost all the known ordinary and special functions of mathematical physics were obtained since Heine's basic hypergeometric series was introduced. W.N. Bailey's and Lucy J. Slater's monographs are the early books which contain these extensions. The renowned Rogers-Ramanujan identities have been shown to follow from a series transformation formula which contains the q-extension of the very well-poised 2-balanced ${}_7F_6(1)$ sum.

5.2 In the Notebooks of Ramanujan

In Chapters XII, XIII and XV of the first Notebook and Chapters X and XI of the second Notebook, Ramanujan's Entries are on hypergeometric series. His chapters X and XI of the second Notebook have, respectively, 35 and 36 Entries on the hypergeometric series, for each one of these, Berndt in his "Ramanujan's Notebooks", Part II, has provided a proof. The only difference is that Bruce Berndt has chosen to present the Entries in the modern day Barnes's notation (not used by Ramanujan). Ramanujan wrote down the first few terms in each series and it was easy for him to discern the elegance in the formulas involving the series. In this

[6] In ${}_AF_B(a_1, a_2, \cdots, a_A\ ;\ b_1, b_2, \cdots, b_B\ ;\ z)$, if the sum of the denominator parameter exceeds the sum of the numerator parameters by 1, i.e. $b_1 + b_2 + \cdots + b_B = a_1 + a_2 + \cdots a_A + 1$, then the series is said to be a Saalschützian. If $A = B + 1$ and $1 + a_1 = b_1 + a_2 = b_2 + a_3 = \cdots = b_B + a_B + 1$, then the series is said to be well-poised. The nomenclatures was introduced by Whipple in 1925, after Ramanujan's time.

context, Richard A. Askey's article entitled, "Ramanujan and Hypergeometric and Basic Hypergeometric Series" in the Proceedings of the Ramanujan International Symposium on Analysis, held on December 26–28, 1987 at Kandy, Sri Lanka, to commemorate the centenary of Ramanujan's birth, is a good starting point for the interested student who wants to learn about Ramanujan's comprehensive Entries on ordinary and basic hypergeometric series. The Invited speakers to this Symposium were Professors R. A. Askey, B.C. Berndt, G. Bhamathi, K. Srinivasa Rao, E.C. G. Sudarshan, M. Washington and D. Zagier. The proceedings were edited by N.K. Thakare, K.C. Sharma and T.T. Raghunathan, for Macmillan (India) Ltd. (1989).

Ramanujan obtained results on continued fractions from the three-term recurrence relations he derived for himself for the hypergeometric series. He rediscovered not only many of the classical results of Euler, Gauss, Heine and others on hypergeometric series and continued fractions but also obtained many new results. His "Lost" Notebook contains many results related to Heine's q-series, and some of the q-continued fractions are related to the modular functions. K.G. Ramanathan wrote two papers on "Ramanujan's continued fractions", Indian J. Pure and Applied Mathematics, **16** (1985) 695–724 and "Hypergeometric series and continued fractions", Proceedings of the Indian Academy of Sciences (Mathematical Sciences) **97** (1987) 277–296. In the words of K.G. Ramanathan: "The exploitation of this relationship between modular functions and continued fractions is one of the most beautiful aspects of Ramanujan's work on continued fractions."

Hardy tried to "annotate" one chapter of the Notebooks, as mentioned earlier, and after that daunting experience decided to go back to his own research work and let it to G. N. Watson to take up the job of fully editing the notebooks with B. M. Wilson. However, as pointed out in an earlier section, this work had to wait for more than five decades, after the death of Ramanujan in 1920, before Bruce C. Berndt started working on the Entries of Ramanujan in his Notebooks, from February 1974 to May 1977, when Part I and Part V of this work were published by Springer-Verlag. This untiring 22-year efforts of Berndt with his students (19 Ph.D. theses contained these researches) and collaborators culminated in the publication of the five-volume work entitled Ramanujan's Notebooks.

The general impression that one has from the various writings on the Notebooks of Ramanujan by the experts is that Ramanujan obtained generalizations from examples or special cases. Chapter XII of the second Notebook and the corresponding Chapter X of the first Notebook are an exception to this, in that the first Entry in both these is on the most general theorem for a ${}_7F_6(1)$ summation theorem, known in literature today as the Dougall-Ramanujan summation theorem, discovered independently by Dougall in 1909. From this first Entry, Ramanujan derived scores of special cases establishing thereby the Gauss, Chu-Vandermonde, Kummer, Pfaff-Saalschütz and Dixon summation theorems.

It is a well-known fact that Ramanujan did not publish any of his results from his Chapters XII and XIII in the first Notebook (and the corresponding Chapters X and XI of the second Notebook) on hypergeometric series either from India or

from Cambridge, as Ramanujan in a letter to his friend Gopala Rao wrote that he was discovering new results and had no time to get his theorems in his Notebooks published.

As stated earlier, the second Notebook is considered as a "revised, enlarged" version of the Entries in various Chapters in the first Notebook. However, Example 7, after Entry 43, in Chapter XII of the first Notebook does not find a place in the "corresponding" examples after Entry 10 in Chapter X of the second Notebook and is therefore not discussed by Berndt, who mentions in Part IV, p. 410, that this example is "incorrect". Although this observation of Berndt is, in a literal sense, true, it is **not** the whole truth. For, this author (K. Srinivasa Rao), Vanden Berghe and Krattenthaler have shown in their publication entitled: *An Entry of Ramanujan on Hypergeometric Series in his Notebooks*, which is presented in the next section, that it is correct except for a missing multiplication factor.

5.3 An Entry in Ramanujan's Notebook

This section is intended to illustrate how it is possible to get new results from the Entries of Ramanujan in his Notebooks. It is a section which is intended for the mathematicians and not for other readers, who will get a flavour of the mathematics in a period when discovering formulas was considered as the main thrust area for research in mathematics—at the beginning of the twentieth century. The reader should also know that the experts estimate that the "elementary" mathematical discoveries of Ramanujan were only ∼5% of the 4000 theorems discovered by Ramanujan—3254 in his Notebooks, in addition to the theorems in his 39 published research papers and the 57 Questions and/or Answers to Questions which he proposed in the Journal of the Indian Mathematical Society—in his short lifespan of 32 years 4 months and 4 days. It is the author's conjecture that the Tamizh epic *Naalaayira Divya Prabhandam (Four thousand divine-verses)* and the Carr's *Synopsis*, which is a catalog of about 4000 mathematical theorems, might have set the impressionable mind of the child prodigy Ramanujan to set his aim as leaving behind about 4000 theorems as a legacy for posterity to remember him and his contributions to Mathematics.

The author, with Vanden Berghe and Krattenthaler,[7] provides a proof for Example 7, after Entry 43, in Chapter XII of the first Notebook of Ramanujan, and derives a new summation theorem for

$$ {}_3F_2(a, a, x; 1 + a, 1 + a + N; 1), \text{ where } N \text{ is a non-negative integer.} $$

In the Notebooks of Ramanujan, identities for hypergeometric series occupy a prominent part, and he discovered by himself all the "classical" summation

[7] K. Srinivasa Rao, G. Vanden Berghe and C. Krattenthaler, J. Comput. Appl. Math. **173** (2) (2005) 239–246.

theorems—viz. those referred to as the Chu-Vandermonde, Kummer, Pfaff-Saalschütz, Dixon and Dougall summation theorems—with only the hint of the Gauss summation theorem from Carr's *Synopsis*, as stated earlier. Dougall's summation theorem is the most general, as all others mentioned are special cases when the parameters in the ${}_7F_6(1)$ are restricted. While Dougall's theorem[8] was published as a paper in (1907), no date can be assigned to the Entry 1 in Chapter X of the second Notebook of Ramanujan, as there are no dates anywhere in the Notebooks of Ramanujan. In any case, Ramanujan rediscovered not only all that was known in Europe on hypergeometric series at that time but also discovered several new theorems. In particular, theorems on products of hypergeometric series[9] as well as several asymptotic expansions.

It is also well known that these results have not been published by Ramanujan. The Chapters on hypergeometric series, Chapters X and XI in his second Notebook, have been extensively studied by Hardy[10] and Berndt.[11] However, the Example 7, after Entry 43, in the Chapter XII of the first Notebook does not find a place in the examples after Entry 10 in Chapter X of his second Notebook. Surprisingly this escaped the attention of Bruce Berndt and his collaborators who provided a proof for every one of the (3254) Entries of Ramanujan in his second Notebook as it was considered as a revised, enlarged version of the first Notebook. The author with Vanden Berhe and Krattenthaler[12] has provided a proof for Example 7, after Entry 43, in Chapter XII of the first Notebook of Ramanujan, using well-known transformation and summation theorems of hypergeometric series. In fact, they proved a more general identity, which perhaps Ramanujan had and from which he recorded the most elegant specializations in his Notebooks—a conjecture consistent with a widespread belief that is the style of Ramanujan!

In the non-standard notation of Ramanujan, Example 7, after Entry 43, in Chapter XII of his first Notebook is:

$$\frac{\pi}{\tan(\pi x)}\frac{\underline{|2x}}{(2^x\underline{|x})^2\,(1-2x)}\left(\sum\frac{1}{2x}-\frac{1}{2}\sum\frac{1}{x}+\frac{1}{1-2x}-\frac{\pi}{2}\tan(\pi x)\right)$$
$$=\frac{1}{1^2}+\frac{x}{\underline{|1}}\cdot\frac{1}{3^2}+\frac{x(x+1)}{\underline{|2}}\cdot\frac{1}{5^2}+\&c,\qquad (XII,43,Ex.7)\qquad (5.33)$$

[8] J. Dougall, *On Vandermonde's theorem and some more general theorems*, Proc. Edinburgh Math. Soc. **25** (1907) 114–132.

[9] See, for instance, L. J. Slater, *Generalized hypergeometric functions*, Cambridge Univ. Press, Cambridge (1966).

[10] G.H. Hardy, *A chapter from Ramanujan's Notebook*, Prof. Camb. Phil. Soc. **21** (1923), 429–503. Also, Chapter VII in G.H. Hardy, *Ramanujan: Twelve lectures on subjects suggested by his life and work*, Chelsea, New York (1944).

[11] Bruce C. Bendt, *Ramanujan's Notebooks*, Part II, Springer-Verlag, New York (1987), Chapter X, pp. 7–47.

[12] K. Srinivasa Rao, G. Vanden Berghe and C. Krattenthaler, *An entry of Ramanujan on hypergeometric series in his Notebooks*, J. Comput. Appl. Math. **173 (2)** (2005) 239–246.

where $\underline{|x}$ is the notation of Ramanujan[13] for the gamma function $\Gamma(x+1)$, which for him was a function over real numbers x. Here, we use the contemporary notation and regard the gamma function as a function over complex numbers.

We observe that the factor on the left-hand side of Entry 43, Ex. 7 in Chapter XII:

$$\frac{\pi}{\tan(\pi x)}\ \frac{\underline{|2x}}{(2^x\underline{|x})^2\ (1-2x)} \tag{5.34}$$

is the same fact that appears on the left-hand side of the Entry 43, Ex. 4 in Chapter XII, which in Ramanujan's first Notebook is

$$\frac{\pi}{\tan(\pi x)}\ \frac{\underline{|2x}}{(2^x\underline{|x})^2\ (1-2x)}\ =\ 1+\frac{x}{\underline{|x}}\cdot\frac{1}{3}+\frac{x(x+1)}{\underline{|2}}\cdot\frac{1}{5}+\&c. \tag{5.35}$$

This is given in Ramanujan's second Notebook as:

$$\frac{\sqrt{\pi}\underline{|n}}{2\underline{|n+\frac{1}{2}}}\ =\ 1-\frac{n}{\underline{|1}}\cdot\frac{1}{3}+\frac{n(n-1)}{\underline{|2}}\cdot\frac{1}{5}+\&c. \tag{5.36}$$

The Example 4, after Entry 10, in Chapter X (or, Ex.4, Entry 43, Chapter XII) is the special case of the corresponding Entry 10, or, Entry 43, provided we do the replacements $n\to\frac{1}{2}$ and $x\to n$. The factor in (5.35) which occurs on the left-hand side of Ex. 4, Entry 43, Chapter XII, can be shown to be equal to:

$$\frac{\sqrt{\pi}\Gamma(1-x)}{2x\Gamma(\frac{3}{2}-x)}\ =\ \frac{\sqrt{\pi}\ \underline{|-x}}{2x\ \underline{|-x+\frac{1}{2}}}, \tag{5.37}$$

after using the reflection formula

$$\Gamma(z)\ \Gamma(1-z)\ =\ \frac{\pi}{\sin(\pi z)}, \tag{5.38}$$

and the duplication formula

$$\Gamma(2z)\ =\ 2^{2z-1}\ \pi^{-1/2}\Gamma(z)\Gamma(z+\frac{1}{2}), \tag{5.39}$$

and some algebraic simplifications.

When we use the standard notation for the hypergeometric function and the Pochhammer symbol

$$(\alpha)_k\ =\ \alpha(\alpha+1)(\alpha+2)\cdots(\alpha+k-1),\quad k>0,\quad (\alpha)_0=1, \tag{5.40}$$

[13]Refer Ramanujan's first letter to G.H. Hardy, as quoted in *Ramanujan: Letters and Commentary*, Bruce C. Berndt and Robert A. Rankin, (Eds.), AMS-LMS (1995).

the series on the right-hand side of (5.35)

$$ {}_2F_1(\frac{1}{2}, x; \frac{3}{2};\ 1), \tag{5.41} $$

by the Gauss summation theorem

$$ {}_2F_1(A, B; C; 1) \ = \ \frac{\Gamma(C)\Gamma(C-A-B)}{\Gamma(C-A)\Gamma(C-B)}, \text{for } \mathrm{Re}(C-A-B) > 0, \tag{5.42} $$

becomes

$$ {}_2F_1(A, B;\ C;\ 1) \ = \ \frac{\sqrt{\pi}\Gamma(1-x)}{2\Gamma(3/2-x)} \ = \ \frac{\sqrt{\pi}}{2}\ \frac{\lfloor -x}{\lfloor -x+\frac{1}{2}}. \tag{5.43} $$

If we compare the right-hand side of this equation with (5.37), which is equal to the left-hand side (5.35), it is clear that Ramanujan missed a multiplicative factor x on the left-hand side of (5.35), while his Entry (5.36) is correct. The same applies to the Entry (5.37). To be precise, the factor (5.34) on the left-hand side of that Entry must also be replaced by the correct value (5.43).

The series on the right-hand side of Entry 43, Example 7 of Chapter XII is, in the hypergeometric notation

$$ {}_3F_2\left(x, \frac{1}{2}, \frac{1}{2};\ \frac{3}{2}, \frac{3}{2};\ 1\right). \tag{5.44} $$

There are two factors on the left-hand side of (5.33). Besides the factor (5.34), the other factor in (5.33) is

$$ \sum \frac{1}{2x} - \frac{1}{2}\sum \frac{1}{x} + \frac{1}{1-2x} - \frac{\pi}{2}\tan(\pi x). \tag{5.45} $$

Ramanujan used the notation $\sum \frac{1}{x}$ to indicate the extension of the function

$$ 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}, \tag{5.46} $$

representing the harmonic numbers, from positive integers n to real x. In other words, $\sum \frac{1}{x}$ is Ramanujan's notation for the *digamma function*

$$ \psi(x) := \frac{\Gamma'(x)}{\Gamma(x)}, \tag{5.47} $$

the logarithmic derivative of the gamma function, or more precisely, for $\psi(x+1)+\gamma$, where γ is the Euler-Mascheroni constant, that is

$$\psi(x+1) \;=\; \sum \frac{1}{x} - \gamma. \tag{5.48}$$

In fact, Ramanujan's very first research on Bernoulli numbers,[14] the digamma function occurs as

$$\frac{d}{dn}\log\Gamma(n+1) \;=\; 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\gamma \;=\; \sum\frac{1}{n}-\gamma. \tag{5.49}$$

Thus, the expression (5.45) becomes

$$\gamma(2x+1)-\frac{1}{2}\psi(x+1)+\frac{1}{1-2x}-\frac{\pi}{2}\tan\pi x+\frac{\gamma}{2}. \tag{5.50}$$

The digamma function satisfies the relation[15]

$$\psi(z+1) \;=\; \psi(z)+\frac{1}{z}, \tag{5.51}$$

the reflection formula[16]

$$\psi(-z) \;=\; \psi(z+1)+\pi\cot(\pi z) \tag{5.52}$$

and the duplication formula

$$2\psi(2z) \;=\; \psi(z)+\psi\left(z+\frac{1}{2}\right)+2\log 2. \tag{5.53}$$

We remark that the duplication formula appears explicitly in Ramanujan's first Notebook. Namely, a comparison of Entry 43, Example 6, in Chapter XII, with $n \to x-1$ in Entry 10, Example 6, Chapter X, shows that Ramanujan obtained

$$\sum\frac{1}{x+\frac{1}{2}}-\sum\frac{1}{x}-2\sum\frac{1}{2x}-2\sum\frac{1}{x}+\frac{1}{x}-2\log 2, \tag{5.54}$$

[14] S. Ramanujan, *Some properties of Bernoulli's numbers, J. Indian Math. Soc.* ***3*** *(1911), 219–234.*

[15] A. Erdelyi, V. Magnus, F. Oberhettinger and F. Tricomi, *Higher transcendental functions*, McGraw Hill, New York (1953), 1.7.1(8).

[16] A. Erdelyi, V. Magnus, F. Oberhettinger and F. Tricomi, *Higher transcendental functions*, McGraw Hill, New York (1953), 1.7.1(11).

which by (5.49) and (5.51) is equivalent to (5.53). Furthermore, the digamma function has the special values

$$\psi\left(\frac{1}{2}\right) = -\gamma - 2\log 2 \qquad \text{and} \qquad \psi(1) = -\gamma. \tag{5.55}$$

We now use the duplication formula (5.53) to convert (5.50) into

$$\frac{1}{2}\psi\left(x+\frac{1}{2}\right) + \log 2 + \frac{1}{1-2x} - \frac{\pi}{2}\tan(\pi x) + \frac{\gamma}{2}. \tag{5.56}$$

The recurrence relation (5.51) implies that

$$\psi\left(x+\frac{1}{2}\right) = \psi\left(x-\frac{1}{2}\right) + \frac{2}{2x-1}, \tag{5.57}$$

while we know from (5.55) that

$$\log 2 = -\frac{1}{2}\psi\left(\frac{1}{2}\right) - \frac{1}{2}\gamma. \tag{5.58}$$

If this is substituted in the last equation, and if we then apply the reflection formula (5.52), we obtain

$$\frac{1}{2}\left(\psi\left(\frac{3}{2}-x\right) - \psi\left(\frac{1}{2}\right)\right) \tag{5.59}$$

for the expression (5.45). Therefore, if we recall that the factor (5.34) on the left-hand side of Ramanujan's Entry 43, Example 7 of Chapter XII (5.33), must be replaced by (5.35), this Entry can be rewritten in contemporary notation as:

$$\begin{aligned} {}_3F_2\left(x, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; 1\right) &= \frac{\sqrt{\pi}}{4}\frac{\Gamma(1-x)}{\Gamma(\frac{3}{2}-x)}\left\{\psi\left(\frac{3}{2}-x\right) - \psi\left(\frac{1}{2}\right)\right\} \\ &\frac{1}{2}\left\{\psi\left(\frac{3}{2}-x\right) - \psi\left(\frac{1}{2}\right)\right\} {}_2F_1\left(x, \frac{1}{2}; \frac{3}{2}; 1\right) \qquad \text{for} \quad \Re x < 2. \end{aligned} \tag{5.60}$$

In view of (1.2.13) in Andrews et al.:[17]

$$\psi(x) - \psi(y) = \sum_{k=0}^{\infty}\left(\frac{1}{k+y} - \frac{1}{k+x}\right), \tag{5.61}$$

the first line of (5.60) is completely equivalent with the second displayed line on p. 410 of Berndt.[18]

[17] G.E. Andrews, R.A. Askey and R. Roy, *Special Functions*, Encycl. Math. and its Applns. **71**, Cambridge Univ. Press, Cambridge (1999).

[18] B.C. Berndt *Ramanujan's Notebooks*, Part IV, Springer-Verlag, New York (1994).

5.4 A New Summation Theorem and its Proof

We now state and prove a theorem which is a generalization of (5.33), which is Entry 43, Example 7, in Chapter XII of Ramanujan's Notebook 1.

Theorem *Let N be a non-negative integer and a be a complex number. If $\Re x < N+2$, then*

$$
\begin{aligned}
&{}_3F_2(a, a, x;\ a+1, a+N+1;\ 1) \\
&\quad = \frac{a\Gamma(a+N+1)\Gamma(1-x)}{N!\,\Gamma(a-x+1)}(\psi(a-x+1)-\psi(a)-\psi(N+1)-\gamma) \\
&\quad - \frac{a\Gamma(a+N+1)\Gamma(1-x)}{N!\,\Gamma(a-x+1)}\sum_{k=1}^{N}\frac{(a)_k(-N)_k}{k\cdot k!(a-x+1_k)}.
\end{aligned}
\tag{5.62}
$$

Proof To evaluate the ${}_3F_2(1)$ series on the left-hand side of (5.62), we introduce a parameter ε, and consider the series

$$
{}_3F_2(a, a-\varepsilon, x;\ 1+a-\varepsilon, 1+a-\varepsilon+N;\ 1) \tag{5.63}
$$

First we apply the non-terminating transformation formula—Ex. 3.6, $q \to 1$, reversed[19]

$$
\begin{aligned}
&{}_3F_2(A, B, C;\ D, E;\ 1) \\
&= \frac{\Gamma(A-B, D, E, S)}{\Gamma(A, D-B, E-B, S+B)}\,{}_3F_2(B, D-A, E-A;\ 1+A-B, S+B;\ 1) \\
&+ \frac{\Gamma(B-A, D, E, S)}{\Gamma(B, D-A, E-A, S+A)}{}_3F_2(A, D-B, E-B;\ 1+A-B, S+A;\ 1),
\end{aligned}
\tag{5.64}
$$

where, the parameter excess $S = D+E-A-B-C$ and we have used the notation

$$
\Gamma(x, y, z, \cdots) = \Gamma(x)\Gamma(y)\Gamma(z)\cdots . \tag{5.65}
$$

and obtain the result

$$
\begin{aligned}
&\frac{\Gamma(1+a-\varepsilon, \varepsilon, 1+a-\varepsilon+N, 2-\varepsilon+N, \varepsilon+N-x)}{\Gamma(a, 1+N, 2+a-2\varepsilon+N-x)} \\
&\qquad \times {}_2F_1\left[\begin{matrix} a-\varepsilon, 1-\varepsilon+N \\ 2+a-2\varepsilon+N-x \end{matrix};\ 1\right]
\end{aligned}
$$

[19] G. Gasper and M. Rahman, *Basic hypergeometric series*, Encyclopedia of Math, and its Applns. **35**, Camb.. Univ. Press, Cambridge (1990).

$$+\frac{\Gamma(1+a-\varepsilon,-\varepsilon,1+a-\varepsilon+N,2-\varepsilon+N-x)}{\Gamma(1-\varepsilon,a-\varepsilon,1-\varepsilon+N,2+a-\varepsilon+N-x)}$$
$$\times {}_3F_2\left[\begin{matrix}a,1,1+N\\1+\varepsilon,2+a-\varepsilon+N-x\end{matrix};1\right] \tag{5.66}$$

Clearly, the convergence of the hypergeometric series on the right-hand side will only be guaranteed if $\operatorname{Re} x < 1$. Therefore, for the moment we suppose $\operatorname{Re} x < 1$.

To the ${}_3F_2(1)$, we apply the transformation formula[20]

$${}_3F_2(A,B,C;D,E;1)$$
$$=\frac{\Gamma(E,S)}{\Gamma(E-A,S+A)}{}_3F_2(A,D-B,D-C;D,S+A;1), \tag{5.67}$$

to get the expression

$$\frac{\Gamma(1+a-\varepsilon+N,2-\varepsilon+N-x)}{\Gamma(a,1+N,2+a-2\varepsilon+N-x)}\,{}_2F_1\left[\begin{matrix}a-\varepsilon,q-\varepsilon+N\\2+a-2\varepsilon+N-x\end{matrix};1\right]$$
$$+\frac{\Gamma(1+a-\varepsilon,-\varepsilon,1+a-\varepsilon+N,1-x)}{1-\varepsilon,a-\varepsilon,1-\varepsilon+N,1+a-x}$$
$$\times {}_3F_2\left[\begin{matrix}a,\varepsilon,\varepsilon-N\\1+\varepsilon,1+a-x\end{matrix};1\right] \tag{5.68}$$

The ${}_2F_1$-series is summed by means of the Gauss summation theorem (5.42), while the ${}_3F_2(1)$-series is written as a sum over k, and subsequently, split into the ranges $k=0$ $k=1,2,\ldots,N$, and $k=N+1,N+2,\ldots$. This yields the expression

$$\frac{1}{\varepsilon}\left(\frac{\Gamma(1+a-\varepsilon,1+\varepsilon,1+a-\varepsilon+N,1-x)}{a,1+N,1+a-\varepsilon-x}\right.$$
$$\left.-\frac{\Gamma(1+a-\varepsilon,1+a-\varepsilon+N,1-x)}{\Gamma(a-\varepsilon,1-\varepsilon+N,1+a-x)}\right)$$
$$-\frac{\Gamma(1+a-\varepsilon,1+a-\varepsilon+N,1-x)}{\Gamma(a-\varepsilon,1-\varepsilon+N,1+a-x)}\sum_{k=1}^{N}\frac{(a)_k(\varepsilon-N)_k}{k!(k+\varepsilon)(1+a-x)_k}$$
$$-\frac{\Gamma(1+a-\varepsilon,1+a-\varepsilon+N,1-x)}{\Gamma(a-\varepsilon,1-\varepsilon+N,1+a-x)}\sum_{k=N+1}^{\infty}\frac{(a)_k(\varepsilon-N)_k}{k!(k+\varepsilon)(1+a-x)_k} \tag{5.69}$$

[20] W.N. Bailey *Generalized hypergeometric series*, Camb. Univ. Press (1935), p. 98, Ex. 7.

Now we take the limit $\varepsilon \to 0$. Thus, our original ${}_3F_2$-series becomes:

$$ {}_3F_2(a, a, x;\ a + a, a + a + N;\ 1). \tag{5.70}$$

On the other hand, the four-line expression which we obtained for this ${}_3F_2$-series simplifies significantly. The last term simply vanishes because of the occurrence of the factor $(\varepsilon - N)_k$, which is equal to

$$ (\varepsilon - N)_k \ = \ (\varepsilon - N)_N (1 + \varepsilon)_{k-N-1}, \quad \text{for} \quad k \geq N + 1, \tag{5.71}$$

which becomes 0 as $\varepsilon \to 0$ for N a non-negative integer. On the other hand, the limit of $\varepsilon \to 0$ of the first two lines can be easily calculated by means of the l'Hôpital's rule to obtain the result (5.62).

As it stands, the assertion is only demonstrated for Re $x < 1$. However, by analytic continuation, Eq. (5.62) is true for any value of x for which the ${}_3F_2$-series on the left-hand side converges, i.e. for Re $x < N + 1$.

The authors (KSR-GvB-CK) conclude the article with the following observations:

- The theorem (5.62) reduces to Ramanujan's Example 7, after Entry 43, in Chapter XII, for $a = 1/2$ and $N = 0$.
- For $x = 1$, in (), the ${}_3F_2(1)$ is a special case of Dixon's theorem,[21] and it has the value $\frac{\pi^2}{8}$.
- For $x = \frac{3}{2}$, the left-hand side of () is:

$$ {}_3F_2(3/2, 1/2, 1/2; 3/2, 3/2; 1) \ = \ {}_2F_1(1/2, 1/2;\ 3/2; 1) \ = \ \frac{\pi}{2}, \tag{5.72}$$

  by the Gauss summation theorem (), which is the result for the right-hand side evaluate by l'Hôpital's rule.
- For $x = -k$, a negative integer, in (5.60), we get:

$$ {}_3F_2(1/2, 1/2, -k;\ 3/2, 3/2;\ 1) \ = \ \frac{\sqrt{\pi}}{2} \frac{\Gamma(k+1)}{\Gamma(k+3/2)} \sum_{j=1}^{k+1} \frac{1}{2j-1}. \tag{5.73}$$

- For N=0, the identity (5.60) reduces to

$$ {}_3F_2(a, a, x;\ 1 + a, 1 + a;\ 1) \ = \ \frac{\Gamma(a+1, 1-x)}{\Gamma(a-x+1)} (\psi(a - x + 1) - \psi(a)). \tag{5.74}$$

[21] See Ex. III.8 of [L.J. Slater, *Generalized hypergeometric series*, Cambridge Univ. Press (1966)], for $a = 1,\ b = c = 1/2$.

In view of (5.61), this is equivalent to Example 5 of Section 10 of Chapter 10 of Ramanujan's second Notebook.[22]

- The ${}_3F_2(a, a, x;\ 1+a, 1+a+N; 1)$ series can be related to the 3-j coefficient

$$\begin{pmatrix} -\frac{x}{2} & -\frac{x}{2} & 0 \\ a-\frac{x}{2} & -a+\frac{x}{2} & 0 \end{pmatrix} \tag{5.75}$$

provided x is a negative integer and $-x \leq a \leq 0$. It also corresponds to the dual Hahn polynomial[23] $S_n(0;\ a, b = 1, c = 1+N)$, for $x = -n$. Finally, as already mentioned, in Entry 43, Example 4, Chapter XII, Ramanujan missed a multiplicative factor x on the left-hand side of his Entry 43, Example 7, of Chapter XII, of Notebook 1.

[22] See, B.C. Berndt, *Ramanujan's Notebooks*, part II, Springer-Verlag, New York (1987), Chapter X, 7–47.

[23] G.E. Andrews, R. Askey and R. Roy, *Special Functions*, Encycl. Math and Its Applns. **71**, Camb. Univ. Press (1999).

Chapter 6
Chandrasekhar (Chandra) and Ramanujan

In this chapter, the role of Dr. S. Chandrasekhar, the Astrophysicist—who was awarded by the Swedish Royal Academy of Sciences, the Nobel Prize in Physics, for the year 1983, along with Dr. William Alfred Fowler (1911–1995), "for his theoretical studies of the physical processes of importance to the structure and evolution of the stars"—in the life of Ramanujan is presented.

6.1 S. Chandrasekhar: Nobel Laureate

Dr. Subrahmanyan Chandrasekhar (19 October 1910–21 August 1995), the renowned Indian-American Astrophysicist, who won the 1983 Nobel Prize in Physics, with Dr. William A. Fowler, for his discovery of the (later day) "Black Hole", had a great admiration for Ramanujan. Ramanujan's fame had its impact and influence on Chandrasekhar who was responsible for:

- The authentic passport photograph of Ramanujan reaching Hardy
- A role in the founding and continuance of the Ramanujan Institute
- A role in the making of the bronze bust of Ramanujan by Paul Granlund

S. Chandrasekhar, born in Lahore on 19 October 1910, was the third child of Mrs. Seethalakshmi and Mr. C. Subrahmanya Iyer, brother of Sir C.V. Raman. He was educated at home through his primary and lower secondary school years and later joined the Hindu High School, in Triplicane, Madras, in Form IV (present-day Standard VIII). He came to know of the death of Ramanujan (on April 26, 1920) from his mother when he was 9 years and 6 months old. He was a distinguished student of the Physics Honours class of Presidency College, Madras, when he came into contact with Professors Arnold Sommerfeld (1928) and Werner Heisenberg (1929)—winners of the Nobel Prize for Physics, in 1933 (Erwin Schrödinger and

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
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Paul Adrian Maurice Dirac) and 1932 (W. Heisenberg), respectively—when they visited India.

The books of Arnold Sommerfeld on *Atomic Structure and Spectral Lines* and A.H. Compton on *X-rays and Electrons*, inspired him to publish his first paper on *The thermodynamics of Compton effect with reference to the interior of the Stars*, in the Indian Journal of Physics, October 15, 1928 issue. Even before he completed his studies for his B.A. Honours degree at the University of Madras, he published several papers including the one in the Proceedings of the Royal Society (London) on *Compton scattering and the new Statistics*. With that start, he continued publishing an incessant stream of research papers, some running to more than hundred pages, in journals like the Proceedings of the Royal Society, Monthly Notices of the Royal Astronomical Society, the Astrophysical Journal and the Physical Review. He won Sir Arthur Eddington's book entitled *The Internal Constitution of Stars* (Cambridge University Press, 1926), as a Prize in a Physics essay contest and this was the beginning of his life-long association with Eddington. In 1930, he was offered a special scholarship to continue his studies and research at Trinity College, Cambridge, where he worked with Professors Ralph H. Fowler, his Ph.D. Supervisor and Paul A.M. Dirac and S.H. Milne. The award of "Sheep-Shanks" Prize was an early recognition of his abilities as an Astrophysicist. His was a meteoric rise to the pinnacle of glory marred by just one unfortunate incident which we now detail.

Chandrasekhar studied the problem of what happens to a star in its final stages of evolution in great detail. Stars vary greatly in size and luminosity. They range from those about the size of our planet to those hundreds of times larger than the Sun. The star nearest to the Earth is the Sun and compared to others it is a medium-sized star. Its mass and radius are the yardsticks for measuring the sizes of other stars, which are classified according to their surface temperature or colour. The smallest star known, called a white dwarf, is only about 1200 miles in diameter and has a density of nearly 1000 tones for cubic inch. This enormous density was explained, in 1925, by Ralph H. Fowler to be due to the tight packing of free electrons and ionized nuclei under extreme gravitational pressure referred to as a state of degenerate matter. Eddington's perfect gas model of the stars led him to believe that the white dwarf stage is the terminal stage in the evolution of all stars. It was at this juncture that Chandrasekhar took into account the special relativistic variation of the mass with velocity and Fermi-Dirac statistics to arrive at the *exact* equation of state for a complete degenerate gas. The prediction of the theory led him to the *Chandrasekhar limit—1.44 times the solar mass—for stars in their last stage of evolution, a detail worked out by him on his long voyage to Trinity College, Cambridge, in 1930*. He derived the dramatic result that stars with a mass greater than 1.44 times the solar mass would eventually collapse past the white dwarf stage into an object of enormous density and speculated the existence of (what are called today as) extremely dense neutron stars and black holes, discovered many years later. This is an over-simplified account of an extremely careful and beautiful state-of-the-art analysis of stellar evolution, by Chandrasekhar.

Chandrasekhar's confidence, based on the irrefutable relativistic theory, however had to confront the established authority of Sir Arthur Eddington who not only

refused to accept the result but felt intuitively: *Your [Chandrasekhar's] mathematics may be all right but I don't think your Physics is correct*. The human side of this episode is recorded in the biography *Chandra* by Kameshwar C. Wali (The University of Chicago Press, Chicago, 1991 and Penguin Books India, New Delhi, 1990) and in the extremely well-written vignette for the Physics student *Chandrasekhar and his limit* by G. Venkataraman (Universities Press, 1992). Also, the interested reader may refer to a biographical article *Dr. S. Chandrasekhar: the Astrophysicist*, in Physics Education (October–December issue, 1985, p. 7).

When Chandrasekhar wanted to present his results, before the Royal Astronomical Society in London, Eddington arranged for a 30 min. presentation, instead of the customary 15 min., on Jan. 11, 1935 but scheduled for himself to talk too. After Chandrasekhar's presentation, Eddington started with: "I do not know whether I shall escape from this meeting alive, but the point of my paper is that there is no such thing as relativistic degeneracy (!)—Dr. Chandrasekhar had got this result before, but he has erased it in his last paper; and, when discussing it with him, I felt driven to the conclusion that this was almost a *reductio ad absurdum* of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way".

Eddington literally tore Chandrasekhar's work to shreds with jokes and rhetoric and ridiculed the result and Chandrasekhar himself later said about this incident: "I had gone to the meeting thinking that I would be proclaimed as having found something very important. Instead, Eddington made a Fool of me. I was distraught. I did not know whether to continue my career."

"I returned to Cambridge late that night, probably around one o'clock. I remember going into a common room. There was still a fire burning, and I remember standing in front of it and repeating to myself, *This is how the world ends, not with a bang, but with a whimper*."

Chandrasekhar's work was appreciated soon by several papers—Niels Bohr, Wolfgang Pauli, Ralph Fowler, Wilson—and Rosenfeld, who read the comments of Eddington, said: "After having courageously read Eddington's paper twice, I have nothing to change in my previous statements, it is the wildest nonsense".

As the controversy was raging, Chandrasekhar took a decision to ignore it after asking for himself: "Am I going to continue the rest of my life fighting . . . or change to other areas of interest. I said, well, I will write a book and then change my interest. So, I did".

In his book *An Introduction to the study of Stellar Structure*, published in 1939, Chandra asserts: "Since the theory is a straight forward consequence of the quantum mechanics and, further, uses Dirac's theory of the electron only in that phase of its application which has been confirmed by laboratory experiments (Klein–Nishina formula, production of Cosmic ray showers, etc.), there can be little doubt that it is essentially correct".

The confrontation between Eddington and Chandrasekhar on this work was hotly debated for several years and at several international conferences. In 1939, at a Paris Conference on White Dwarfs and Super Novae, Chandrasekhar got an opportunity

to state openly that Eddington was wrong. This historical controversy hindered the progress of research on neutron stars and the collapse of stars for three to four decades. It is also a rare instance where Eddington went astray in his judgment based on intuition. Chandrasekhar's theory was completely vindicated, by the 1950s, and hundreds of white dwarf stars are known today whose measured masses fall neatly on the now famous mass-radius relationship curve predicted by him. No white dwarf star has been found with a mass greater than 1.44 times the solar mass.

In October 1982, on the occasion of Eddington's birth centenary, Chandrasekhar was invited by the Cambridge University to deliver a series of lectures. His lectures were entitled *Eddington: The Most Distinguished Astrophysicist of His Time*. In these two lectures[1] which are great classic masterpieces, Chandrasekhar is totally objective in his assessment of the scientific works of Eddington—the astrophysicist who could be considered Chandrasekhar's greatest scientific "adversary". There was never any rancour between Eddington and Chandrasekhar despite this controversy.

The announcement of the award of the Noble Prize in Physics, to Dr. Subrahmanya Chandrasekar, along with another American Physicist William A. Fowler of the California Institute of Technology, was made on the occasion of Chandra's 73rd birthday—on October 19, 1983—for the discovery of the Chandrasekhar limit.

Chandrasekhar wrote several books and gave several lectures in his inimitable style: it is said that Chandra spoke in paragraphs and that he chose to read his special named lectures such as the Eddington Centenary Lectures. Chandra won innumerable awards, in 1968, read: *Dr. Chandrasekhar not only played a leading role in modern India's contribution to scientific research, but is also a link between India and the West*.

Many wonder as to what has contributed to Chandrasekhar's astounding productivity over 50 years, in the light of Thomas Huxley's remark, that a man of sixty in science does more harm than good! We can only quote Chandrasekhar's own observation on Lord Rayleigh, the nineteenth-century physicist, who remained steadily productive in a variety of fields for five decades and turned out some of his best known work (such as the discovery of the Argon gas) in the later part of his career: "his influence was enormous because he added to the great body of knowledge, constantly inventing many things that were not spectacular but were always important". This is equally true of Chandrasekhar himself, who effortlessly switched fields and worked as hard he only could until he had the *personal satisfaction* of having gained a clear *perspective* of the field. He continued to enrich the field of Astrophysics and had a quest for *Truth and Beauty* and elegance and mathematical order, throughout his life.

[1] *Truth and Beauty*, S. Chandrasekhar, Penguin Books, 1987. In Chapter 6 of this book are reprinted: *Arthur Stanley Eddington Centenary Lectures* entitled: *Eddington: The Most Distinguished Astrophysicist*; and *Eddington: The expositor and the exponent of general relativity*.

6.2 Chandrasekhar on Ramanujan

On every one of his visits to India, since July 1936 when he first met Mrs. Janaki Ramanujan to acquire the passport photograph of Ramanujan for Hardy, he made it a point to call on the lady to pay his respects to her. However, Chandrasekhar was seldom willing to talk about Ramanujan or his work on any occasion.

In 1987, on December 22, the birth centenary celebrations of Ramanujan were inaugurated by the Prime Minister of India, Sri Rajiv Gandhi, in a crisp 30 min program, the schedule for which was followed with clock-work precision. The strict security enforced required the participants to be seated by 8.15 AM, for the inauguration at 9.15 AM, by the Prime Minister. The Hall, with a seating capacity of 3000, was packed to the full and the dais had a provision for several hundred politicians—all the MLAs, MPs, and the VIPs vying for a place to be on the dais, and the first row itself was one of the longest, with nearly 50–60 chairs for the Ministers and VVIPs. To the author's surprise, after the inauguration, the PM and his convoy of security personnel, left and the hall became emptied of the VVIPs and VIPs, barring the reputed mathematicians and was reduced to about 400 persons. There were no further deliberations for the day barring the only public lecture for the Birth Centenary to be delivered at 2.30 P.M. by none other than Dr. S. Chandrasekhar the Nobel Laureate Astrophysicist. The author was shocked and so did many others when Dr. Chandrasekhar preceded his talk with the statement that what little he knew about Ramanujan he had spoken at the Banquet at the June 3, 1987, International Conference held at the University of Illinois, Urbana-Champagne, and therefore he said, as he would not like to repeat himself—this, coming from a reputed teacher who was proud of his whole class, of two students, winning the Nobel Prize for Physics—he would choose to speak on Newton's *Principia*, which was his chosen topic of study for the last decade of his life. Since most of us in India were absent at that august meeting on June 3, 1987, the following article which appeared in *Ramanujan Revisited*, edited by G.E. Andrews, R. Askey, B.C. Berndt, R.A. Rankin and K. G. Ramanathan (Academic Press, 1988), is reproduced here below:

"I cannot clearly say anything that will relate to Ramanujan as a Mathematician, particularly in this company which includes among others, Professors Richard Askey, Bruce Berndt, and George Andrews, who have devoted years to exploring and following his many trails. But I do share with Ramanujan the same cultural background in our early formative years: both of us originate in a common social background—he from Kumbakonam and I from Tanjore, both ancient centres of Tamil culture and not very far apart. Besides, Ramanujan's parents and my own grandparents lived in very similar social and financial circumstances. On this account I can probably visualize Ramanujan's background better than even my younger Indian colleagues of later generations.

"With this common background, I can perhaps throw some light on some conflicting statements that have been made about Ramanujan and 'God' by some

of his Indian contemporaries. I refer here particularly to the colourful stories concerning Ramanujan's devotion to the Namakkal goddess.

"Quite generally, it may be stated that among those who were brought up in south India during the first two decades of this century, there was (and probably still is) very little correlation between observance and belief. In particular, I can vouch from my own personal experience, that some of the 'observances' that one followed were largely for the purposes of not offending the sensibilities of one's parents, relations, and friends.

"I can say a good deal on these matters, but I shall only state that I do not accept what has commonly been said and written about Ramanujan's religious beliefs. I corresponded with Hardy on this matter while he was preparing for his Harvard Lectures; and I am personally much more inclined to accept his view as expressed in a letter to me dated February 19, 1936.

> . . . And my own view is that, at bottom and to a first approximation, R. was (intellectually as sound an infidel as Bertrand Russell or Littlewood . . . .
>
> One thing I am sure. R was not in the least an 'inspired idiot' that some people seem to have thought him. On the contrary, he was (except for a period when his mental equilibrium was definitely upset by illness) a very shrewd and sensible person : very individual, of course, and with a reasonable allowance of the minor eccentricities of genius, but fundamentally normal and sane.

"And this view of Hardy's is corroborated by K. Ananda Rao, himself a mathematician with distinction, who had been Hardy's student and Ramanujan's contemporary in Cambridge. Ananda Rao is well known and remembered for his contributions to the theory of Tauberian theorems, function-theory and the theory of Dirichlet series. He has written:

> In his nature he was simple, entirely free from affectation, with no trace whatever of his being self-conscious of his abilities. He was quite sociable, very polite and considerate to others. He was a man full of humour and a good conversationalist, and it was always interesting to listen to him. On occasions when I met him, we used to talk in homely Tamil. He could talk on many things besides mathematics . . . .

"This view of Ananda Rao is not surprisingly the same as Hardy's. He has written,

> . . . the picture which I want to present to you is that of a man who had his peculiarities like other distinguished men, but a man in whose society one could take pleasure, with whom one could drink tea and discuss politics or mathematics.

"Let me now turn to the role of Ramanujan in the development of science in India during the early years of this century.

"Perhaps the best way I can give you a feeling for what Ramanujan meant to the young men going to schools and colleges during the period 1915–1930 is to recall for you the way in which I first learnt of Ramanujan's name.

"It must have been a day in April 1920, when I was not quite 10 years old, when my mother told me of an item in the newspaper of the day that a famous Indian mathematician, Ramanujan by name, had died the preceding day; and she told me further that Ramanujan had gone to England for some years earlier, had collaborated

with some famous English mathematicians, and that he had returned only recently, and was well known internationally for what he had achieved. Though I had no idea at that time of what kind of a mathematician Ramanujan was, or indeed what scientific achievement meant, I can still recall the gladness I felt at the assurance that one brought up under circumstances similar to my own, could have achieved what I could not grasp. I am sure that others were equally gladdened. I hope that it is not hard for you to imagine what the example of Ramanujan could have provided for young men and women of those times, beginning to look at the world with increasingly different perceptions.

"The fact that Ramanujan's early years were spent in a scientifically sterile atmosphere, that his life in India was not without hardships, that under circumstances that appeared to most Indians as nothing short of miraculous, he had gone to Cambridge, supported by eminent mathematicians, and had returned to India with every assurance that he would be considered, in time, as one of the most original mathematicians of the century—these facts were enough—more than enough—for aspiring young Indian students to break their bonds of intellectual confinement and perhaps soar the way that Ramanujan did.

"It may be argued, perhaps with some justice, that this was a sentimental attitude: Ramanujan represents so extreme a fluctuation from the norm an Indian must be considered to a large extent as accidental. But to the Indians of the time, Ramanujan was not unique in the way we think of him today. He was one of others who had, during that same period, achieved, in their judgement, comparably in science and in others areas of human activity. Gandhi, Motilal and Jawaharlal Nehru, Rabindranath Tagore, J.C. Bose, C.V. Raman, M.N. Saha, S.N. Bose, and a host of others, were in the fore front of the fermenting Indian scene. The twenties and the thirties were a period when young Indians were inspired for achievement and accomplishment by these men who they saw among them.

"I do not wish to leave the impression that Ramanujan's influence was only in this very generalized sense. I think it is fair to say that almost all the mathematicians who reached distinction during the three or four decades following Ramanujan were directly or in directly inspired by his example.

"But Ramanujan's name inspired not only ambitious young men planning scientific careers; it also stimulated to action those with public concern. Let me give one example.

"When I was a student in Madras one of my classmates (who came from a very wealthy family) was one Alagappa Chettiar. We became good friends; but our lives diverged along different paths after 1930. In the years before and during the second world war, Alagappa Chettiar prospered as an entrepreneur and became a noted philanthropist. He was in fact knighted by the British Government.

"During the late forties after the war, Sir Alagappa Chettiar (as he was then) wrote to me inquiring if it might be useful for him to found a mathematical Institute in Madras named after Ramanujan. I enthusiastically supported the idea; and when I returned to India briefly in 1951, the Ramanujan Institute had been founded a few months earlier. Its first director, T. Vijayaraghavan, was one of the most talented among Hardy's former students; he died at a comparatively young age in 1955. C.T.

Rajagopal, a student of Ananda Rao, took over the directorship from him. Already at that time the financial status of the Institute was shaky, since Alagappa Chettiar's fortune was melting away.

"In April 1957, when Alagappa Chettiar died, the fate of the Institute hung in the balance; Rajagopal wrote to me that the institute *will cease to exist on the first of next month*, whereupon I wrote to the Prime Minister [Jawaharlal Nehru], explaining the origin of the Institute and the seriousness of its condition. Nehru's prompt answer was refreshing: *Even if you had not put in your strong recommendation in favour of the Ramanujan Institute of Mathematics, I would not have liked anything to happen which put an end to it. Now that you have also written to me on this subject, I shall keep in touch with this matter and I think I can assure you that the Institute will be carried on*. And it was; but haltingly and precariously for the next 12 years. It is at this Institute in Madras that Ramanujan's centennial will be celebrated by an International Conference in December.

"There is very little more I can say. My own view, 66 years after my first knowing of his name, is that India and the Indian scientific community were exceptionally fortunate to having before them the example of Ramanujan. It is hopeless to try to emulate him. But he was there even as the Everest is there."

On Ramanujan Institute, founded by Alagappa Chettiar, in 1951, which was absorbed in 1957 by the University of Madras to become the Centre for Advanced Study in Mathematics and it moved into a modern building in 1972, which is situated on the Wallajah Road, on the other side of which is the famous M.A. Chidambaram Stadium for the national sport Cricket, known formerly as 'Chepauk, Madras Cricket Club Ground'. Attempts to get this street renamed after Ramanujan have met with no success till date. It is unfortunate that Robert Kanigel recoded in his popular book entitled *The Man who Knew Infinity: a life of the Genius Ramanujan* recorded that the Ramanujan Institute ". . . does not specialize in areas of mathematics Ramanujan pursued, nor does its name bear anything like the lustre of its namesake." This ought to make us sit up, having left it to George E. Andrews, Richard A. Askey and Bruce C. Berndt to make Ramanujan and his mathematics vibrant with life ever and anon, since they came into contact with the genius and his work.

6.3 The Passport Photograph of Ramanujan

Strange to say, the British government ruling India did not insist on Ramanujan to have a passport when he left India in 1914, but when he was returning to India, they insisted on his having a passport! So this is one of a very few authentic photographs of Ramanujan which is available today. This authentic passport photo was first published in the book of Hardy on *Ramanujan: Twelve Lectures on subjects suggested by his life and work*. It was taken in February 1919, when Ramanujan showed signs of improvement in deteriorating his health condition, after prolonged treatment in several nursing homes and sanatoria where he was confined

since the diagnosis at that time was that he was suffering from tuberculosis. The passport photo was taken in February 1919, in a nursing home in Cambridge, where Ramanujan was undergoing treatment.

The letters of Chandrasekhar to Hardy on the Photograph of Ramanujan, written on August 4 and December 28, 1937, are reproduced here below:

The University of Chicago
Yerkes Observatory
WILLIAMS BAY, WIS
1937 August 4

Dear Hardy,

I remember your telling me that when Ramanujan's collected papers were being edited, it was your original intention to include a portrait of R., but eventually you had to abandon the plan as no good photograph was available.

So, when I was in India last summer, I made an effort to find a reasonably good photograph. I met Mrs. Ramanujan—his wife, who incidentally is having a rather difficult life, some of her unscrupulous relatives having swindled her out of such financial resources as R. had left her—and it transpired that the only photograph of R. (other than the one where he is "cap and gown") available is his passport photograph taken prior to his leaving England.

I had a negative taken of his passport photograph (which is not in my possession) and had an enlargement professionally made. I am sending you by separate cover an enlarged photograph of R, and as you will see it is a reasonably good one. I am not in a position to judge how 'true' it is, but I have it on Mrs. Ramanujam's authority that the rather worried look R has in the picture was extremely frequent during his last year.

I do not know if you would want to include this photograph in your book on R—I refer to your Harvard lectures and I do not know either if the enclosed enlargement would be good enough for reproduction purposes. In any case I think a good professional must be able to use the original negative I have and if it should be needed, I shall be only too glad to loan it. I have some more spare copies of the enlargements of R and if you should know others who may like to have a copy of R's photograph perhaps you could ask them to write to me.

With kindest regards,
Yours very sincerely, ,
Chandrasekhar.

P.S. In any case, I should be glad to know what you think of the photograph.

Some of the copies of the photograph referred to in this letter of Chandrasekhar to Hardy are still in the collection of papers of Ramanujan at the Wren Library of the Trinity College, Cambridge.

The University of Chicago
Yerkes Observatory
WILLIAMS BAY, WIS
1937 August 4

Dear Hardy,

Many thanks for your letter of Dec. 15. I am very glad to learn of your intention to include the photograph in your book.

I am enclosing the negative from which the enlargement I sent you was made. Also a rather good copy of the same size as the negative and four rather poor copies of the enlargement. I am afraid that my own efforts at copying from the enlargement proved less successful than I had hoped! I am however certain that a professional photographer can make even better enlargements from the negative than the one I sent you.

My impression is that the college has no portrait of R. in its collection of fellows—perhaps you would be so kind as to pass on one of the four copies I am sending.

With the seasons greetings and with kind regards,

Yours very sincerely,
Chandrasekhar.

Jawaharlal Nehru, the first Prime Minister of Independent India, reputed for his book "Discovery of India", was genuinely interested in Science and Technology. We have already seen that Chandrasekhar appealed to him for the continuance of the Ramanujan Institute. The letter of Chandrasekhar to Nehru and the latter's reply are preserved in the Nehru Memorial Museum[2] *by Bruce C. Berndt and Robert A. Rankin.*[2]

We reproduce here the reference to Ramanujan contained in Nehru's *Discovery of India*:

"Mathematics in India inevitably makes one think of one extraordinary figure of recent times. This was Srinivasa Ramanujam. Born in a poor Brahmin family in South India, having no opportunities for a proper education, he became a clerk in the Madras Port Trust. But he was bubbling over with some irrepressible quality of instinctive genius and played about with numbers and equations in his spare time. By a lucky chance he attracted the attention of a mathematician who sent some of his amateur work to Cambridge in England. People there were impressed and a scholarship was arranged for him. So he left his clerk's job and went to Cambridge and during a very brief period there did work of profound value and amazing originality. The Royal Society of England went rather out of their way and made him a Fellow, but he died 2 years later, probably tuberculosis, at the age of 33. Professor Julian Huxley has, I believe, referred to him somewhere as the greatest mathematician of the century."

[2]The author's efforts to get copies of these letters for inclusion in the Indian edition of the book on *Ramanujan: Letters and Commentary*, Bruce C. Berndt and Robert A. Rankin, did not meet with success.

"Ramanujan's brief life and death are symbolic of conditions in India. Of our millions how few get any education at all, how many live on the verge of starvation; of even those who get some education and have nothing to look forward to but a clerkship in some office on a pay that is usually far less than the unemployment dole in England. If life opened its gates to them and offered them food and healthy conditions of living and education and opportunities of growth, how many among these millions would be eminent scientists, educators, technicians, industrialists, writers and artists, helping to build a new India and new world?"

6.4 A Remarkable Story

The Srinivasa Ramanujan Medal of the Indian National Science Academy was conferred on Dr. Chandrasekhar, in 1968. So, that year, Chandrasekhar came to India and gave the *Nehru Memorial Lecture* and the *Ramanujan Lecture*. Besides he had a meeting with Mrs. Ramanujan. What happened on that occasion is recorded to us in the chapter on "Conversations with Chandra" by Kameshwar C. Wali in his book on *Chandra* (The University of Chicago Press, Chicago, 1991 and Penguin Books India, New Delhi, 1990, p. 262), which is interesting reading indeed and so it is reproduced below:

I learned of the incident from a conversation I had with Hardy at dinner in Trinity during the spring of 1936. Hardy arrived a little late but bandaged, and he sat at the table opposite me. The fact that he was bandaged naturally aroused the concern of those around him, and when questioned, he told a remarkable story. It's rather elaborate. You may want to reproduce it from the talk I gave to a private club at the University of Chicago.

The following is from S. Chandrasekhar, Notes and Records of the Royal Society 30 (1976): 249. This account is now in print in *Littlewood's Miscellany*, ed. by Bela Bollobas (London: Cambridge University Press, 1986). Also, the Madras city based popular newspaper *The Hindu* carried the story, on November 17, 1968, under the heading: "When U.K. Policeman winked at an offence".

It appeared that he [Hardy] was in London during the day and while crossing Piccadilly Circus a motorcycle hit him and dragged him along. He was bruised but only superficially. Nevertheless, Hardy was escorted to the Scotland Yard by the policeman who had arrested the cyclist in order that Hardy could report what had happened. After Hardy had given the appropriate evidence and was about to leave, a messenger came to him and told him that a senior officer of the Scotland Yard wanted to see him. Hardy was slightly surprised. But the officer treated him with great courtesy, asked him to be seated and said, "Professor Hardy, I have been wanting to see you for many years. In fact, I have waited for this occasion for 17 years. Do you know that we have had evidence in our files here to arrest you for giving false evidence?" Hardy was a little surprised and the officer continued. "Do you remember, Professor Hardy, that in February 1918, an Indian mathematician had tried to commit suicide by falling before the train in an underground tube station?

His intention was thwarted and he was arrested and brought to Scotland Yard. I was in charge of that case. And you arrived later to give evidence." At this point I should digress, even as Hardy did while narrating his encounter with the officer of the Scotland Yard.

Ramanujan had been quite ill during the winter of 1917 and apparently in a state of extreme depression. Ramanujan in fact had tried to commit suicide in the manner described by the officer; but by a series of miracles (like the switch being turned off by a guardsman, and the train coming to a stop just a few feet ahead of where Ramanujan had fallen) he had been saved. To continue with the story.

"When you arrived at Scotland Yard, Professor Hardy, you told us that Ramanujan was a Fellow of the Royal Society and as such could not be arrested. We released Ramanujan and you left apparently believing that you had bluffed us; but in fact you had not. You knew as well as we did that Fellows of the Royal Society are not immune from arrest. But you also told a lie. At the time of this incident Ramanujan was not in fact a Fellow of the Royal Society. But you knew that he would be elected a month later but that is not the same thing. Nevertheless, on inquiry we found that the man whom we had arrested was in deed reputed to be a great mathematician and we in Scotland Yard did not want to spoil that life. And so we let you believe that you had convinced me. But I have always hoped that an occasion would arise when I could tell you that we knew all along that you were telling a falsehood and had perjured yourself. The occasion has now come; but I am not going to arrest you."

"I cannot recall whether it was on this occasion, or on a later occasion, that Littlewood referred to Ramanujan's attempt to commit suicide in connection with his (Ramanujan's) election to a Fellowship at Trinity. There is apparently a rule that one who is medically insane cannot be elected to a Fellowship at Trinity. And Littlewood (who was one of the electors during the year that Ramanujan was a candidate) was afraid that Ramanujan's "insanity" in having attempted suicide might be brought up to disqualify his election; and on that account Littlewood said that he had gone to the electors meeting with a medical certificate to the effect that Ramanujan was not afflicted by insanity. And Littlewood added that he was extremely glad that no occasion arose which required him to produce the certificate."

Our conversation continues

You know there is a sequel to that story. In 1968 I gave the Nehru Memorial Lecture and the Ramanujan Lecture at the Indian National Academy of Sciences. They now call it the Indian National Science Academy. And, since it was the Ramanujan Memorial Lecture, I told the stories which Hardy and Littlewood had told me about Ramanujan's election to Trinity Fellowship and the Fellowship of the Royal Society. In that context, I mentioned Ramanujan's attempted suicide. But it appeared in the newspaper the following day with the headlines, "Ramanujan tried to commit suicide", and told the story in a garbled way. I was astonished at the opposition it created. Everybody thought I was ill-advised in telling that story. I went to Bangalore to see Raman, and one of the first things he said was, "Why do you want to defame that man?" Then there was a letter in the Times of India saying that my motive in recounting the incident was to enhance my own personal reputation at the expense of Ramanujan's. I was very depressed by that reaction. A

few days later when I went to Madras, I was met at the library by a young man who introduced himself as a neighbour of Mrs. Ramanujan; and he told me that Mrs. Ramanujan was anxious to see me. I was naturally worried that she would be very upset about it. However, she told me that my account of Ramanujan's attempted suicide had cleared up certain things about her life, which was so tragic. I have written up the things she told me in the Royal Society notes, but not for the public, because I don't know whether it is the kind of thing one should write without giving proper account of the whole story. Just one remark, Mrs. Ramanujan told me, for instance, that Ramanujan's mother put every kind of obstacle in their married life. . . .

It is known (recorded, for example, by Hardy) that during Ramanujan's years in England, he received very few letters from his family in India. And Hardy attributed Ramanujan's depression during his later years in Cambridge to his 'misunderstandings' with his family. Ramanujan used to write regularly to his wife, but apparently she never saw any of those letters since his mother apparently did not pass them on. Ramanujan's wife could not write to him because she did not have the money for postage. . . .

It throws an entirely different light. Yet he made some of his great contributions to mathematics during his 4 years in England. So I did write of my visit with Mrs. Ramanujan and sent it to the Royal Society Archives in London with his photograph and other things. They asked me whether they should publish it in their notes and records. I have not given them permission, but I have a feeling that it should be published sometime.

Today, Mrs. Ramanujan is no more. The time is ripe for the Royal Society to publish the above-mentioned write-up of Chandra on his visit to meet Mrs. Ramanujan in 1968 and other documents which Chandra left with the Royal Society in its Notes and Records section. Professor Bruce C. Berndt and Robert A. Rankin did look into these Notes left behind by Chandra, before they completed their work on *Ramanujan: Letters and Commentary* and I understand that these Notes are details of what we already know (private communication from Prof. Berndt).

6.5 Sources for the Books on Ramanujan

There are two aspects of interest to biographers and mathematicians regarding Ramanujan: his life and work. Mathematicians, who are interested in his work, have to contend with not only his publications in established journals, but also with his Notebooks, and his 'Lost' Notebook, discovered in Spring 1976, by George E. Andrews, who wrote about its contents as ". . . that chaotic collection of thousands of formulas". However, the Entries made in his Notebooks (I and II) are in a neat handwriting and well presented. Those who attempt to write biographies or articles on Ramanujan have to surmount the time barrier to reconstruct a story from all the indirect information accessible to them, as stated earlier, Hardy on Ramanujan is like James Boswell on Samuel Johnson. The challenge to the mathematicians who work

on any of this thousands of recorded results, which are still shrouded in mystery, is to prove the same with what was accessible to Ramanujan in those days in the form of books and publications. While the individual writer's perception of Ramanujan will depend on his/her background and imagination, the task of the mathematician is perhaps, unenviable in comparison.

Anyone who ever heard of Srinivasa Ramanujan reads the compelling rags-to-intellectual-riches story of Ramanujan contained in the two notices, one by G.H. Hardy and the other by Dewan Bahadur Ramachandra Rao and P.V. Seshu Iyer, published at the beginning of the *Collected papers of Srinivasa Ramanujan*, would be moved by the achievements of the unorthodox mathematical genius under adverse circumstances. The lack of formal education, lack of appreciation and a job, in the beginning of his career, and ill-health during the last few years of his life did not prevent him from being creative in mathematics. This is indeed something not easy to comprehend, for often one would buckle under similar trying circumstances.

Creativity at its best under trying and difficult circumstances, achieved by the peerless Ramanujan, brings to mind the superb musical compositions, the creations of evergreen songs in praise of the mythological Hindu Gods, by the Carnatic music composer par excellence, Saint Thyagaraja, who despite financial stress and compelling pressures of day-to-day existence, especially from his own sibling, refused to accept the Royal gifts sent with a request that he should sing in praise of the local King. The composer remained focussed due to his devotion to Lord Rama which erupted into songs in praise of Him. Besides, he had the courage of conviction that after singing in praise of the Lord and his divine acts depicted in the epic Ramayana, which gave him supreme joy, he would not and could not bring himself to sing in praise of any human being, even if it be the King himself, for pelf. Similarly, even though Ramanujan was running from pillar-to-post in search of a benefactor, his mind could remain focussed on his only passion in life, mathematics, and dwell on mathematics which made him leave behind for posterity his Notebooks with 3254 Entries, each an important theorem as yet unknown to mathematicians, besides his 39 research publications and 59 Questions/Answers in the JIMS. The surrender of Ramanujan to Mathematics is akin to the surrender of the renowned peerless Carnatic music composer Saint Thiagarajan (4 May 1767–6 Jan. 1847) to Lord Rama of the epic Ramayana of harbinger-poet Ādi Kavi Valmiki (5th Century B.C.).

The articles, Notices, are the earliest biographical sketches of the life of Ramanujan. An authentic version of not only the life but also a critical appraisal of the work of Ramanujan is contained in the most valuable contribution of Hardy, his book *Ramanujan: Twelve Lectures on subjects suggested by his life and work* (CUP, 1940, reprinted by AMS, 1999), from which we have culled extensively information and presented the same in the earlier sections of this book.

After the 75th Birth Anniversary of Ramanujan, in 1962, celebrated at the Port Trust, a teacher of Mathematics, P.K. Srinivasan, of Muthialpet High School, Madras, put in a stupendous effort to bring out a two-part Memorial publication:

Ramanujan Memorial Number, Vol. 1: *Letters and Reminiscences* and
Ramanujan Memorial Number, Vol. 2: *An Inspiration*,

both published by The Muthialpet High School, Number Friends Society, Old Boys' Committee, Madras-1, in 1968. A year before this, the renowned Library Scientist, of the University of Madras, S.R. Ranganathan, who had contact with G.H. Hardy in the 1920s, brought out a biography entitled "*Ramanujan: The Man and the Mathematician* (Asia Publishing House, 1967).

6.6 The Centenary Year, 1987: Ramanujan Revisited

In the Birth Centenary Year of Ramanujan, 1987, celebrated all over the world, there were several meetings whose theme was the work of Ramanujan and its continuing influence and impact on the growth and development of Mathematics. The Annamalai University and the Anna University had conferences preceding an International Conference organized by the National Board for Higher Mathematics, at the Institute of Mathematical Sciences, in Madras, from December 22 to December 27, 1987. It was inaugurated by the Prime Minister Rajiv Gandhi. In this meeting, Professors G.E. Andrews, R.A. Askey, B.C. Berndt, S. Chandrasekhar, R.A. Rankin, A. Selberg, M.V. Subba Rao, D. Zagier and others from abroad, and Professors R.P. Agarwal, S. Bhargava, S. Raghavan, S.S. Rangachari, C.S. Seshadri, A. Verma and several others from India took part. Unfortunately, the death of the Chief Minister of Tamilnadu, M.G. Ramachandran, occurred on December 24, 1987, which brought all human activities to a stand still for the next 2 days, until the burial took place at a hastily selected site, a part of the famous Marina Beach, in what could be called as a characteristic behaviour of the mass of people of a country which is prone to excesses even on such solemn occasions for mourning the loss of a life.

All the visiting professors for the Ramanujan Centenary celebrations were trapped in a five star hotel of the city, while most of the participants were huddled inside the IIT Madras Hostels. The speakers and audience were separated by deserted streets, symbolic of the State mourning, declared by the Government. After hectic negotiations, the International Conference itself was extended for one more day, though 2 days were lost due to the mourning. I had a role to play in this, since I had to get the help of Air India to cancel the tickets booked for the scheduled departure date after the Madras meeting and get them booked for the next day to go for the next meeting in Kandy, Sri Lanka. Thanks to my good friend, Mr. V. Kumar, of Air India, I could meet the operations manager and get this done on the phone itself, since movement on the roads was ruled out.

The International Conference on Number Theory, held at Kandy, was hosted jointly by the Institute of Fundamental Studies (IFS), Kandy, and the Institute of Mathematical Sciences, Madras. I had been entrusted this task of conducting this conference by Professor E.C.G. Sudarshan, who was a close friend of the Director of IFS. The delegation from Madras consisted of Professors A. Selberg, R.A. Rankin, B.C. Berndt, E.C.G. Sudarshan, D. Zagier, G. Bhamathi and K. Srinivasa Rao (the author). It was also the first time that I spoke in that meeting about the Life and

Work of Srinivasa Ramanujan, a precursor to innumerable lectures I have delivered at most of the centres in the countries I visited during the next 25 years, till date.

The Centenary meeting at Madras was significant for it was at this meeting that the "Lost" Notebook was released by the Prime Minister of India, Sri Rajiv Gandhi, who gave the first copy of the "Lost" Notebook to Srimathi Janaki Ramanujan. This significant publication contains: The "Lost" Notebook of Ramanujan, discovered by George E. Andrews, in the Spring of 1976, had been earlier "found" in November 1965 by Dr. J.M. Whittaker, lying loose on the floor among the papers of G.N. Watson. These were obviously a part of the papers despatched to Hardy from India, by Ramanujan's younger brother, in August 1923, more than 3 years after the death of Ramanujan. Though Hardy had entrusted the task of studying these papers to G.N. Watson and B.M. Wilson, the untimely death of B.M. Wilson, brought their effort to a premature halt. However, Robert A. Rankin took up the classification of all the papers and completed this work by 1968. Thanks to this effort, when I requested Robert A. Rankin for details about his work, he wrote to the Cambridge University Librarian, who reserved a table in the Wren Library for 2 days for me with permission to see the contents of the three large foolscap size Card Board boxes, each about 2 inches thick, containing the papers of Ramanujan in the Archives of the Wren library. At that time, I was told that each page can be copied for a nominal cost (of about 5–10 pence) if an official request for the same is made. For example, one item accessible with Add.Ms.a.94 is:

S. Ramanujan: "Lost Notebook" on q-series and similar types, n.d.

In an Appendix at the end of this book, the reader will find all the papers of Ramanujan at the Wren Library, the National Archives of our country, in New Delhi, and the Tamilnadu Archives in Chennai.

Srinivasa Ramanujan—The Lost Notebook and Other Unpublished Papers (Narosa Publishing House, New Delhi, 1987) has an introduction by George E. Andrews. The following publisher's note is self-explanatory as regards the contents of this invaluable compilation:

"In this collection of unpublished manuscripts of Srinivasa Ramanujan, we have first reproduced a major portion of the 'Lost Notebook', consisting of 90 unpaginated sheets representing his work on q-series and other topics. The "Lost Notebook" was brought to light in the spring of 1976 as "part of the Watson bequest" by Professor G.E. Andrews whose introduction precedes the text of the "Lost Notebook" reproduced here. This is followed by letters written by Ramanujan to G.H. Hardy during his stay in English nursing homes, on many mathematical topics including "coefficients in the $1/g_3$, and $1/g_2$ problems" as well as the only available remnant of his famous letter dated 12 January 1920 on mock theta functions. Various sheets in Ramanujan's handwriting seemingly related to these letters have been inserted close to them.

"Next, we have provided a hitherto unpublished manuscript of Ramanujan's on "Properties of $p(n)$ and $\tau(n)$ $\cdots$ dealing with congruence relations satisfied by these arithmetic functions. One may find, thereafter, 28 sheets copied from the "Loose Papers" of Ramanujan held in the Trinity College Library; these include a note on "Reciprocal functions", another concerning "Approximate summation

of series involving prime numbers", Ramanujan's marvellous discoveries on Euler products of Dirichlet series associated to modular forms and his famous "forty identities", with relevant sheets in Ramanujan's handwriting juxtaposed. . . .

"The subsequent 117 pages include Ramanujan's unpublished work related to various papers of his, especially in continuation of the one entitled "Highly composite numbers" and "On certain trigonometrical sums" . . . with Hardy's noting there on; besides, one may find here class invariants listed by Ramanujan and a host of interesting identities of an arithmetic nature.

"At the end, we have taken care not to miss out on interesting letters from J.E. Littlewood to G.H. Hardy, G.H. Hardy to Ramanujan, G.H. Hardy to G.N. Watson, etc. with a bearing on Ramanujan's work and various other letters of significance. Fragments in Ramanujan's handwriting have all been arranged as appropriately as possible. This last section contains also E.H. Neville's letter on Ramanujan extracted from *Nature* of 20 January 1921."

From the above publisher's note it is clear that the volume brought out on the occasion of the Ramanujan birth centenary as a companion volume to the two-volume facsimile edition of Ramanujan's Notebooks contains a lot more information which would be of interest to the mathematicians. Narosa Publishing House has thus the set of three volumes on the Notebooks of Ramanujan which is a must for any good mathematics library anywhere in the world.

The following are the other significant meetings throughout the Ramanujan Centenary Year, 1987, held in centres in India and some other countries:

- In December 1986, year-long celebrations at the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras were inaugurated by Professor R.P. Agarwal, the Vice Chancellor of Rajasthan University.
- Professor R.P. Agarwal conducted a 3-week National Instructional Seminar on "Ramanujan and Special Functions", from May 15 to June 6, 1987, at Mt. Abu, Rajasthan. This Seminar funded by the National Board for Higher Mathematics (NBHM) was inaugurated by Professor M.S. Raghunathan, the Chairman of NBHM. Professor E.C.G. Sudarshan, the Director of IMSc, directed me to go to this meeting to present my early work on relating hypergeometric series to the coupling and recoupling coefficients of angular momentum. Professor Ratan P. Agarwal appreciated warmly my work and said that he was very happy to find my genuine application of the hypergeometric series in theoretical physics and became my mentor for Special Functions. I also took the responsibility of preparing his Lecture Notes and these were brought out as a Report of that Mt. Abu Meeting.
- About 125 Mathematicians from the world gathered at the University of Illinois, Urbana-Champagne to celebrate the Centenary of Ramanujan's birth, due to the initiative of Professor Bruce C. Berndt.
- A National Instructional Conference on Ramanujan's Circle Method, sponsored by the Central University of Pondicherry and the NBHM, was held at Pondicherry, from September 21 to October 5, 1987.

- A "*Srinivasa Ramanujan Centenary Seminar on Algebra and its Applications*" was held at the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras, from October 26 to 31, 1987, as a part of its year-long celebrations of the centenary.
- The Association of Mathematics Teachers of India (AMTI) held its XXIII Annual Conference as a National homage to Ramanujan, at Kumbakonam Town Higher Secondary School, where Ramanujan had studied. A Hall in this school is named after its famous alumnus as "Ramanujam Hall" on this occasion.
- The Ramanujan Mathematical Society, which came into existence during this period due to the initiative of E.S. Sampath Kumar, R. Balakrishnan and others, in 1985, conducted a Ramanujan Centennial Conference at the Annamalai University, Annamalai Nagar, from December 15 to 18, 1987. I was one of the invited speakers.
- An International Conference on Mathematics was organized by the Anna University, Madras, in collaboration with the Tamilnadu Academy of Sciences and Committee for Science and Technology Development, at the University's campus in Guindy, from December 19 to 21, 1987. The Computer Center of the Anna University has been named after Ramanujan.
- The *Ramanujan Birth Centenary Year International Symposium on Analysis* was organized by the Department of Mathematics of the University of Pune, from December 26 to 28, 1987.
- The Indian Mathematical Society's 53rd Annual Conference was devoted to the celebration of the Ramanujan Birth Centenary and was held at the University of Gorakhpur, Gorakhpur, from December 30, 1987 to January 2, 1988.
- Several institutions held 1-day Symposia to mark the centenary:
  - Bharathiar University, Coimbatore, on August 12, 1987.
  - The Indian Institute of Science, Bangalore, on September 16, 1987.
  - The S.N. Bose Institute for Advanced Study and Research in Mathematics and Mathematical Physics, Calcutta, on December 31, 1987.

As a consequence several conference proceedings, published in 1987–1988 are today available covering a whole gamut of work suggested or inspired by the work of Ramanujan. Most notable among these is *Ramanujan Revisited*, edited by George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin. It is on this occasion that Dr. S. Chandrasekhar gave a Banquet address "On Ramanujan" on June 3, 1967. In another fascinating article in this Proceedings, Freeman J. Dyson, the reputed Physicist, makes revealing statements about the natures of G.N. Watson and George E. Andrews: *A walk through Ramanujan's garden*:

I am grateful to the organizers of this conference, and to Bruce Berndt and George Andrews, in particular, for bringing me here and giving me a chance to enjoy all the new flowers that have been growing in recent years in Ramanujan's garden. George Andrews is now the chief gardener and is doing a magnificent job. He is, if I may say so, doing a better job than George Watson who used to be chief gardener in the old days. Watson was chief gardener in the 1930s and worked hard to develop and

elucidate Ramanujan's ideas, especially the theories of singular moduli and mock theta functions.[3] But Watson was the sort of gardener who liked to keep human visitors as far as possible away from the flowers. He stood at the entrance to his garden like the goddess Prosperine. Prosperine stands at the entrance to her garden in the poem of Swinburne which Watson quoted at the end of his famous lecture, "The Final Problem: an account of the mock theta functions" (*J. of London Math. Soc.* ***11*** *(1936)* 55-80):

> "Pale, beyond porch and portal,
> Crowned with calm leaves, she stands
> Who gathers all things mortal
> With cold immortal hands."

Watson was like that. My good friend Oliver Atkin had misfortune to work on Ramanujan's discoveries during the years when Watson was in charge of the garden. Atkin discovered and proved many new congruence properties of partitions, extending and developing Watson's own work.—ref. For example, A.O.L. Atkin, *Proof of a Conjecture of Ramanujan*, *Glasgow Math. Jour.* **8** (1967) 14–32. Atkin wrote letters to Watson, keeping him abreast of the progress and asking him for help and advice. When Watson died in 1965, Atkin wrote to me, "I expect you saw that Watson died. My correspondence with him is thus closed. Final score of letters: twenty – nil."

George Andrews is different. He does not stand like Prosperine, gathering all things mortal with cold immortal hands. He answers letters. He even limes to have live human beings in his garden, trampling over the flower-beds. He gives help and encouragement to anyone who comes visiting, whether it is an elderly professor like me or a raw student. As a result, the garden is now blooming and sprouting more vigorously than ever. George Andrews also enlarged the territory of the garden by finding the famous lost notebook of Ramanujan, the notebook which Watson certainly had in his possession for 30 years but apparently never examined. Andrews not only examined the notebook for the first time but also explained and displayed for all to see the treasures that it contains."[4]

Another noteworthy publication is the Proceedings of the International Centenary Conference, held at the Anna University, Madras, from December 19 to 21, 1987, edited by Krishnaswami Alladi and brought out as a part of the Lecture Notes in Mathematics, No. 1395, series of Springer-Verlag (1989). That same year, 1987, a biography of Ramanujan, by the Tamil writer, T.V. Rangaswami, under his pseudonym "Ragami", appeared as a serial in the weekly edition of the popular Tamil newspaper "Dinamani Kadir". These articles, 31 in number, were based on a

[3] J.M. Whittaker, *George Neville Watson*, *Biographical Memoirs of Fellows of the Royal Society*, **12** (1966) 512–530.

[4] G.E. Andrews, *An Introduction to Ramanujan's Lost Notebook*, Am. Math. Monthly **86** (1979) 89–108 and a series of papers in *Advances in Mathematics*, 1981–1987.

book by the author Ragami himself entitled "Ganithamedhai Ramanujan" (Pooram Publications, Madras, 1985).

My interest in the Life and Work of Ramanujan was triggered by Professor E.C.G. Sudarshan, the Director of the Institute of Mathematical Sciences, who walked into my office room at the Institute, with Dr. Jasjit Singh, a railway official and also a writer. Jasjit Singh wanted me to help with the translation of an article of Ragami in a Tamizh newspaper. After reading the article, I told Jasjit Singh that is a part of a serial and no purpose would be served if I do not translate the full set of articles of Ragami in that daily newspaper. I suggested that he should approach the Editor of the newspaper himself to get the whole set of articles from their offices. Within a few weeks, I was confronted by Jasjit Singh who succeeded in this and sent me all the 31 articles. I set myself the task of translating them for Jasjit Singh, an Indian Railways officer, who published a series of four articles under the title: "Srinivasa Ramanujan: A short biography", in Mathematics Education, a UGC (University Grants Commission) magazine, vol. **4** (1987), pp. 5, 61, 117 and 187, without even an acknowledgment that he was inspired by the series of articles of Rangaswamy Iyengar (who wrote under the pseudonym Ragami), translated into English for him by me, at the instance of (late) Professor E.C.G. Sudarshan, the Director of the Institute of Mathematical Sciences, who due to his brilliance as a speaker and conversationalist had many friends from all walks of life in India and abroad. Also, most unfortunately, this is not atypical of Indian bureaucracy![5]

Mrs. Mini Krishnan, an Editor of the four UGC magazines, on Physics, Chemistry, Mathematics and Biology education, accepted my idea of bringing out a special issue in 1983, soon after Dr. S. Chandrasekhar won his Nobel Prize, with a cover showing Chandra receiving the Nobel Prize for Physics,[6] from the King of Sweden and including an article by me entitled *S. Chandrasekhar: the Astrophysicist*, written at the suggestion of Ms. Mini Krishnan, who wanted the cover photo to be accompanied by an article about the Noble Laureate.[7]

I wanted to see the reaction to my efforts, by sending a copy of the same to Dr. S. Chandrasekhar himself!

[5]Note: Or, for that matter, bureaucrats in general. Alistair Cook in his *Letter from America*, speeches broadcast over the Radio, which I used to hear during my stays in Bonn, Germany, at Noon on Sundays, during the late 1970s, quoted a statement of John F. Kennedy and said that those words would be ringing in the ears of many and that they were the words of the ghost writer for the Kennedy family, Sorensen—"Theodore Chaikin Sorensen (1928–2010) was an American lawyer, writer, and presidential adviser. He was a speech-writer for President John F. Kennedy, as well as one of his closest advisers. President Kennedy once called him his *intellectual blood bank*".—Wikipedia.

[6]I wish to express my sincere thanks to Mrs. Savithri Sarma and Mr. Sarma, sister and brother-in-law of Dr. S. Chandrasekhar, but for whose help I would not have got this photograph and the valuable suggestion of Ms. Mini Krishnan that the photo on the cover should go with a cover story, an article on Dr. Chandrasekar and his work.

[7]Incidentally, this was my first attempt at writing a biographical article, that too of my distinguished senior at Presidency College, Madras.

I was delighted when on receiving a copy of this issue from Macmillan (India) Ltd., Dr. S. Chandrasekhar (Laboratory of Astrophysics and Space Research, University of Chicago, Chicago, Illinois, USA) wrote me the following short letter, dated April 23, 1986:

Dear K.S. Rao,

Thank you for your letter of March 26th.

The issue of Physics Education which includes your article has arrived meantime. I am grateful for what you have written. I do not know if I can be objective in your assessment; but frankly I find it greatly exaggerates! But I do appreciate the efforts you have taken in writing the article.

With warm regards,

Yours sincerely,

(sd.) S. Chandrasekhar.

I considered this as a favourable augury for me to enter into the field of biography writing. The occasion arose and I was forced into a situation where I was asked to lead a delegation to Sri Lanka, to celebrate the Centenary of Ramanujan at the Institute of Fundamental Studies, Kandy, Sri Lanka, in December 1987. I could not induce anyone at a short notice to give a talk on the life and work of Srinivasa Ramanujan, as a keynote address in the 2-day meeting.[8] I geared up to give my first lecture in the presence of the renowned Ramanujan followers: Professors Robert Rankin, Richard Askey, George Andrews, Bruce Berndt, Don Zagier, not to mention the Director of my Institute, Dr. E.C.G. Sudarshan and Dr. G. Bhamathi of the University of Madras. I think I came out successful in this acid test when I was asked to provide a write-up of the article by Professor Sudarshan, in particular. Thus, I began my efforts as a biographer of the life and work of Srinivasa Ramanujan, a pet theme from then till date, and I have lost count of the number of centres in India and abroad where I have lectured on this topic, often for 2 hours at a stretch, with a tea break in the middle.

My first effort at popularization of Ramanujan's work in writing, for the students of mathematics, especially in India, was through a special issue, which ought to have been an issue of the UGC magazine, Mathematics Education, published by Macmillan (India) Ltd. Request letters were sent to chosen contributors by me stating this fact, but eventually due to a stranger than fiction turn of events, which usually happen in India, the article of the present author, with expository articles which I requested from Professors G.E. Andrews, R.A. Askey, B.C. Berndt, R.P. Agarwal, A. Verma, S. Bhargava, M.V. Subbarao and including an article by T. Soundararajan, as a monograph was brought out by Macmillan as a special issue

[8]This meeting had to be rescheduled, as the popular Chief Minister of the State of Tamilnadu and film actor Dr. M.G. Ramachandran, died on Dec. 24 and everything including all transportation came to a grinding halt until the State decided on and found his final resting place on the Marina Beach! Thanks to my good friend Mr. V. Kumar, of Air India, I was able to get all seven flight tickets re-reserved for the conference to the end of that week.

and without my name as its sole Editor but instead the names of K.R. Nagarajan and T. Soundararajan of Madurai Kamaraj University as that University's special issue. When I protested, I was requested by Ms. Mini Krishnan not to raise this issue since it was she who had requested the Vice Chancellor of the Madurai Kamaraj University, Dr. S. Krishnaswamy for its sponsorship and it was his suggestion to get the MKU involved by getting the Mathematics Department head involved. Thus, the principal editor was discarded and his name was replaced with the names of two others who had no role to play in corresponding with authors to get their consent to send their articles for the special issue or in editing the articles.

Later, on receiving my article on Ramanujan (October 17, 1988), in this special issue, Dr. Chandrasekhar wrote:

Dear K. S. Rao,

Thank you for your letter of October 4 and for your copy of the publication "Srinivasa Ramanujan (1887–1920): A Tribute". I appreciated your article and I am grateful for your sending a copy to me. It was good of you also to remember my birthday.

With best wishes also from my wife,

Yours sincerely,
(sd.) S. Chandrasekhar.

A Technical Report No.2 on the *Development of Elliptic Functions according to Ramanujan* by Dr. K. Venkatachaliengar was also brought out by Madurai Kamaraj University, in 1987.

Chapter 7
Books on Ramanujan and Busts of Ramanujan

The most authentic sources about the life and work of the mathematical genius Srinivasa Ramanujan are the articles of Ramachandra Rao and Seshu Aiyer at the beginning of the *Collected Papers by Srinivasa Ramanujan*, first published in 1928[1] and G.H. Hardy's[2] *Ramanujan: Twelve Lectures on subjects suggested by his life and work*.

On the occasion of the 75th Birth Anniversary of Ramanujan two volumes edited by P.K. Srinivasan, the Mathematics Teacher of Muthialpet High School, Madras, also provided rich source material for the subsequent books on Ramanujan. This Mathematics Teacher, who was a Fulbright Scholar in the United States of America, for 2 years in the early 1960s, deserves all praise for his service to the Mathematics and mathematicians. For, after his return from the United States, he left no stone unturned and sent his students in all directions on missions to seek and find letters to and from Ramanujan. This culminated in his editing and publication of two volumes entitled: *Ramanujan: Letters and Reminiscences*[3] and *Ramanujan: An Inspiration*.[4]

[1] *Collected Papers by Srinivasa Ramanujan*, edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, Chelsea, New York (1962), first published by Cambridge University Press, Cambridge (1928), and more recently reprinted on the occasion of the 125th Birth Anniversary of Ramanujan, in 2012, by the Am. Math. Soc. and London Math. Soc. (2012).

[2] G. H. Hardy, *Ramanujan: Twelve Lectures on subjects suggested by his life and work*, Chelsea, New York (1940). This book has also been reprinted by the AMS-LMS in (2012).

[3] P.K. Srinivasan, Editor, *Ramanujan: Letters and Reminiscences*, Memorial Number, Vol. 1, Muthialpet High School, Madras (1968).

[4] P.K. Srinivasan, Editor, *Ramanujan: An Inspiration*, Memorial Number, Vol. 2, Muthialpet High School, Madras (1968).

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
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7.1 S.R. Ranganathan's Biography (1967)

S.R. Ranganathan, the renowned Librarian of the University of Madras who introduced the colon system of classification of books and periodicals in a Library, in the Introduction to his book[5] entitled *Ramanujan: The Man and the Mathematician*, revealed his role in the first biographical Notices on Ramanujan which appeared in the *Collected Papers of Srinivasa Ramanujan*:

In 1923, it was decided that a biography of Ramanujan should be given at the beginning of his Collected Papers. The University of Madras appointed a Committee to write the biography. It consisted of E.M. MacPhail, the Vice-Chancellor, P.V. Seshu Ayyar, Professor of Applied Mathematics in the Presidency College and Secretary of the Indian Mathematical Society, and R. Ramachandra Rao, Education Secretary of the Government of Madras and former President of the Society. As a junior member of the staff of the Department of Mathematics of the Presidency College, it fell to my share to prepare a draft of the biography. As approved by the Committee, it was published in the Collected Papers in 1927.

The inaccessibility of this biography in the *Collected Papers by Srinivasa Ramanujan* and the lack of awareness about Ramanujan and his achievements as a mathematician among University graduates, "who had not even heard of his name" and whose number "had increased considerably" between 1950 and 1960, were the stated motivations of S.R. Ranganathan to write this brief biography about *Ramanujan: The Man and the Mathematician*. In his All India Radio talk, on the occasion of the 75th Birth Anniversary of Ramanujan, in 1962, he produced responses in the form of "letters from a couple of the surviving friends of Ramanujan". This spurred S.R. Ranganathan to get reminiscences from all of them, including from Mrs. Janaki Ramanujan, over a period of 2 years, and he embarked on the task of writing biography concerning the human aspects of Ramanujan.

S.R. Ranganathan himself points out in his biography that "the Contents Page forms a clear Conspectus of the book". So, we give below the table of contents of his book "Ramanujan: the man and the mathematician":

A	Introduction
B	Ramanujan: A Puzzle
C	Super-Activity Period 1: 1907 to 1911
D	Economic Sufficiency
E	University Setting
F	Ramanujan's Life Abroad
G	Super-Activity Period 2: 1914 to 1918
H	Precipitation of Honours
J	Ramanujan's Health Fails

(continued)

[5] S.R. Ranganathan, *Ramanujan: The Man and the Mathematician*, Asia Publishing House (1967).

K	Memorial
L	Ramanujan's Notebooks
M	Reminiscences of Friends
N	Ramanujan the Man
P	Ramanujan the Mathematician
Q	Works By and On Ramanujan
R	Up-to-date Bibliography
S	Bio-data
	Index

An anecdote in this book reveals the simplicity of Ramanujan to mundane details, as told by the renowned Indian Statistician Dr. P.C. Mahalanobis, F.R.S., who was a student of King's College, Cambridge (from October 1913):

I was fortunate in forming a good friendship with Ramanujan very soon. It came about in a somewhat strange way. One day, soon after his arrival, I went to see Ramanujan in his own room in Trinity College. It had turned quite cold. Ramanujan was sitting very near the fire. I asked him whether he was quite warm at night. He said that he was feeling the cold though he was sleeping with his overcoat on and was also wrapping himself up in a shawl. I went to his bedroom to see whether he had enough blankets. I found that his bed had a number of blankets but all tucked in tightly, with a bed cover spread over them. He did not know that he should turn back the blankets and get into the bed. The bed cover was loose; he was sleeping under that linen cover with his overcoat and shawl. I showed him how to get under the blankets. He was extremely touched. I believe this was the reason why he was so kind.

It is an interesting fact that after Ranganathan left the University of Madras, he became the Chief Librarian of the Benaras Hindu University. It was he who drew the attention of Dr. K. S. Krishnan, a student and collaborator of Sir C.V. Raman, regarding the Notebooks of Ramanujan. At that time, Dr. K. S. Krishnan was a neighbour of Pundit Jawaharlal Nehru in Allahabad and he induced Ranganathan to write to Dr. Homi J. Bhabha at the Tata Institute of Fundamental Research in Bombay. Subsequently, Ranganathan, Krishnan and Bhabha called on Pundit Nehru, and as a consequence the facsimile edition of the Notebooks of Ramanujan were published without commentary.

7.2 Robert Kanigel's[6] *The Man Who Knew Infinity*

In late 1981, when magazines and newspapers in the world were full of articles marketing the birth centenary of Srinivasa Ramanujan, Robert Kanigel asked: "Ramanujan Who?" of an agent of the publisher (Charles Scribner's Sons), who was interested in bringing out a biography of Ramanujan. Inspired by the earlier

[6]Based on the author's review of the book which appeared in The Hindu, in October 1993.

writings on Ramanujan mentioned above, and intrigued by the Hindu culture, Kanigel travelled to places that figured in Ramanujan's life in South India for five weeks. During this period, he interviewed the aging Janakiammal—the then living link to the legend—the relatives and friends of Ramanujan and mathematicians. He was encouraged by the "Gang of three mathematicians"—George E. Andrews, the discoverer of the "Lost" Notebook of Ramanujan; Bruce C. Berndt who took upon himself the arduous and challenging task of editing the three Notebooks of Ramanujan containing about 3254 Entries without proofs, from May 1977, and Richard Askey who spearheaded the commissioning of the bust of Ramanujan by Paul Granlund to present it to Janakiammal Ramanujan. Kanigel brought out, in 1991, a delightfully readable biography entitled *The Man Who Knew Infinity: A Life of the Genius Ramanujan*. In this 438 pages book, Kanigel has intertwined into a braid three stories: the story of Ramanujan, the inscrutable intellect with a simple heart; the story of G.H. Hardy, the mathematician par excellence who not only recognized the genius "in the tattered garb in which it was clothed" but also strove to bring Ramanujan to the attention of the world and a silhouette of the mathematics they did together.

"The Man Who Knew Infinity" consists of eight chapters, a prologue, an epilogue and some 40 pages of notes in the end in small print. The first three chapters deal with Ramanujan's childhood in the South Indian scenario, his early mathematical exploits and his search for patrons. These portions rely heavily upon the source material referred to earlier on, especially in the books of S.R. Ranganathan, P.K. Srinivasan and the articles of *Ragami*. Kanigel has exercised the liberty of an experienced writer and author to describe vividly the events of the period 1887–1913, as if he were an eye witness.

Kanigel gives a good description of Kumbakonam, "the Cambridge of South India", the home town of Ramanujan, flanked by the river Cauvery and one of its tributaries. Very little is said about Ramanujan's father. His mother is portrayed as a shrewd, cultured lady, whose obsession about Ramanujan's genius made her use her powerful personality to zealously protect and project his interests all through her life.

Kanigel agrees with E.H. Neville who referred to Ramanujan's early years, the five solid years after his academic failure, as the carefree days, before his marriage, in 1909, and that in some ways, they were the most productive years of his life. This is debatable in view of Hardy's observation that during those formative years Ramanujan's "genius was misdirected, side-tracked and to a certain extent distorted". Added to this is the fact that the excellent work published by Ramanujan from Cambridge, without recourse to his results written down in his Notebooks, including his collaboration with Hardy, could very well be considered as Ramanujan's most productive 5-year period in life!

A brief description is given of the marriage of 21-year-old Ramanujan to 9-year-old Janaki, arranged by Komalathammal, not attended by Ramanujan's father, at Rajendram, in July 1909. Kanigel portrays the inauspicious incidents which marred the wedding, foreboding the tragic death to follow 11 years later, on April 26, 1920. Ramanujan and Janaki lived together only for a year in Madras before Ramanujan

went to Cambridge in 1914, and for a year after he returned to India, in March 1919. Kanigel wrote that *scarcely off the boat, Ramanujan had dropped into the family snake pit*. Komalathammal does not come out with flying colours through this last phase of Ramanujan's life, when as a typical tyrannical mother-in-law, she was quarrelling with her daughter-in-law, tried her best to keep the couple separated, forcing Ramanujan to even rebel against her. And *the family drifted back into something like the obscurity that was, save for the accident of Ramanujan's birth, its natural lot*.

Like in two of the four major novels of my favourite author, Sir Arthur Conan Doyle, on Sherlock Holmes—viz. the *Valley of Fear* and the *Sign of Four*—Kanigel is in his elements when he starts his Chapter IV with a completely different venue and describes an Ivory Tower of British Science—Cambridge on the banks of the river Cam. The pivotal character, whose stature was that of a mathematical colossus in the early decades of the twentieth century, is none other than G.H. Hardy.

Kanigel provides a fascinating account of the venerable Cambridge institution called the Tripos which referred to a three-legged stool on which sat a man whose job was to dispute, sometimes humorously and sometimes aggressively, for the degree in mathematics. Tripos examinations for proficiency in mathematics consisted of two parts. The disputant who *wrangled* over points of logic and who came at the top of the test of successful candidates was named a Senior Wrangler, an honour which clung to the recipient like an aura for a lifetime.

Kanigel vividly detailed the state of English mathematics and the turn given to it by Hardy, who chartered a course to the future and became a spokesman for the new rigour, with his textbook *A Course of Pure Mathematics*, completed in 1908—the first rigorous exposition of difficult mathematical concepts, in England.

It was at this stage of Hardy's life that the letter from the unknown Indian clerk reached him. Kanigel delightfully states that it was Hardy's willingness to stray from safe, familiar paths, his penchant for the unorthodox and unexpected, which enabled him to open his heart and mind to the wildly new and alien, mathematics-dense letter from India: "*The wheels of Ramanujan's career, for ten years barely creaking along, now greased by Hardy's approval, began to whirr and whine like a finely tuned race car engine.*"

The story is well told of how Hardy and Neville, the Cambridge mathematicians; Sir Francis Spring and Narayana Iyer, of the Madras Port Trust; Ramanujan's mother and her devotion to the goddess Namagiri of Namakkal; Lord Pentland, the Governor of Madras and his private Secretary, C.B. Cotterell; Professor Littlehailes and Francis Dewsbury, the Registrar of the University of Madras, in an orchestrated effort found the funds to send Ramanujan sailing to England, on a unique historic mathematical adventurous voyage of the twentieth century.

Kanigel's effort to make the mathematics of Ramanujan and Hardy accessible to the non-expert is highly commendable. The first two formulae in Carr's Synopsis and their generalization are illustrated; a generalization of the roots of the quadratic equation is given as an example of Ramanujan's non-formal notation, the ubiquitous nature of π stitched across the whole tapestry of mathematics, the definition of a modular equation, the distribution of prime numbers, the meaning of a

complex number, Ramanujan's definition of a highly composite number, partitions of a number, the Rogers-Ramanujan identities, as well as the magic of some of Ramanujan's formulae sent to Hardy with his first letter, have all been generously interspersed with the stories of the lives of the two heroes of this book—Ramanujan and Hardy.

In Kanigel's rendering, Ramanujan emerges as one untouched by the seductive charms of the West, successfully labouring under an impossible handicap—*a poor Hindu pitting his brains against the accumulated wisdom of Europe*—self-willed, every bit the product of India and its customs; holding strong religious beliefs; impetuous when overcome by shame; finicky and demanding in sickness; warming to the kindness of Hardy; who was not discovered by Hardy or anyone else but "like all other great men he invented himself": a "Swayambhu"[7] or self-born genius.

Kanigel states that the 140 pages of material pertaining to "mock" theta functions, discovered in the Spring of 1976 by George Andrews, who christened these as the "Lost" Notebook of Ramanujan, was part of a batch of papers Dewsbury had sent Hardy, in 1923, and that in 1928 Hardy handed over this "Notebook" and the manuscripts of Ramanujan to G.N. Watson—an explanation for how the "Notebook" was not "lost" but found by G.E. Andrews while he was going through the estate of G.N. Watson with the Trinity College.

In the Epilogue, Kanigel completes the story of Hardy, who won several honours and who died on December 1, 1947, the day he was to be presented the Copley Medal, the Royal Society's highest honour.[8]

The 40 pages of Notes giving the sources for the quotations and statements contain interesting snippets of information and show the thorough analysis of all the material which Kanigel gathered. This separation of the Notes, without marking in the text, contributes to the smooth flow in the reading of this delightfully written first full-length biography of Ramanujan, which was published by Scribner's.[9]

7.3 Review of *Ramanujan: Letters and Commentary*[10] by Berndt and Rankin

Ramanujan: Letters and Commentary by Bruce C. Berndt and Robert A. Rankin, American Mathematical Society and London Mathematical Society, 1995, reprinted by the AMS in 1999, has been brought out as an Indian Edition with a Preface,

[7]Kanigel should be complemented for finding this most apt word in Sanskrit to describe the genius of Ramanujan, which he uses as a heading to a section in his book *The Man Who Knew Infinity*.

[8]The Copley Medal was awarded to Dr. S. Chandrasekhar, the Indian-American Astrophysicist.

[9]Robert Kanigel, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Charles Scribner's Sons, New York (1991), was priced at $ 27.95. In 1994, Rupa & Co. brought out an Indian paper back edition at a cost of only Rs. 195/-.

[10]Bruce C. Berndt and Robert A. Rankin, *Ramanujan: Letters and Commentary*, (Am. Math. Soc. and London Math. Soc. 1995, reprinted by the AMS in 1999).

Additions to the Indian Edition and Errata, by the present author of this book (K. Srinivasa Rao). This Indian edition was published by Affiliated East West Press Pvt. Ltd. (1997) and was released at a formal function at the Institute of Mathematical Sciences.

This book is a classical work containing the first letter from Ramanujan to Hardy, "one of the most famous letters in the history of mathematics", and Ramanujan's last letter—containing until the discovery of the "lost" Notebook, "the only information available about Ramanujan's work in 1919–1920". The authors have left no stone unturned to acquire as many letters as possible, since most of these letters, especially those exchanged by Hardy and Ramanujan (when the latter was for about 2 years in different sanatoria in England) contain interesting mathematical theorems. In fact, the two Memorial Numbers edited by P.K. Srinivasan—*Ramanujan: Letters and Reminiscences* and *Ramanujan: an Inspiration*—and published by Muthialpet High School, Madras, in 1968, are the precursors to this book of Berndt and Rankin, though not acknowledged as such. In fact, most of the writers on Ramanujan have been greatly benefited by the singular efforts of Mr. P.K. Srinivasan, soon after his return to India after being a Fulbright Scholar in the United States for 2 years, when he probably conceived of this idea to publish this two-volume compilation.

After a brief biography of Ramanujan, transcripts of the available 172 letters are reproduced with commentaries at the end of each of them, by Berndt and Rankin, in eight chapters, in this book whose Special 1997 Edition has 363 + xviii pages and was priced at Rs. 295/-. In Chap. 1, the letters regarding Ramanujan's first position at the Madras Port Trust and the correspondence with Professor M.J.M. Hill, referred to earlier in this book, are reproduced. In Chap. 2, the first two historic letters of Ramanujan to Hardy are reproduced in full and the commentary is on each one of the formulae contained in them. This is the first time that the second letter of Ramanujan to Hardy appears in full. In the "Collected Papers of Srinivasa Ramanujan" there is missing material marked by. . . at certain places. There are biographical sketches (of a few lines or a few paragraphs length) of almost all the characters who wrote to Ramanujan or who are mentioned in his letters in this book. Hardy's list of six New Year wishes, contained in a post card to a friend of his in the 1920s, is an example of a delightful snippet in one of the commentaries (p. 33):

(1) prove the Riemann hypothesis; (2) make 211 not out in the fourth innings of the lost test match at the Oval; (3) find an argument for the non-existence of God which shall convince the general public; (4) be the first man at the top of Mt. Everest; (5) be proclaimed the first President of the U.S.S.R., Great Britain, and Germany, (6) murder Mussolini.

The next four chapters are devoted to the letters pertaining to Ramanujan's *Preparing to go to England, Ramanujan at Cambridge*, Ramanujan's illness and those written after Ramanujan returned to India. Berndt and Rankin have brought out in this volume, for the first time, Ramanujan's letters from sanatoria (Matlock House, Hill Grove, Fitzroy House). Ramanujan who was confined to sanatoria off and on for nearly half the time of his 5-year sojourn in England, wrote several notes/letters to Hardy and received notes/replies from Hardy. These contain mathematical insights into the problems of interest to the two of them. These

letters and commentaries on them, therefore, make this book significant to the mathematicians. The letters and Ramanujan to his kith and kin and his friends reflect the human angle of Ramanujan. The incisive commentaries by the editors provide a proper perspective of the persons with whom Ramanujan corresponded. The last letter written by Ramanujan to Hardy, on January 12, 1920, is reproduced and a portion of the letter following the first paragraph of the letter has been missing: "*For almost 60 years, the only information available about Ramanujan's work in 1919–1920 was contained in his last letter to Hardy. Now we know that the results Ramanujan described in this letter are part of a much longer collection, called the 'lost' notebook.*"

In this letter, Ramanujan (as pointed out earlier) did not even hint to Hardy that he had not only not recovered from his suspected tuberculosis but that his health was on the decline. However, it contained the heart-warming news to Hardy that he had found new results on "mock" theta functions.

The minor role of this author in providing Bruce Berndt and Robert Rankin, the editors of this book, with letters written by Ramanujan to his boyhood friend, Mr. E. Vinayaka Rao, is explicitly acknowledged in Chap. 4 of their book, *Ramanujan: Letters and Commentary*.[11] An Indian edition with a few additions and errata by the author was also brought out:[12]

The three letters below that Ramanujan wrote to Mr. E. Vinayaka Row on June 11, 1914, 24 March 1915 and 10 September 1915 have been made available through the courtesy of Professor K. Srinivasa Rao of the Institute of Mathematical Sciences, Madras. They were given to him by Mr. V. Madhava Rao, the son of E. Vinayaka Rao. The latter was a lawyer and resident in Mylapore and a friend of K. Srinivasa Rao's father [late Mr. K. Vallabheswar Rao (22 Dec. 1899–10 Jan. 1983)]. The letters, parts of which have become somewhat moth-eaten, were discovered in a suitcase in Mr. Madhava Rao's attic. . . . Hardy first reported Ramanujan's illness in the Spring of 1917. In this letter [dated 24 March 1915], we learn that already during Ramanujan's first winter in England he was not feeling well.

The originals of these letters have since been handed-over by Mrs. Saraswathi Dole, eldest sister of Madhava Rao, to the Ramanujan Museum at Royapuram (a brief description of which will follow).

In Chap. 7, of Bruce Berndt and Robert Rankin's book, the letters announcing the death of Ramanujan to Hardy, settling of the balance amount in the account of Ramanujan with the Barclays Bank Ltd. in Cambridge, correspondence between the relatives of Ramanujan (especially, his mother and his wife) wrangling over the pension with the Government of Madras and the Port Trust were reproduced. In Chap. 8, the letters pertaining to the publication of the Collected Papers and manuscripts of Ramanujan and some correspondence between Hardy and G.N. Watson and from

[11]Bruce C. Berndt and Robert A. Rankin, *Ramanujan: Letters and Commentary*, AM–LMS publication (1995).

[12]Bruce C. Berndt and Robert A. Rankin, *Ramanujan: Letters and Commentary*, Indian edition was brought out by K. Srinivasa Rao, through Affiliated East West Press Pvt. Ltd., in 1997.

G.N. Watson to B.M. Wilson are dealt with. In Chap. 9, a very brief account of the family history and the family record of Ramanujan with the National Archives in New Delhi is presented.

Berndt and Rankin were interested in getting an Indian edition of this book originally published by the American Mathematical Society and the London Mathematical Society, in 1995, and they graciously permitted me to write a Preface to the Indian edition and include a few additions to the Indian edition, published by the Affiliated East West Press Ltd. (1997):

In this Indian edition, which coincides with the second printing of this book published by the American Mathematical Society (AMS) and the London Mathematical Society (LMS), an Errata to the first edition is appended at the end of this volume, in a section entitled Additions to the Indian Edition. The letter of Mr. E.S. Montagu, referred as No. 22—Financial dated February 24, 1921, in the AMS-LMS edition, and a photocopy of Ramanujan's letter to Mr. E. Vinayaka Row (one of the three reproduced, on pp. 113–117) now deposited with the Ramanujan Museum in Royapuram, are included. ...

To the excellent collection of photographs in the AMS-LMS edition, are added three photographs of the busts of Ramanujan: that made by Paul Granlund, that by N. Masilamani and that on the beautiful white marble Saraswathi temple tower ("gopuram") of the Birla Institute of Technology.

7.4 Busts of Ramanujan

Any artist, I believe, would agree that to draw a figure or paint a portrait of a person is not beset with difficulties and the quality of the outcome depends mainly on the skills of shading and closuring of the artist. However, to sculpt a bust or statute of an individual from a (two-dimensional) photo or portrait depends not only on the skills of the sculptor but also on his fertile imagination and knowledge connected with the subject. In the case of Ramanujan, a bust was commissioned almost seven decades after his death and the sculptors had only one authentic passport photo (in addition to the photos of Ramanujan either in the graduation gown or with others in that attire in a group photograph taken on that occasion). In what follows, the bust of Ramanujan on the Saraswathi Temple in the Birla Institute of Technology and Science, Pilani, perhaps the earliest sculpted, and the later busts sculpted by Paul Granlund and N. Masilamani and the full length bronze statute made in 2010 by Mr. K. Ravi are described.

7.4.1 On the Temple Tower of BITS, Pilani

While there are many institutions in India where different portraits of Srinivasa Ramanujan can be found, perhaps the first sculpted bust of Ramanujan is the one

found on the Gopuram ("temple tower") of the Saraswathi (goddess of learning) Temple in the Birla Institute of Technology and Science, (BITS), Pilani, Rajasthan. The work on this beautiful white marble temple dedicated to goddess Saraswathi, called the Sharada Peeth, was started in January 1956 and was completed in June 1959. The temple was consecrated by Shri Morarji Desai, on February 2, 1960. The imposing marble Gopuram is 109 feet in height, and there are in all 959 idols and statues on the outer Prakara ("wall"), at the base of the temple tower, made by Shri Narain Mal Jaimini of Jaipur. Thanks to Dr. S. Venkateswaran, Professor of Mathematics and Director of BITS, my attention was drawn to this sculpted bust of Ramanujan which was shown to me by Dr. K.R.V. Subramanian, Dean, Distance Learning Programme Division of BITS, who also provided me with the details about the temple stated above. This bust of Ramanujan is one of several in the middle row of three rows of figurines and is one of several other eminent persons including Sir C.V. Raman, Mahatma Gandhi, Rabindranath Tagore, Jawaharlal Nehru, John F. Kennedy and Vladimir I. Lenin.

7.4.2 Bust Made by Paul Granlund

In the *Professor Srinivasa Ramanujan Commemoration Volume*, edited by Ramananda Bharathi and published by *Professor Srinivasa Ramanujan International Memorial Bust Committee*, Madras, in 1974, details of the formation of that Committee can be found. However, no bust was made!

At the beginning of this book, it has been pointed out that Professor Richard A. Askey learnt from newspaper reports which appeared soon after the discovery of the "Lost" Notebook of Ramanujan, by Professor George Andrews, in the Spring of 1976, that Mrs. Ramanujan was lamenting over the absence of even of a bust of Ramanujan—promised to her but not made. This was sufficient for his efforts, detailed earlier, to get the bust made and presenting the same to Smt. Janakiammal Ramanujan, in 1985, at a function in the University of Madras.

The challenge of transforming the passport (two-dimensional) photograph of Ramanujan into a three-dimensional bust was taken up by Paul Granlund, sculptor-in-residence at the Gustavous Adolphous College, Saint Peter, Minnesota, USA. On February 6, 1985, while presenting a copy of the bronze bust of Ramanujan to the Indian Academy of Sciences, Bangalore, Professor S. Chandrasekhar and Mrs. Lalitha Chandrasekhar, the following tribute to Ramanujan by Professor Askey, (reproduced below, from *Patrika*, Newsletter of the Indian Academy of Sciences), was read out:

In the spring of 1976, Andrews went to Europe for a meeting and stopped in Cambridge to see what old manuscripts he could find. One find was not a manuscript but 140 pages of formulas in Ramanujan's handwriting.

The story of the thread from these sheets to the bust is simple. Andrews has done a lot of very deep work trying to understand what Ramanujan discovered. Eventually, "The New York Times" heard about it and interviewed him.

The Hindu followed with a more extensive interview and also published an interview with Ramanujan's widow, Janaki Ammal. She lamented the fact that a statue of Ramanujan had never been made, although one had been promised. Andrews sent me copies of these interviews, and after a couple of months my subconscious mind finally got through to my conscious mind and it was clear that a bust should be made. Since Janaki Ammal was 80, time was important, so it was up to individuals rather than governments or societies, since institutions move slowly. My first reason for wanting a bust was simple; if Ramanujan's widow wanted one she should have it. That was the least we could do to show our appreciation of Ramanujan, and while she did not understand his mathematics, she knew that he was one of the few whose work will last. As long as people do mathematics, some of Ramanujan's work will be appreciated. Fame is a strange thing and is often fleeting. An interview on a television program is now accepted form of honour.

In Ramanujan's case a permanent memorial is appropriate: one which can be appreciated by those who do not understand his mathematics should be added to the memorial Ramanujan made for himself and his work.

I am pleased to have played a role in this, and would like to thank the more than hundred mathematicians and scientists who contributed money for the bust which was presented to Janaki Ammal. The bust being dedicated today was donated by a couple who are now friends, Subrahmanyan and Lalitha Chandrasekhar. When I asked Chandra about the appropriateness of a bust of Ramanujan, he immediately replied that it was a good idea and they would do all they could to help. They did. Finally I want to thank the sculptor, Paul Granlund.

Thanks to Professor Askey, I am able to record here that the ten copies of the bust (to make the cost viable, at about 6000 $ per copy) made by Paul Granlund are at:

- Raman Research Institute, Bangalore
- The Indian Academy of Sciences, both in Bangalore
- The Tata Institute of Fundamental Research, Mumbai
- Inter-University Consortium for Astronomy and Astrophysics, Pune
- Ministry of Defence of the Government of India, New Delhi

The two copies in England are with:

- The Library of Cambridge University, Cambridge
- The Royal Society, London

The four copies in the United States are with:

- The Gustavous Adolphous College, St. Peter, Minnesota
- The Vaughn Foundation, Houston, Texas

and two are in private collections of individuals in America. The copies at the Indian Academy of Sciences and the Cambridge University were gifted by the Chandrasekhars; five bust are in the India including the original with the foster family of Janakiammal, four in the United States and two in England.

7.4.3 Bust Made by N. Masilamani

On the occasion of the Birth Centenary of Srinivasa Ramanujan, on December 22, 1987, Dr. B. Ramamurthy, the first Neurosurgeon of India, and the then acting Vice Chancellor of the University of Madras, was requested by me that a fitting memorial should be set up for the "natural genius" in Madras, that an appeal should be made, through media, for artefacts and letters pertaining to Ramanujan and that these could be preserved along with Ramanujan's original Notebooks and other manuscripts that can be retrieved from the Cambridge University through diplomatic channels, in a National Museum. Dr. Ramamurthy did constitute the Ramanujan Museum Committee with the Director of the Ramanujan Institute for Advanced Study in Mathematics, of the University of Madras, as its ex-officio Chairman. I was also in the committee at its meetings. A bust of Ramanujan was commissioned and its was sculpted by Mr. N. Masilamani, who belongs to a hereditary sculptor ("Sthapathi") family and was trained in traditional sculpting by his father, who was felicitated by the Government and other agencies with several gold medals for excellence in Arts. To cope with the modern evolution in arts, he joined the Kumbakonam College of Arts and Crafts and graduated with credit. Back in Madras, he came into contact with Mr. Nagappa, a renowned artist, painter and sculptor, whose works of art, numbering more than 100, can be found at several prominent places, including the Theosophical Society's headquarters in Chennai, in the High Court of Madras, at the Ripon Buildings in Madras and in several other cities in India. Mr. Masilamani belongs to this school of Nagappa. In fact, a majestic bronze lion and a bronze bust of Napoleon III were acquired by my father as he was the legal advisor of Nagappa, who executed many orders and produced exquisite sculpted statues/busts. Mr. Masilamani's fascination for Srinivasa Ramanujan and the enthusiastic support extended to him by Professors K. S. Padmanabhan, M.S. Rangachari, P.S. Rema and other Directors of the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras, resulted in his creation of a bust of Ramanujan which was unveiled on March 26, 1993, on the occasion of a Special Symposium on "Approximation Theory and its Applications", convened and conducted by Professor Geetha Srinivasa Rao and organized as a part of the Silver Jubilee celebrations of the Ramanujan Institute, founded in 1950 by the illustrious philanthropist and educationist, Sri Alagappa Chettiar. This bust of Ramanujan is now permanently housed in the foyer of the Ramanujan Institute, at Chepauk, Chennai.

On the occasion of the centenary of the Madras Port Trust, its Chairman Thiru V. Selvaraj, IAS, sanctioned to Janakiammal a lifetime pension of Rs. 300/- per month. The first monthly pension amount was given to Mrs. Janakiammal Ramanujan by the then Union Minister of Shipping and Transport, Mr. Veerendra Patil, on December 5, 1981. A newly acquired water barge was named as *Srinivasa Ramanujan* and the then Chairman of the Madras Port Trust presented a framed photograph of the ship to Mrs. Janakiammal Ramanujan, on January 6, 1983, and honoured the lady at that function. The bust of Ramanujan sculpted by Mani Nagappa was unveiled

at the Madras Port Trust, on January 17, 1990. The inscription on the pedestal of the bust reads: "*Thiru Srinivasa Ramanujan, F.R.S. (London). Born: 22-12-1887. Died: 26-4-1920. An Employee of Madras Port Trust in the year 1912. Bust unveiled by Hon'ble Minister Thiru Murasoli Maran, Minister of Urban Development, in the presence of Thiru K.P. Unnni Krishnan, Hon'ble Union Minister of Surface Transport and Communication, on 17th Jan. 1990. – A. Balraj, I.A.S., Chairman.*"

7.4.4 Bronze Statue in Ramanujan IT City by K. G. Ravi

In March–April 2010, the author was pleasantly surprised when the Registrar of the Society for Electronic Transactions and Security (SETS), in the Ramanujan IT City, Chennai, Dr. K. Thiagarajan approached me and asked me whether I can help the SETS organization acquire a bust of Ramanujan. This was some time in March/April 2009. We were in contact from that time when I came up with a suggestion that in the Ramanujan IT City it would be appropriate to have a Ramanujan–Hardy combined statue, like the Gauss–Weber monument in Göttingen in Germany. I showed him the photograph of the Gauss–Weber monument and the morphed composite of Ramanujan (seated) and Hardy (standing) photographs taken from the edited volume of P.K. Srinivasan and these were into the statue I had in my mind for the Ramanujan IT City. The one change I incorporated, with the approval of all concerned, was to use the face with that in the renowned, recognizable passport photo of Ramanujan. However, though there were no budget constraints, the Chairman Dr. R. Chidambaram of SETS was for only a full statue of Ramanujan (and ruled out Hardy's statute being combined with that of Ramanujan's). Who is to make it?

A full length statue of the Tamil actor who won international renown (Chevalier award winner), Sivaji Ganesan, was made and it was in the news since its location was being discussed—the location of the Kannagi statue had become a controversy and it was first located on the marina and then removed and waited for a relocation for a long time. I thought that the right person to sculpt the Ramanujan statue would be the one who made this true to life statue of the versatile thespian and Tamizh films actor, Ganesamoorthy, honoured for his innumerable characters acted in 288 films (in Tamizh, Telugu, Kannada, Malayalam and Hindi), as Nadigar Thilakam Sivaji Ganesan (1928–2001).

My father, a lawyer by profession, was a great admirer of Emperor Napoleon.[13] He got a woodcut statue of Emperor Napoleon Bonaparte (1769–1821) made by Curzon and Company, in Mumbai, in the 1930s and had it on an ornate teakwood

[13]On my first visit to Paris, France, in 1972, my father told me that I should not miss seeing the tomb of Napoleon at the Invalides, in France, and the Shakespeare Museum at Stratford-upon-Avon in England—and I did visit these two places and brought souvenirs for my father, a small statue of Napoleon on a horse after one of his battles on the Alps, and an illustrated complete works of William Shakespeare, as a gift from the Curator of the Museum!

bureau on which the quotation of Napoleon was painted in silver letters, which read: "*After all, a man ought to fulfill his destiny*", a statement of Napoleon in his last days when he was imprisoned in Elba. My father acquired two bronze busts—a bronze Lion (replica of the one at the site of the Battle of Waterloo, in Belgium) and the bust of Napoleon III—which I had grown accustomed to seeing from my childhood days. These bronze busts were presented to my father by a sculptor of repute in Madras, the renowned Mani Nagappa, who was my father's client, when I was a child.

I was aware that Mani Nagappa had a studio as a professional sculptor on Mount Road. I recalled that he was a client of my father, an Advocate. This made me contact Mani Nagappa on the phone and request whether he could make a full length statute of Ramanujan. He told me that he was very old and that he has stopped sculpting but that he could suggest a good student of his, K. G. Ravi, and put me into contact with him.

A quotation for the making of a life size bronze statue was obtained and I got the approval of Dr. R. Chidambaram, Chairman of the Board of Governors of SETS and its Director. This took naturally its own time and by the beginning of 2010 a firm order was placed with Mr. K. G. Ravi. A model statue in clay of the life size statue was first made for our approval, based on the photograph of Ramanujan seated, taken immediately after he got his BA degree (by research) in Mathematics, in 1918, from the Cambridge University. However, since the face of Ramanujan which became renowned was the passport photograph taken at the time of his return to India,[14] on my suggestion, it was decided to use the passport photo of Ramanujan for his face and the seated photograph of his (when his face was flush and cheeky, as opposed to his passport photograph in which he became leaner) for the posture.

A group of four of us—Mr. S. Thiagarajan Registrar, Dr. P. Nageswar Rao and Mr. R. Uppili of Society for Electronic Transactions and Security (SETS), Ramanujan IT City, Taramani, Chennai—made a couple of visits to the studio in Royapuram where the sculptor had his workshop and made a few suggestions for improvement. The approval of the statue was sought from the concerned authorities and obtained.

7.5 The Notebooks of Ramanujan

The Notebooks of Ramanujan and the manuscripts concerning him are now scattered in different libraries and archives. The publications of his *Collected Papers*, Hardy's lectures on *Ramanujan*, the facsimile editions of his Notebooks and Bruce Berndt's five-part publication "*Ramanujan's Notebooks*" are undoubtedly the most valuable collection of the mathematical works of Ramanujan and most

[14]The British government, which did not insist on a passport when he left for England, insisted on his getting one on his return to India, with a photograph, in March 1919, when he was emaciated after being in different sanatoria for nearly 2 and 1/2 years!

reputed research institutions the world over have them in their libraries. This chapter contains brief accounts of where the original Notebooks and other documents are, besides short notes on the relevance of Ramanujan to students and mathematicians in the new millennium.

Let us recall the history of the Notebooks: Ramanujan had noted down the results of his researches, without proofs (as in the *Synopsis of Elementary Mathematical Results*, by G.S. Carr), in three Notebooks, between the years 1903 and 1914, before he left for England. These were the Notebooks which he showed to his benefactors to convince them about his abilities as a mathematician. The results in these Notebooks were organized by him. The first Notebook has 16 Chapters and 134 pages. The second is a revised, enlarged version of the first, containing 21 Chapters in 252 pages. The third Notebook contains 33 pages of unorganized material. Ramanujan took his Notebooks with him to Cambridge. But, in one of his letters to a friend, he wrote that he had no time to look into them and most probably he did not put them to use during his 5-year stay abroad.

When Ramanujan left for India, he left behind one of his Notebooks with Professor Hardy. Three days after Ramanujan died, his younger brother S. Lakshminarasimhan, in a letter to Hardy, dated 29 April 1920, informing him about his demise, wrote: All his Mss that were in his trunk were handed over on the day of his death to Mr. Ramachandra Row's son-in-law, since the former was at Nilgiris. Not only those but also the journals, magazines—all he possessed except the books which he had, were taken away by them. In a letter to Prof. Hardy, dated 3 December 1920, Mr. R. Ramachandra Row, wrote: "Ramanujan's M.S.S. whatever they are with me and will be handed over intact to the University of Madras who I understand is already in correspondence with you regarding the methods of publication".

After Ramanujan's death, Hardy spent 3 to 4 months on a chapter in the Notebooks, on Hypergeometric series, and in the introduction to the paper he published on this topic he wrote: "a systematic verification of the results would be a very heavy undertaking". He tried to persuade the University of Madras and others to undertake such a task. He passed on the Notebooks to Professor G.N. Watson, who along with B.M. Wilson started editing different chapters in the Notebooks. However, unfortunately, due to the premature death of Wilson, the joint effort got disrupted and Watson did not complete the joint work.

In March 1925, Professor Hardy gave the first Notebook of Ramanujan with him to Dr. S.R. Ranganathan, the then Librarian of the University of Madras, with the remark: *Ramanujan belongs to your country, the proper place for his Notebook is your own (Madras) University Library*.

The second and third Notebook of Ramanujan were donated to the Madras University's Library after his death. T.A. Satagopan at the University of Madras made three handwritten copies of all the three Notebooks and one copy of each of these was sent to Hardy.

In 1957, at the instance of Professor K. Chandrasekhar and Professor Homi J. Bhabha (1909–1966) of the Tata Institute of Fundamental Research, Bombay, brought out the facsimile edition of these Notebooks in two volumes, without any commentary. These were reprinted by Narosa Publishing House, in 1987.

A resurgence of interest in the work of Ramanujan occurred, in the Spring of 1976, when Professor George E. Andrews of the Pennsylvania State University, while going through the Estate of G.N. Watson, who died in 1965, with the Trinity College, made the historical discovery of 140 pages of Ramanujan's paper in a box, containing some 600 theorems mostly on "mock" theta functions. These were results which Ramanujan had noted down during the year when he was deathly ill, in India, after his return from Cambridge. To these papers, Professor Andrews gave the romantic name, the *Lost* Notebook of Ramanujan and wrote a series of four articles: G.E. Andrews, *An Introduction of Ramanujan's 'Lost' Notebook*, Am. Math. Monthly **86** (1969) 89–108; G.E. Andrews, *Ramanujan's 'Lost' Notebook*:

I. *Partial Theta Functions*, Advances in Mathematics **41** (1981) 137–172; *Ramanujan's 'Lost' Notebook*.
II. "Theta-function expansions", ibid p. 173–185; *Ramanujan's 'Lost' Notebook*.
III. *The Roger's Ramanujan continued Fraction*, ibid p. 186–208.

George Andrews set for himself the task of proving several of the theorems. The "Lost" Notebook was published and released along with the reprints of the Notebooks of Ramanujan, by Narosa Publishing House on the occasion of the Ramanujan Birth Centenary, held in Madras, in December 1987.

Of course, there have been many individual mathematicians, all over the world, who worked on some aspects of the results in the Notebooks and several hundreds of research revealed that between 1973 and 1984 over 200 research papers refer to Ramanujan in their titles and abstracts. But there was no concerted or dedicated effort, in the direction as suggested by Hardy, till the 1980s. Professor Bruce C. Berndt, at the University of Illinois, Urbana-Champagne, took up the *heavy undertaking or formidable task* of editing the Notebooks of Ramanujan in right earnestness and set for himself the goal "to prove each of Ramanujan's theorems", if they were new and for the known results refer to literature where proofs may be found. The details of the contents of the Notebooks and other manuscripts is now recorded in the authentic work of Bruce C. Berndt.

The original Notebooks are now in the main Library of the University of Madras, in Chepauk, and are not easily accessible. The pages are stuck in between translucent paper and this should have been done at a time when lamination of the pages was not possible. A few tracks indicating moth holes are beginning to appear. The Notebooks should not be left in this condition and should be better preserved. They are invaluable possessions of this country.

7.6 The Wren Library

King's College, Trinity College and St. John's College of Cambridge University, Cambridge[15] are next to each other on Kings parade, which narrows into Trinity Street. The spacious quadrangles with lush green and velvety lawns behind the picturesque main facades of these colleges come as a total surprise to any first-time visitor to Cambridge, who would be greeted by numerous "PRIVATE", "NO ENTRY" sign boards, warding him off from the premises. These magnificent courtyards are open to the public for two hours on working days (12 Noon to 2 p.m.). The visitors are requested to be conscious of the studying atmosphere and the fact that some of the quadrangles are also residences of the scholars. These three colleges and the Queen's, Clare and the St. Catherine's Colleges have the willow-tree lined curving the Cam river flowing through great green pastures at their rear. A suggested walk in Cambridge (for about an hour and a half) takes one through these tradition-bound institutions, as well as the majestic King's Chapel, criss-crossing six bridges and one can also see a seventh one called "Mathematical Bridge", an intricate wooden bridge, spanning the river Cam at Queen's College.

"Henry VIII took over two colleges [the King's College and Michael house, of the fourteenth Century] to create Trinity College in 1546, giving it substantial endowments from monasteries he had dissolved. But its present grandeur is largely due to Thomas Neville, Master from 1593, who created the Great Court. The cloistered Neville's Court was later completed by the building of Sir Christopher Wren's magnificent Library. The largest of Cambridge colleges, Trinity College, boasts some twenty-eight Nobel Prize-winners," proclaims *A Jarrold Guide to the University city of Cambridge* (Jarrold Publishing, 1995).

In 1995, the Master of Trinity College was the reputed mathematician Sir Michael Atiyah, F.R.S. Like most of the 31 colleges in Cambridge, Trinity College had (about 660) undergraduate students, (300) graduate students and some (140) Fellows. Some of the renowned Trinity alumni are: Lord Byron, Dryden, Macaulay, Tennyson, Sir Isaac Newton, James Clerk Maxwell, Lord Rutherford, Sir Arthur Eddington, S. Chandrasekhar, G.H. Hardy, Bertrand Russell, Jawaharlal Nehru and Srinivasa Ramanujan. While the Cambridge University is a central body which provides coordinating administration, lectures, laboratories, libraries, examinations and degrees, the 700-year-old colleges provide educational supervision, tuition and residence for its members. However, the colleges are separate foundations having their own libraries and are largely autonomous.

The Wren Library, completed in 1695, is the older part of the Trinity College library, named after its designer, Sir Christopher Wren, containing the collection as in 1820 with a few later additions. Its internal size is 191 ft. × 40 ft. × 38 ft.—a very high ceiling indeed of 38ft.! Four statues on the roof of the Wren library

[15]The author visited Trinity College, Cambridge University, as a guest of Prof. Macfarlane and wishes to express his thanks for this opportunity which enabled him to ask for and see the cardboard boxes in their archives—more about this later.

represent Divinity, Law, Physics and Mathematics. The interior carvings in lime-wood and oak panelings. The stained glass window at the South end of the Wren Library shows Sir Isaac Newton being presented by lady "Fame" to King George III, with Francis Bacon in the foreground. At this end are also the marble busts of Bacon and Newton. Twenty-eight busts on the top of the wooden book cases are plaster casts of ancient and modern writers (most with no connection with Trinity). A full-length statue of Byron on a high pedestal is near the southern end and beyond this are the (eight) Reader's Places.

Professor Robert A. Rankin of Glasgow University, whose 80th Birthday was celebrated with a two-day conference on the 27th and 28th of October 1995, when requested by me, readily wrote to Dr. David McKitterick, the Librarian of Trinity College and obtained for me a Reader's place for two days, October 17 and 18, 1995, in the hallowed Wren Library. This introduction provided me access to the boxes containing papers of Ramanujan preserved by them in their Archives.

The Wren Library has a total stock of about 300,000 volumes of which 2500 are early manuscripts—1250 medieval manuscripts, 750 are fifteenth-century printed books—75,000 are pre-1820 printed books. It also has the Capell collection of Shakespeariana, many books from the library of Sir Isaac Newton and the Rothschild collection of eighteenth-century English Literature.

Visitors to the Library breathe in the air of the Wren Library and view some of the rare exhibits are placed in glass covered tables: the first bound volume of the plays of Shakespeare, the handwritten sonnets of Milton, some beautifully illustrated pages of Pilgrim's Progress, an open page from Winnie the Pooh, as well as the eighth-century manuscripts of the Epistles of St. Paul are some of the displayed exhibits.

With the help of Professor Rankin, the Wren Library has preserved and catalogued the papers of Ramanujan. These have the reference addresses as Add.Ms. a.94, with s superscript which refers to the specific document, and as Add.Ms.a.94^{1--18}, are contained in a large sized (A3-size) two inches high cardboard box. The references Add.Ms. b.100–107C refer to the five bound (foolscap size) volumes and, in addition to these, the papers and letters are preserved in two large (A3) sized thick cardboard boxes. In Appendix B are given the relevant details of the Ramanujan Manuscripts as maintained in the Archives of the Wren Library of Trinity College. In fact, Appendix B is a more detailed version of the Supplement II in the article of K. Venkatachaliengar entitled "Ramanujan's Manuscripts-II", which appeared in the Mathematics Student **52** (1984) 215.

In 1980, K. Venkatachaliengar, a Professor of mathematics at Mysore University, quoted Hardy's remarks to Professor S.R. Ranganathan that *Ramanujan belongs to your country, the proper place for his Notebooks is your own University Library* and added: "Naturally what applies to Ramanujan's Notebook applies equally to the other manuscript material which are passed on by Professor Hardy in subsequent years to Professor Watson. I request our younger colleagues and the Madras University to request our younger colleagues and the Madras University to request the University of Oxford and Cambridge to deliver all the Ramanujan manuscripts to our country keeping photostat copies of all of them in England. This would naturally take time; hence I request the University Colleges and institutes of higher learning in

our country to obtain photostat copies of the manuscripts in Oxford and Cambridge [The cost may be a couple of thousand rupees] and make them available to interested scholars of our country."

In his Presidential address to the Forty-fifth and Forty-sixth Conferences of the Indian Mathematical Society, held at Ranchi, in December 1979, and at Bangalore, in December 1980, respectively, Professor Venkatachaliengar spoke on the "Ramanujan Manuscripts". His address in 1979 ended with the poignant appeal: "The task of getting the originals or at least photo copies of all the Ramanujan manuscripts must be undertaken by our countrymen, who should also face the formidable task of editing them, taking at first those parts which are comparatively easy to edit; this is true of his elliptic function formulae which Ramanujan has given in nearly half of his note books. $\cdots$ May I hope that the University Grants Commission will engage scholars to perform this task as early as possible."

In Appendix C, we give the papers of Ramanujan passed on by the Madras Port Trust to the National Archives at New Delhi where they are preserved. A scrutiny of the Appendices B and C will inevitably reveal that the most interesting material is in the Wren Library but then through the book "Ramanujan: Letters and Commentary" (AMS-LMS 1995; Indian Edition, with a Preface, Additions to the Indian Edition and Errata, by K.. Srinivasa Rao, published by Affiliated East West Press Pvt. Ltd., 1997), Bruce C. Berndt and Robert A. Rankin have made this public.

The original Notebooks of Ramanujan are in the Madras University Library. The author had the opportunity of seeing them after a great effort, since the then Director of the Ramanujan Institute for Advanced Study had given strict instructions to the Librarian of the University of Madras not to show them to anyone without his approval. This author came to know, after a denial of their presence in the Library, that they were not in the Trinity College Library nor in the National Archives in India. He confronted the Librarian with the deductive logic that since they were brought back to India by Dr. S.R. Ranganathan, with the statement by Prof. Hardy that "Ramanujan is your country man and so his Notebooks belong to your country" they must be in the Madras University Library. Only then were the Notebooks shown, in 1986, to this author. A facsimile edition of the Notebooks were made in 1962, by T.I.F.R., Mumbai.

The author had the pleasure of getting the permission from the concerned authorities of the University of Madras[16] for scanning these Notebooks and also accompanying several visitors to the Institute of Mathematical Sciences, including Professors Richard Askey, Bruce Berndt, Robert Rankin, Bela Bolabas, E.C.G. Sudarshan, to see these Notebooks when they were in a reasonably good condition, around the Birth Centenary year, 1987. Despite repeated requests to the concerned authorities (Librarian, Registrar and even the Vice Chancellors of the University) that they should be preserved in carefully controlled conditions—since paper is hygroscopic and in a tropical country, if they are not kept in an air

[16]This was easy for me, since my wife, Dr. Geetha Srinivasa Rao, was a professor at the Ramanujan Institute for Advanced Study of the University of Madras and so was well-known to all those in the Registrar's office.

conditioned environment, they will become worn out—no action was taken. What is worse, they have been kept away from the public view[17] and "preserved" in a steel bureau whose key is zealously guarded by the Librarian! As a consequence, the Notebooks which I saw at the time of the Birth Centenary of Ramanujan and which were then in a reasonably good condition, were at the time of the 125th Birth Anniversary in a brittle state when turning the pages to browse through the Notebooks resulted in their breaking!

The apathy to preserve the Notebooks as a National Heritage, and keeping them locked-up in a Godrej bureau, is appalling!

My classmate in M.Sc. at the Presidency College, Madras, and good friend, Dr. T.S. Radhakrishnan, Former Head of the Material Sciences Division of the Indira Gandhi Center for Atomic Research (IGCAR), Kalpakkam, is willing to design a box—like the one I got made for display at the 98th Indian Science Congress Exhibition in Chennai—which could be maintained with an A/C unit. The request is made again now, in this the Ramanujan Remembrance Centenary year, 2020, with the fond hope that the preservation is done at least now, in the spirit of "better late, than never!"

The Ramanujan papers are better off where they are now, in the cardboard boxes in the Archives of the Trinity College, Cambridge. However, it is time that the efforts of getting them and preserving them, along with the original Notebooks of Ramanujan, should be made sooner than later, by the concerned authorities, Ramanujan admirers and heritage activists[18] like Dr. S. Muthiah.

[17]In the Wren Library, the works of Newton and the originals of Winnie the Pooh, have been preserved as antiques and exhibited prominently by the Curators of the Library.

[18]Subbiah Muthiah, MBE (1930–2019), is an Indian writer, journalist, cartographer, amateur historian and heritage activist known for his writings on the political and cultural history of Chennai. He is known for his books on the History of Chennai—*Madras Discovered*, East West Books (Madras) Pvt. Ltd. (1981) and a collection of articles under the column "Madras Miscellany", in the daily newspaper The Hindu, from November 1999. On 7 March 2002, Muthiah was made an "Honorary Member of the Civil Division of the Most Excellent Order of the British Empire", the citation for which read, for: "service by those who are not British citizens but who have pursued ideals which Britain values and shares". He is the founder of the fortnightly newspaper Madras Musings and the principal organizer of the annual Madras Day celebrations. Muthiah is the founder-President of the Madras Book Club.- Wikipedia.

Chapter 8
Ramanujan Birth Anniversaries and Documentaries on Ramanujan

The 75th, 100th and 125th Birth Anniversaries of Ramanujan were celebrated in different ways on December 22, 1962, 1987 and 2012. In this chapter are presented some of the significant events to commemorate those significant anniversary years.

8.1 The 75th Birth Anniversary of Ramanujan

In 1962, on the occasion of the 75th Birth Anniversary of Ramanujan, (late) Mr. P.K. Srinivasan, an ardent, enthusiastic Mathematics Teacher then at the Muthialpet High School, started an effort, with courage, conviction, enthusiasm and earnestness, which culminated in the release, in 1968, of

"*Ramanujan: Letters and Reminiscences*"

and

"*Ramanujan: an Inspiration*",

two Memorial Volumes containing the first ever collection of letters to and from Ramanujan. In an editorial to the first of these two volumes, P.K. Srinivasan wrote about the formation of a "Ramanujan Memorial Foundation, with an object of setting up a permanent memorial to Ramanujan in the shape of a multi-story building in Madras, housing a planetarium, mathematics exhibition wings, auditorium, library and show rooms displaying applications of mathematics in Industry. It will be a house of entertainment 'par excellence' for the layman and it will strive to make mathematics as popular as dance and music."

His dream had its first implementation at the Avvai Kalai Kazhagam where a room was set apart as the Ramanujan Museum with exhibits prepared by P.K. Srinivasan and some formulae of Ramanujan from his Notebooks which could be of interest to primary, middle and high school students. The origins of this Museum—"*a museum with a mission*"—is best described in what Srinivasan, its Founder and

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
https://doi.org/10.1007/978-981-16-0447-8_8

Resident Curator, said on April 26, 1996, the 76th remembrance day of Ramanujan, in the presence of relatives and well-wishers of Ramanujan:

Way back in May 1968, when Rajaji released the 2 volume Ramanujan Memorial Numbers, at Gokhale Hall, it was announced that there should be a museum to inspire people with the creative genius of Ramanujan and his singular life of dedication to mathematics. The announcement was received with thunderous applause. Since 1968, all the attempts to find a patron for housing the museum did not bear fruit. Nor was any museum started. Avvai Academy, a premier organization founded in 1992 in North Madras with its redoubtable secretary Mr. A.T.B. Bose, came forward to give the museum a place as one of its units. On 18th March 1993, the Museum was declared open by our elder statesman Sri C. Subramaniam, who has been awarded the country's highest honour of Bharath Rathna, in February 1998.

It would be appropriate to observe that the Museum is still in the making and it presents Ramanujan as a mathematician in the making of a super student star. The museum started with a few original and xeroxed letters, more than 8 decades old, of and to Ramanujan in their facsimile edition, panels of photographs of buildings associated with Ramanujan, of foreigners who played a role in the recognition and promotion of research career of Ramanujan and of Indians who had something or other to do in this passage from obscurity to fame.

With financial assistance of The Hindu, thanks to Mr. N. Ravi, distinguished editor who readily granted the request of Srinivasa Ramanujan Museum Associates represented by Sri S. Balathandapani of National Law School, a former student of the curator, sixty one custom made commercial art produced, laminated and framed bilingual (Tamil & English) charts of uniform size were procured and placed in the museum in 1995. They contain the jottings of interest to school students—jottings culled by the curator from his [Ramanujan's] Notebooks. This became the second phase.

Schools started sending their children to the museum. XI standard Math students simply marvel at the creative contribution of Ramanujan even at School mathematics level and enquire why they were not included in textbooks and taught in the class room.

It is gratifying to make it known that Prof. C. G. Swaminathan donated, in August 1994, one original letter of Ramanujan to his late father, Prof. C.N. Ganapathy Iyer of Presidency College. This was an important addition to the museum as it tells us where Ramanujan saw Prof. Hardy's book: 'Orders of Infinity' that triggered their correspondence.

Now we have gathered to witness on this 76th Remembrance Day of Ramanujan a historic event of Mrs. Saraswathi Dole donating to the museum three original letters written by Ramanujan to her late father Mr. E. Vinayaka Row, thanks to the recommendation of Prof. K. Srinivasa Rao of MATSCIENCE.

$\cdots$ Announcement has already been made regarding the need to launch a massive 10 acre 10 crore 10 year project of setting up a Ramanujan Mathemagic Plaza of international standard that will not only portray Ramanujan's genius with all its splendour but also present the vast expanse, the great beauty and the tremendous

power of mathematics, Today and Tomorrow. This alone will do full justice to the great memory of Ramanujan whose stature is still rising to the wonderment of the entire mankind.

··· Attached to the museum an active Math Education Center is functioning to take up the neglected areas of math education such as teacher growth through credit courses in intensive practicals, indiginization of math curriculum through incorporation of Kolam and folk mathematics, inter school primary math Olympiad, help to schools in organizing math club, math lab and math expo, in building up the math section of the libraries and display of wall paintings of mathematical interest, publication of recreational math books and alternative instruction strategies for teachers. Culturisation of mathematics through greeting school days with date magic square, magic square dances and inclusion of mathematical themes in school annuals and school dramas, production of math learning kits, summer math camps, reference library for journalists, etc.

8.2 Notebooks of Ramanujan and Their Accessibility

The fact that the five-part work of Bruce C. Berndt[1] entitled *Ramanujan's Notebooks* were expensive and not affordable to most institutions in India made this author get the approval of the Department of Science and Technology, Government of India, through the Secretary Dr. V.S. Ramamurthy, to fund the production of a pair of CD ROMs with the author as the sole content provider, and the help of two assistants—Rajkumar, B.E. in Computer Science, and M.Sc. in Mathematics—to help him create software for the CD ROMs at the Institute of Mathematical Sciences. The same was then taken to the National Multimedia Resource Center (NMRC),[2] at Pune, of the Center for Development of Advanced Computing (C-DAC), at New Delhi, for implementation. This was a two-and-a-half year project (from Dec. 2002 to April 2005), with periodic visits to C-DAC at New Delhi, to report on the progress of the project to the concerned funding authorities.[3] In Chap. 9, we present details about the contents of these CD ROMs.

[1]Bruce C Berndt, *Ramanujan's Notebooks*, Springer-Verlag, New York, Part I(1985), Part II (1989), Part III (1991), Part IV (1994), Part V (1997).

[2]Mr. Dinesh Katre was the Group Coordinator of NMRC and his able assistant Mr. Sudhir Patel, the one who implemented all the contents onto the CD ROMS.

[3]A Project Planning & Monitoring Group (PPMG) of DST had the following Members: Dr. B.D. Acharya and Dr. H.K.N. Trivedi both representing DST; Prof. K. Paranjape, Institute of Mathematical Sciences; Prof. S. Bhargava, Mathematics Department, University of Mysore; Prof. R.J. Hans Gill, Panjab University, and Dr. Asha Rani Singhal, Meerut University, besides the Project Coordinator K. Srinivasa Rao of IMSc as the Content Provider and Dinesh Katre of NMRC as the Production Manager of the Project.

8.3 Awareness Today

One hundred and twenty-five years after the birth of Ramanujan, it is no exaggeration to say that the interest in Ramanujan is still alive and unabated not only in the world of mathematics but also in the world of the media. One may ask: How? As has been stated earlier, as long as people do mathematics Ramanujan's work will have a pride of place through publications of researchers in International journals of repute. There are, in fact, three journals named after Ramanujan: *The Hardy-Ramanujan Journal*, edited by Professors K. Ramachandra and R. Balasubramanian and published by K. Ramachandra (formerly of the Tata Institute of Fundamental Research, Bombay) of the Hardy–Ramanujan Society and having only a limited circulation: *The Journal of the Ramanujan Mathematical Society*, started in 1985, with the initiative of Professors E. Sampath Kumar, R. Balakrishnan and others; and the *The Ramanujan Journal*, started in 1987 with a galaxy of 27 mathematicians and Professor Krishnaswami Alladi as its Editor-in-Chief—an international journal devoted to the areas of Mathematics influenced by Ramanujan –and published by Kluwer Academic Publishers. This is a tribute befitting the greatest. Also in popular magazines there are articles—for instance in the March 1998 issue of *Resonance* there is an article entitled *Ramanujan and I* by the renowned Hungarian mathematician, late Professor Paul Erdös, hailed as the *Western Ramanujan*. However, I have a whimsical tale to report regarding what happened in the media world:

Resonance reprinted in its December 1996 issue an article entitled *Reflections Around the Ramanujan Centenary*, by the Fields Medalist Atle Selberg who had participated in the International Conference held in Madras in December 1987 and in the Number Theory Conference which followed this at Kandy, in Sri Lanka. This was the occasion on which I had the pleasure of delivering a Keynote address on the Life and Work of Srinivasa Ramanujan, for the first time.

In the Saturday editions of two of the reputed English newspapers of India, on March 7, 1998, two articles appeared. One of these was a well-written article about the Ramanujan Museum (in the Avvai Kalai Kazhagam) in Royapuram, the realization of the pioneering effort of P.K. Srinivasan. The other was an article entitled "The Ramanujan factor" with an eye-catching photograph in the centre of the article. But, alas! The photo was not that of Srinivasa Ramanujan! Needless to say, in the next week's magazine section the newspaper carried a brief regret for their bloomer. Instead of brushing this aside as just another occasional journalistic blunder, let me ask: Would this have happened if there is a public awareness of the photo of Ramanujan, as we have of Mahatma Gandhi? If not, how can this be achieved?

In passing let me mention the genesis of the article referred in the above paragraph. Mr. C.A. Reddi, a friend of Bruce Berndt, who is also known to me, was requested to enlarge on the "*Ramanujan Family Record*", supposed to have been compiled by the younger brother of Ramanujan, S. Lakshmi Narasimhan, in 40 pages but for which only scanty information is available. Unable to locate the

originals with the relatives of Ramanujan, Mr. Reddi wrote a 11-page article some time in June 1997 entitled "The Early and Last Days of Srinivasa Ramanujan". The author, Reddi, was shocked to see the article of his in an abridged form under a new title and accompanied by the photo of one Mr. S. Rangaswami and not S. Ramanujan!

We now revert to how to make Ramanujan recognizable by every Indian: Most countries have the tradition of honouring their great scientists also by bringing out special commemorative Stamps and First Day Covers on significant days. The March 1998 issue of the *Notices of the American Mathematical Society*, Volume **45**, issue No.3, carries on its cover "A Collection of Mathematical Stamps"—of Ramanujan, Bolzano, Lagrange, Pascal, Kovalevskaya, al-Khuwarizmi, Dedekind, Newton and Cauchy. The Indian Philately Association has been ever vigilant, and on the occasion of the 75th Birth Anniversary of Ramanujan a commemorative stamp was brought out by the Government of India. While there exists a "Stamp Corner" column in the *Mathematical Intelligencer*, the author is unaware of one such for Currency Notes!

The author is aware of at least two countries which have honoured their scientists by using their portraits on Currency Notes. The 10 Deutsch Mark currency note of Germany carries the portrait of the renowned polymath Carl Friedrich Gauss (1777–1855) with the ubiquitous Gauss distribution also on the face of the note. The Austrian 1000 Schilling currency note has the portrait of the Physicist Nobel Prizeman Erwin Schrödinger (1887–1961) one of the founders of Quantum Mechanics, with the symbol Ψ, used to represent the wave function, highlighted on it. **Perhaps, the Indian Government could honour Ramanujan and Raman, the mathematical genius and the winner of the Physics Nobel Prize for this country, by using their portraits on the currency notes!**

8.4 Promising Prospects

Srinivasa Ramanujan is an exception and holds a unique place as a *mathematical genius*. India has no dearth of students with talent. Due to the fact that no laboratory is required for starting postgraduate courses, most colleges in India start with a Master's Degree programme in Mathematics. A large number of students enrol for these courses too. However, the lack of job opportunities at the end of a degree in mathematics is even resulting in a few institutions stop offering the mathematics courses! This is a dangerous trend, since mathematics is the foundation for all sciences.

In my career, I have come across two outstanding students who were interested in Number Theory. One of them was about 9-years-old Alladi Krishnaswami, in 1964, and got over his initial infatuation for aviation, as a school boy accompanying every year his father, Professor Alladi Ramakrishnan, to many of the reputed research institutes and universities of the world, to take to mathematics. As a Vivekananda College Mathematics student, he got interested in Fibonacci numbers

and Farey series. Fortunately, he came into contact with the prodigious Hungarian mathematician Paul Erdös—renowned for his path-breaking works in Probabilistic Number Theory, Combinatorics and Graph Theory—on his visit to Calcutta. His father, Professor Alladi Ramakrishnan, was in Calcutta to give an invited talk, and since his son was unable to accompany him due to his term examinations at that time, the father chose to present his son's work on Farey-Fibonacci series in the student seminar. On hearing this presentation, Prof Erdös told Professor Ramakrishnan that he would change his travel plans to meet Krishna in Madras, in 1974. This was the first of three visits made by Prof. Erdös to the Institute of Mathematical Sciences, Madras, in 1974. Young Alladi Krishnaswami (A. Krishnaswami, in Indian School records, is now K. Alladi), after completing his Bachelor's degree in Mathematics from the Vivekananda College, in Madras, went to the University of California at Los Angeles (UCLA) and within 3 years completed the Ph.D. under the supervision of Professor E.G. Strauss (the last student of Professor Albert Einstein). After a post-doctoral fellowship at the University of Michigan, Ann Arbor, he was for some years (1981–1990) my colleague at the Institute of Mathematical Sciences, Madras, before migrating to the United States where he is now a professor in the Department of Mathematics of the University of Florida, Gainsville. Krishnaswami Alladi is an established Number Theorist, and he collaborated with Paul Erdös and they have five papers together. I have known him as a precocious child, Krishna. Among his collaborators are George E. Andrews and Dave Bressoud. He has been Editor-in-Chief of *The Ramanujan Journal*, since its inception in 1997.

Another promising student of Mathematics, Kannan Soundararajan, was spotted as a precocious student[4] was brought to IMSc, when he was in his VIII Standard, in 1985, by the mathematics teacher of Padma Seshadri School and Professor Sudarshan put him into contact with me. Following my suggestion he came to visit me at the institute and I showed him the journals and books in Mathematics, got him the permission to use the excellent Library of the Institute by introducing him to the Librarian and the staff of the Library. I introduced him to Professor R. Balasubramanian and he in turn brought him into contact with Professor C.S. Seshadri, F.R.S. the Founder of Chennai Mathematics Institute.

Soundararajan found, in 1985, that the only four-digit number with two repeated adjacent digits and which is itself the square of a number with repeated digits is: $7744 = 88^2$. Encouraged by his parents and his school, Sound—the nickname by which he was popular in his school—took the Mathematics Olympiad Tests conducted by the Association of Mathematics Teachers of India (AMTI), consecutively from 1987 to 1991 and stood all-India first in all the tests. Spotted by the National Board for Higher Mathematics (NBHM) of the Department of Atomic Energy, Government of India, he participated in their training camps and took up projects on Gödel's Incompleteness Theorem and Non-standard analysis. Sound

[4]K. Srinivasa Rao, *A Talented and Promising Student: Kannan Soundararajan at Michigan*, Maths. Newsletter of the Ramanujan Mathematical Society, Vol. **3** (1993), p. 45.

was the recipient of the award for the most outstanding student in Mathematics Expos for 1989–1990. In September 1989, he was one of the ten students chosen to participate in a 6-week programme as a research scholar in the Research Science Institute (RSI) sponsored by the Center of Excellence in Education, Virginia, USA, along with 58 American students and a few from Australia, China, France, Italy and Sweden. His paper '*On the behaviour of a sparse series*' was adjudged as one of the five best written projects in the RSI. His outstanding performance in the Scholastic Aptitude Test (SAT)—in which he scored 770 led to his being ranked as *one of the outstanding students* of the 71 who had attended since 1984—got him admission to the University of Michigan to pursue his undergraduate studies. Professor Hugh Montgomery of the University of Michigan made the required provisions to financially support Sound to join the university after his schooling (XII Standard) at Padma Seshadri School, in Madras. During his undergraduate studies at Michigan, he published more than half a dozen papers in international journals, collaborated with R. Balasubramanian on a summer visit to Matscience, and came into contact with several number theorists like Professor Paul Erdös, Andrew Gainville, L.C. Washington, A. Ivic. He completed his doctoral thesis at the University of Princeton, Princeton, under the supervision of Professor Peter Sarnack. As a young mathematician, as an associate professor at Michigan State University, Sound has been awarded the 2003 (Raphael) Salem Prize, initiated in 1968, for outstanding work in the field of Number Theory. At the International Congress of Mathematicians, in Hyderabad, in August 2010, Kannan Soundararajan's name was in the shortlist of those nominated for the Fields Medal.

These are two success stories of youngsters spotted as precocious students of Mathematics, from independent India, whose recognition started from within the country leading to international recognition. Krishna and Sound are now American citizens of Indian origin, at University of Florida, Gainsville, and Stanford University, California, respectively.

With modern-day communications being excellent—especially, in research institutions like the Tata Institute of Fundamental Research (TIFR, Bombay), Indian Institute of Science (IISc, Bangalore), Institute of Mathematical Sciences (IMSc, Madras), Harishchandra Research Institute (HRI, Allahabad), and the most recent Indian Institutes of Information Technology (IIITs at Sri City, Hyderabad and Trivandrum), and the Indian Institutes of Science Education Research (IISERs at Berhampur, Bhopal, Kolkata, Mohali, Thiruvananthapuram and Tirupathi)—we can safely assert that the difficulties faced by Srinivasa Ramanujan, at the beginning of the twentieth century, regarding recognition, venues for studying and even funding, to a certain extent, are things of the past. However, there is no scope to be complacent and a constant vigil must be there to spot and nurture talent. The country has been obsessed with the down trodden, for whose uplift the reservation policy was suggested in the Constitution for a limited period of time. But the Reservation Policy has taken such deep roots that it has now spread to every level—from primary school to the employment even. The road map to a developed country status for India is through the gateways of higher educational institutions, and providing all the tools necessary rather than sops gives the tools to learn and excel and provides

a competitive environment for the bright students to get a name and fame for themselves and the county.

Thus, the urgent need of the hour is to first device methods of spotting the talented, from schools in the country and setting up a fast track to develop the talent of such carefully chosen few and, if need be, take them out of the main stream, and send them to excellent mentors who are willing to explore the unexplored territories of knowledge, with a proper foundation for teaching and research. Only then can excellent teachers hope to spot and catch the talented students at school, set their goals very high to make them achieve excellence in mathematics (and other branches of Science as well) and hope that occasionally, a miracle like Ramanujan may appear, from the blue!

In August 2010, India had the privilege of being the venue for the International Congress of Mathematicians (ICM), a gathering of about 4000 mathematicians. This is the first time that India was chosen to host the ICM, and the International Convention Center at Hyderabad was the venue for the meeting. With this ICM 2010, it is no exaggeration to say that the country is making ever so slowly, the transition from a "developing" to a "developed" country status.

8.5 Video Programmes

During Ramanujan's birth centenary year, 1987, a two-part video programme on "*Letters from an Indian Clerk*" was made by *An Independent Communications Association Production for Channel 4 with WGBH/Boston*. In it Ramanujan's Kumbakonam and Hardy's Cambridge were shown with interviews of Professors Bela Bollobas, a Hungarian Number Theorist at Cambridge; George Andrews, the discoverer of the "Lost" Notebook; S. Chandrasekhar, the Astrophysicist who was an admirer of Ramanujan all his life; and C.J. Hamson, Emeritus Professor of Law at the University of Cambridge, who was then a surviving contemporary of Hardy and Littlewood. This tastefully made documentary (about 55 min duration) was produced and directed by the renowned Christopher Sykes. The visuals of Ramanujan were his passport photograph, the photo of his taken after his graduation (wearing a cap and gown) with other graduates and the bust of his made by Paul Granlund. Mrs. Janakiammal was also interviewed and she recalls how Ramanujan was always interested in writing down his "sums" on paper; that mathematics was everything for him in this universe; and that food was given to him by her by hand while he was writing down his "sums" on paper; and that on his death bed he assured her that whether he is alive or dead, his mathematics would provide for her. It is these pages which he was writing down furiously as he was bed ridden and dying which constitute his Lost Notebook, which is considered by mathematicians today as perhaps the best work Ramanujan did.

In India a 30 min video programme was made entitled *The enigma of Srinivasa Ramanujan* by the *National Council for Science and Technology Communications* and credits were given to Dr. B.D. Acharya and N.K. Sehgal of the Department

of Science and Technology, Government of India, and to the following mathematicians: Drs. M.S. Rangachari, S. Raghavan, S.S. Rangachari, K. Ramachandra and to Mrs. Janakiammal and her foster son W. Narayanan. In this short documentary, the role of Ramanujan was played by a child, a young man and an adult (Raghubir Yadav) while Hardy was portrayed by Tom Alter. Hardy's words that he was neither an "inspired idiot" nor a "psychological creep" but a rational human being are mouthed by the actor. The temple town of Kumbakonam and the Town High School where Ramanujan studied are shown, besides the Goddess Namagiri of Namakkal. In typical Indian filmy style, for a brief moment the young Ramanujan is shown running around the pillars chasing the little girl Janaki.

8.6 Ira Hauptman's Partition

A play based on the famous collaboration of Hardy with Ramanujan has been scripted by Playwright Professor Ira Hauptman of the Department of Drama, Theatre and Dance, Queens College, Flushing, New York. The events scripted are purported to have taken place during the years 1914–1920, when Ramanujan was in Cambridge. The world premiere of this play entitled "Partition" took place in April 2003, in Berkeley. Directed by Barbara Olivier and staged at the Aurora Company, this play was enacted from April 11 to May 18, 2003. A review of this 145 min play by Professor Kenneth A. Ribet is to appear in the Notices of the American Mathematical Society—see http://math.berkeley.edu/ribet. Also, a review of the play by Professor Krishnaswami Alladi appeared in The Hindu newspaper on May 26, 2003. The author had the pleasure of witnessing the Dramatised Readings—a 75 min slick presentation on September 13, 2003, at the Central Lecture Theatre of the IIT Madras and had the full house in rapt attention with periodic ripples of laughter at the witty dialogues delivered with zest, enthusiasm and proper voice intonations.

In Hauptman's play, besides Hardy and Ramanujan, three other characters were introduced, in the revised version presented at IIT Madras: a fictitious Professor of Classics, Alfred Billingford, the renowned French mathematician of the seventeenth Century, Pierre Fermat, and the Goddess Namagiri of Namakkal. It was an enjoyable comedy, if one is either unaware of or willing to ignore the following:

- The Goddess is not omniscient or omnipotent but begs the ghost of Fermat for the solution to Fermat's last theorem!—Fermat wrote, in the margin of his copy of "Arithmeticus" by Diophantus of Alexandria, that: "It is impossible for a cube to be written as the sum of two cubes, or a fourth power to be written as the sum of two fourth powers, or generally for any number which is a power greater than the second to be written as a sum of two like powers". Or, in other words, the Diophantine equation, $x^n + y^n = z^n$, has no integer solutions for $n > 2$. To this theorem he added that "I have a truly wonderful demonstration of this proposition, which this margin is too narrow to contain". This theorem worried

mathematicians for three centuries and Professor Kenneth A. Ribet bonded the Fermat's last theorem to a conjecture of Yutaka Taniyama about zeta functions, in 1955 (see, for instance: www.public.iastate/~kchoi/time.htm).

- That Hardy suggested the Fermat's last theorem to Ramanujan to keep him busy after Ramanujan's attempted suicide, contrary to the fact that Hardy and Ramanujan did not attempt to solve the Fermat's last theorem, which was solved by Andrew Wiles of Cambridge University and Princeton, in June 1993.
- The two incidents based on Hardy's "A Mathematician's Apology", published with a forward by C.P. Snow (Cambridge University Press, 1967):
  - Hardy started a conversation with Ramanujan that a taxi cab number was 153, to which Ramanujan comes up with the observation that $153 = 1^3 + 5^3 + 3^3$ and that only five such numbers exist—five, if one counts the trivial $0^3 + 0^3 + 1^3 = 1^3$ along with the three others, besides 153 itself, being: $370 = 3^3 + 7^3 + 0^3$; $371 = 3^3 + 7^3 + 1^3$; $407 = 4^3 + 0^3 + 7^3$. This detail is from Hardy's "Apology", published in 1940, two decades after the death of Ramanujan– is fiction, while the authentic factual episode about the taxi cab number 1729 being the sum of two cubes in two different ways: $9^3 + 10^3 = 1^3 + 12^3$ is also a part of their play.
  - Hardy's rating of a great mathematical result being of the Hobbs class was corrected by Hardy to the Bradman class, in later years. There can be no evidence that referred to the work of Ramanujan on mock theta functions to be of the Bradman class! For, while Hobbs belonged to Ramanujan's times, Bradman's Test Match career started 8 years after Ramanujan died and it was from 1928/29 to 1948 with no sports during the Second World War period of about 5–6 years which coincided with the peak of the career of Bradman. One can imagine what his achievements would have been if he had played uninterrupted like the later day Brian Lara and Sachin Tendulkar, whose careers were at a time when continuous cricket, day-in-and-day-out, were possible. . . . The playwright proclaims that the work of Ramanujan on mock theta functions is also a gift of the goddess Namagiri of Namakkal.
- Ramanujan is not known to have been at "Addenbrooke's Hospital" as on background PowerPoint presentation slide showed, with a team of doctors.
- The exit line of the Goddess Namagiri of Namakkal: "Ayyo! Ramanujan your mouth is full of blood!" is not in the original text of the playwright—I thank Professor Ira Hauptman for this clarification contained in her e-reply to me, when I pointed out that "Ayyo!" is an expletive of the common man in distress and totally not befitting as an utterance by a God.
- The only letter which Ramanujan wrote to Hardy, on January 1920, did not contain any reference to even his deteriorating health, and certainly no mention was made about the glowing, fluffy hair of Janaki after a bath!
- One wonders why the playwright left out Professor J.E. Littlewood as a character in the play—he would have been appropriate than the fictitious Professor Billingford, especially since Hardy considered Littlewood as better

than himself (on a scale of 1–100, where Hardy gave himself 25, Littlewood 30 and Ramanujan 100).
- The playwright might have used utterances of the Goddess as thoughts which came to Ramanujan instead of portraying the Goddess as a mere mortal accompanying Ramanujan to Cambridge.[5]

In spite of these acts of omission and commission, the readings from the pay "Partition" were entertaining and 75 min pass away quickly thanks to the good effort of the actors.[6] The real with the imaginary and fictitious parts were seamlessly woven in this play of words.

8.7 Web Sites

For those who wish to look up a website for Srinivasa Ramanujan, the following information will be useful:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians

The websites of Professors Richard Askey, George E. Andrews, Bruce C. Berndt, K. Srinivasa Rao are:

http://www.math.wisc.edu/~askey
http://www.math.psu.edu/~andrews
http://www.math.uiuc.edu/~berndt
http://www.imsc.res.ernet.in/~rao/ramanujan

The home page of Professor Richard Allan Askey contains a link to Srinivasa Ramanujan and clicking on it shows the photo of the bust of Ramanujan sculpted by Paul Garnlund. The home page of Professor Bruce C. Berndt has Errata to his book publications, viz. Parts I to V of the Ramanujan Notebooks and lists also the books "Ramanujan: Letters and Commentary" and "Ramanujan: Essays and Surveys", both with Robert A. Rankin, jointly published by the American Mathematical Society and the London Mathematical Society, in 1995, and by the AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, USA, in 2000. There is also a link to the Gateway to "The Ramanujan Journal", a Springer-Verlag, an international journal of repute, with Krishnaswami Alladi, at present its Coordinating Editor.

[5]The sensitive Director, William Wyler, of the 1959 epic religious movie "Ben Hur", produced by Sam Zimbalist, for Metro-Goldwyn-Mayer, starring Charlton Heston as the title character, a movie adaptation of Lew Wallace's 1880 novel *Ben-Hur: A Tale of Christ*, did not show the face of the actor who played the role of Jesus Christ (Claude Heather) throughout the film.

[6]Dr. Vithal Rajan as Ramanujan; former Air Vice Marshall Cecil Parker as Hardy; the great grandson of Rao Bahadur P.V. Seshu Iyer, Ranjan Ranganathan as Professor Alfred Billington; the Director of The Little Theatre of Hyderabad, Shankar Melkote as Pierre Fermat; and Professor of English, Mrs. Sarla Melkote as the Goddess Namagiri of Namakkal.

A website has been designed and created between December 2002 and April 2005 and is at present accessible at:

www.imsc.res.in/~rao/ramanujan

This website includes the scanned images of the Original Notebooks of Ramanujan, the Collected Papers of Srinivasa Ramanujan, biographical articles, the story of the life of Ramanujan through photographs and material collected and presented for the first time in the two-volume publication of P.K. Srinivasan, interviews of Professors George E. Andrews, Bruce C. Berndt and Ratan P. Agarwal. The interview of Bruce Berndt was recorded on video at the University of Kyoto by Professor S. Kanemitsu of the University of Kinki, Fukuoka, while all the three of us were participating in an International Conference on Number Theory, on November 29, 2001. The interview of Professor George Andrews was at the University of Lucknow, while that of Professor Ratan P. Agarwal was at his residence in Lucknow, on January 6, 2002, during a symposium on "Special Functions and their Applications", as a part of the Indian Science Congress, 2002. The website also includes some pictures of the πie Pavilion and the Ramanujan Photo Gallery. It is the hope of the author (then, and even now) that the website will be officially announced and made available at the home page of the Institute of Mathematical Sciences.[7]

8.8 The ISC, πie Pavilion and Ramanujan Exhibitions

The 86th Indian Science Congress 1999 (ISC 99) was inaugurated by the Prime Minister of India, at the Anna University. On this occasion, the Chief Minister of Tamilnadu in his speech stated, as reported in The Hindu, on January 4, 1999:

Steps have been taken to establish a "Ramanujan Museum"—in the name of the mathematics wizard of Chennai jointly by the University of Madras and the Institute of Mathematical Sciences here. M. Karunanidhi appealed to the Prime Minister to consider "making the museum into a national asset".

The idea of having a "π- Room", like the one at the Palais de la Des Couverts, in Paris, France, in a Museum to house the original Notebooks of Srinivasa Ramanujan was raised by the author, K. Srinivasa Rao, on April 26, 1998, at a function in the Avvai Kalai Kazagam, which houses a room as Ramanujan Museum, when Dr. V.S. Ramamurthy, Secretary, Department of Science and Technology, Government of India, suggested that a πie room may be created for the sixth National Children's Science Congress (December 26–29, 1999) and the Indian Science Congress Exhibition (January 3–9, 1999). This is the origin of the πie Pavilion, a Project

[7]The author was a student of Professor Alladi Ramakrishnan, from 1964 to 1972, and a faculty member from 1972 till his superannuation in 2002, after which he was associated with the DST Project for 2 and half years when the two CD ROMs on the Life and Work of Srinivasa Ramanujan were produced (see the section which follows on CD ROMs project).

with the author as a Principal Co-Investigator, to provide the design and contents. The proposal, through the Periyar Science and Technology Center and its Executive Director, Dr. S. Iyamperumal, was approved and the exhibits prepared for the ISC 99 Exhibition were redone and installed by May–June 1999 and released for viewing by the public soon after at the PSTC, Kotturpuram.

A 12 feet Diameter, 2 feet high, wooden, cylindrical structure was designed by the author, to display on its inside surface the first 404 digits of the transcendental number π. The wooden cylinder was erected on wooden pillars at the Hall Number 13 of the Anna University in time for the 6th National Children's Science Congress which preceded the ISC 99 Exhibition. About 40 Posters were designed, enlarged and laminated. Most of the posters were of size A3, while the first 2000 digits of the transcendental number e were exhibited in a larger poster (of size A2). The story of π and e and the most beautiful equation in the whole of mathematics:

$$e^{i\,\pi} + 1 = 0$$

along with the stories of the two ubiquitous transcendental numbers in mathematics, π and e along with the unique complex number[8] $i = \sqrt{-1}$. The Contributions of Aryabhatta, Bhaskara, Euler, Gauss, Jacobi and the several formulae of Ramanujan including his 17 infinite series representations for π are highlighted. Two posters of the Gateway Arch of St. Louis, Missouri, Mississippi, exemplifying the Catenary (given by $\frac{e^x+e^{-x}}{2} = Cosh(x)$) were contributed by Dr. V. Lakshminarayanan of the School of Optometry, University of Missouri, St. Louis, who sent the same (carefully packed) for the exhibition.

A replica of the Ramanujan Museum at Royapuram with laminated posters depicting the number patterns observed or discovered by Ramanujan and contained in Ramanujan's Notebooks were displayed. A bronze bust of Ramanujan made by the renowned sculptor Paul Granlund presented, in 1984, to Mrs. Janakiammal Ramanujan, the original Passport of Ramanujan (with the relatives of Janaki-ammal[9]) were also displayed. These exhibitions, open to the public, were displayed, in specially made cabinets, along with the copies of the facsimile edition of the Notebooks of Ramanujan, from the Library of the Institute of Mathematical Sciences. A pilot CD ROM project, as a precursor to the CD ROMs project, was

[8]The author acknowledges with thanks the help of his colleagues Dr. R. Jagannathan, Dr. G.S. Moni, Library Assistant G. Venkatesan, and a student I. Suresh, volunteers of the Institute at the time of the ISC 99 Exhibition, which was shifted subsequently to the Periyar Science and Technology Centre (PSTC), Kotturpuram, where it is permanently housed since then and which underwent a major expansion with a generous (70 lakh Rs.) grant-in-aid for a Project proposed by the author and for whose initial implementation spent a year in the PSTC, from March 2010–April 2011.

[9]Thanks to the gesture of Mr. W. Narayanan, foster son of Mrs. Janakiammal Ramanujan, the bust and the Passport were on public display for the first time a this ISC 99 Exhibition.

also exhibited. This pilot project, at the instance of the author, was executed with the help of the K-12 group.[10]

The enthusiastic and whole-hearted support of Mr. P.K. Srinivasan and Mr. A.T.B. Bose, Secretary of the Avvai Academy, Royapuram, as Resource Persons and volunteers to the National Science Congress (NSC) 98 and Indian Science Congress Exhibition (ISCE) 99, contributed to the success of this pilot project. Visitors to the πie Pavilion, Pilot CD ROM Project and the Ramanujan Museum Replica, were Dr. R. Chidambaram, Dr. A.P.J. Abdul Kalam, Dr. V.S. Ramamurthy, Dr. V.C. Kulandaiswamy, and many prominent scientists and the public. The then Chairman of the Department of Atomic Energy, Dr. R. Chidambaram, suggested that the project may be replicated at the Nehru Museum of the Tata Institute of Fundamental Research. Dr. A.P.J. Abdul Kalam wanted the exhibitions to be on wheels, as a "Rath Yatra", and taken to all towns and cities of the country.

The PIE Pavilion and the Ramanujan Museum (Replica) were fused into the “RAMANUJAN GALLERY”, by the author, K. Srinivasa Rao, after the ISCE 99 and the same inaugurated by Ms. C.K. Gariyali, I.A.S., Vice Chairperson, Science City, and dedicated to the public at the Periyar Science and Technology Center, Kotturpuram, Chennai, on Science Day, February 28, 1999. Even as she was cutting the ribbon to inaugurate, Dr. Gariyali asked me as to what would happen to this exhibition after the ISC 99. When I said that I had not thought about that as I was busy getting all the Posters ready for display almost single-handedly, Dr. Gariyali sent word for Dr. E. Sargurumurthy the then Executive Director of the Periyar Science and Technology Center (PSTC) in Kotturpuram, waited for his arrival while listening to the details about the poster exhibits, and directed him to provide space in the PSTC for me to set up the exhibition as a permanent part of the Science section in it. As a consequence, a Project was proposed to the DST through the Executive Director, with me as the Principal Investigator. Subsequently, the required financial support asked for was provided through the Mathematical Sciences Division headed by (late) Dr. B.D. Acharya, at the instance of the Secretary of DST, Dr. V.S. Ramamurthy, to set up a RAMANUJAN PHOTO GALLERY by the side of the "PIE Pavilion and the Ramanujan Gallery" in a Hall in the Physical Sciences section of the PSTC. The work on the Gallery was completed by the author, consisting of about 110 laminated (A3-size) exhibits pertaining to the stories of π, i and e, and the life and work of Ramanujan, with the help of Mr. A.T.B. Bose of the Avvai Academy, in Royapuram where there is a permanent Ramanujan Museum, the first in honour of Ramanujan, set up by (late) Mr. P.K. Srinivasan, a Fulbright Scholar in the United States for 2 years in early 19602, and an enthusiastic mathematics teacher of Muthialpet High School. The exhibition was opened to the public by the end of June 1999.

[10]The help of Dr. Uma Swaminathan and Dr. Gaurav Bhatnagar, then with the K-12 group of National Institute of Information Technology, New Delhi, is gratefully acknowledged here.

Besides the ISC Exhibition 1998, at Chennai, and the ISC Exhibition 2012, at Hyderabad, the author helped set up exhibitions at

- The Modern School, Chennai, 2007
- The Ramanujan Mathematics Academy and Mathematics Library, Ramachandrapuram, 2013
- The Indian Institute of Science Education Research (IISER), Tirupathi, 2018
- The Kerala Development Innovation Strategic Council (KSDIC), Trivandrum, 2018

which carried exhibits on the Life and Work of Ramanujan. The "πie Pavilion and the Ramanujan Gallery are permanent exhibits at the PSTC, Kotturpuram, Chennai, 1999, have been enlarged at a cost of 70 Lakh Rupees and opened to public from 2010.

Documentaries on Ramanujan have been made based on the life and work of Ramanujan by Nandan Kudhyadi[11] of Nandan Kudhyadi Productions. He has made a Trilogy of documentaries on Srinivasa Ramanujan:

- "The Enigma of Srinivasa Ramanujan", (38 mins.), in the Ramanujan Centenary year 1987
- "The Genius of Ramanujan", (60 mins.), co-produced by IISER, Pune and Vigyan Prasar, New Delhi, in 2012, on the occasion of the 125th Birth Anniversary of Ramanujan, 2012
- "The Mystique of Ramanujan Conjectures: Ramanujan's Spiritual Realization in Mathematics", in 2014

The following is written by Anuradha Mascrenhas[12] on Nandan Kudhyadi: "Pune-based filmmaker wins 3 awards at National Science Film Festival":

"For his brilliant cinematic rendition of the abstruse work of a mathematical genius, city-based Nandan Kudhyadi has won three awards for the fourth docudrama on Srinivasa Ramanujan: The Mathematician and His Legacy at the seventh National Science Film festival. Vigyan prasar, an autonomous organisation under the Department of Science and Technology and National Council of Science Museums, organised the festival in Kolkata between February 14 and 18, 2012." The three awards include the

- Bronze Beaver Award
- Technical Excellence Award: Direction for creating an absorbing narrative that moves with ease between documentary interviews, vivid reenactments and explanatory graphics
- Technical Excellence Award: Research for its exploration into contemporary applications of Ramanujan's abstract theories, and for his meticulous attention

[11]Nandan Kudhyadi studied at Faculty of Fine Arts, M.S. University, Baroda and worked at the Foundation for Liberal And Management Education (FLAME). He has produced more than 65 quality documentaries from his Production House: Nandan Kudhyadi Productions, in Pune.

[12]In the Indian Express newspaper, dated February 27, 2017.

to its biographical detail. Earlier last December, the same film had won the Jury Award at the India International Science Festival 2016.

In the second documentary, "The Genius of Ramanujan", the author had a role to play as one of the resource persons. It released in March 2013 at the Indian Institute of Science Education and Research, Pune. The third documentary was released by the end of the year 2014 and Telecast on Rajya Sabha TV channel, on Dec. 22, 2014, at 1 P.M. and 5 P.M. ($\sim$ 55 mins.). With it Nandan Kudhyadi completed a trilogy of Documentaries on the life and work of Srinivasa Ramanujan.

The documentaries of Nandan Kudhyadi include excellent ones on the Nobel Prize men Sir C.V. Raman and Dr. S. Chandrasekhar. This quality conscious director's other documentaries have been screened at International Film Festivals in Brussels, Hawaii, Paris, Tokyo, Vancouver and "The Genius of Srinivasa Ramanujan" was screened at the International Conference on Number Theory at New Delhi, in Dec. 2012. It was also screened at the annual joint meeting of the American Mathematical Society and the Mathematical Association of America, at Baltimore, in January 2014. The author has screened the first or the second of the Ramanujan trilogy, after a PowerPoint presentation on the life and work of Ramanujan, at several Schools, and higher education institutions, in India and abroad.

Chapter 9
Relevance of Ramanujan Today

In this chapter the contents of the two CD ROMs on the life and work of Srinivasa Ramanujan, the international recognition and celebrations of the Birth Centenary and the 125th Birth Anniversary of Srinivasa Ramanujan are described. The chapter ends with an appeal that efforts should be made in the setting up of a *Ramanujan Science Museum*, for the life and work of the "natural" mathematical genius, Srinivasa Ramanujan, continues to be relevant in this twenty-first century.

9.1 Contents of the CD ROMs

Ramanujan's work has inspired generations of Mathematicians. The three journals named after him are:

- Journal of the Ramanujan Mathematical Society
- The Hardy-Ramanujan Journal
- The Ramanujan Journal

Hundreds of papers have appeared and continue to appear based on the published papers of Ramanujan, his three Notebooks and the "Lost" Notebook discovered by George E Andrews, in the Spring of 1976. These facts reveal the enduring nature of his remarkably stupendous contributions to Mathematics. The "Collected Papers of Srinivasa Ramanujan", edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson, originally published in 1927, and "Ramanujan: Twelve lectures on subjects suggested by his life and work", by Hardy first published in 1940, have both been reprinted in 1999 by AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, USA. This clearly shows the relevance and importance of the original papers of Ramanujan in this millennium.

The 125th Birth Anniversary of Ramanujan was inaugurated by the Prime Minister, Dr. Manmohan Singh, who declared 2012 as the National Mathematics

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
https://doi.org/10.1007/978-981-16-0447-8_9

Year; released a commemorative stamp and first day cover; and declared from 22 December 2012 onwards, the date of birth of Ramanujan will be celebrated as the National Mathematics Day. Every mathematician would hail these as the best for Mathematics development in our country.

Two CD ROMs were developed by The Institute of Mathematical Sciences (IMSc), Chennai and by the National Multimedia Resource Centre (NMRC) of the Centre for Development of Advanced Computing (C-DAC), Pune. This project was sponsored by the Department of Science and Technology (DST), Government of India, with the concept and design of contents by K. Srinivasa Rao, the author.[1] This was a two-and-a-half-year DST funded, Rs. 20 Lakh Project, at IMSc, during Dec. 2002 and April 2005. The CD ROMs were released by Dr. V.S. Ramamurthy, then Secretary of the DST, at a public function in the Jamia Milia Islamia, New Delhi, on December 22, 2005; and subsequently, also at a special function arranged at the Triple Helix Auditorium of CLRI, Chennai, where Dr. M.S. Ananth, Director, IIT Madras, received the first copy of the CD ROMs from Dr. T. Ramaswami, Secretary, DST.

The main objectives of this successfully completed project were: to spread awareness among the students and the general public on the life and work of Srinivasa Ramanujan; to preserve in digitized form the Collected Papers of Srinivasa Ramanujan and the work contained in the original Notebooks of Ramanujan.
Given below is the list of items that were included in the CD-ROMs:

- Birth and Schooling (1887–1903)
- Years of Adversity (1904–1909)
- Turning points (1909–1914)
- Cambridge Years (1914–1919)
- Return to India (1919–1920)
- After 1920
- Museum (clip using Quick Time Virtual Reality (QTVR))
- Film Clips
- Interviews of Professors G.E. Andrews, R.A. Askey, B.C. Berndt, R.P. Agarwal, Aru Sankaralingam, S. Ramaswamy and W. Narayanan, by the author
- Keyword Index
- Image Index
- Notebooks of Ramanujan
- Collected Papers of Srinivasa Ramanujan (39 published papers and his 59 Questions and Answers to Questions in the Journal of the Indian Mathematical Society)

The source material for the CD ROMs were:

- The two biography articles. The first written by G.H. Hardy, P.V. Seshu Iyer and B. Ramachandra Row, which appeared in the Preface to the *Collected Papers of*

[1] At that time, the author was the Director of the Ramanujan Museum and Math Education Centre, housed in the Avvai Kalai Kazhagam, Royapuram, Chennai.

Srinivasa Ramanujan, edited by the same authors Hardy, Seshu Iyer and Wilson (1927) and the second in Hardy's *Ramanujan: Twelve Lectures Suggested by His Life and Work*.

- E.H. Neville's 1941 All India Radio talk.
- Bruce C. Berndt's article in *Ramanujan's Notebooks*, Part I (1985).
- K. Srinivasa Rao's: *Srinivasa Ramanujan: A Mathematical Genius*, East West Books (Madras) Pvt. Ltd. (1998, 2nd edition 2004).
- Original Notebooks of Srinivasa Ramanujan, with the University of Madras and/or Notebooks of Srinivasa Ramanujan (facsimile edition) 2 Volumes, brought out by the Tata Institute of Fundamental Research, Bombay, 1957 and also by Narosa, New Delhi, 1987.

Note. The writings of G.H. Hardy provided the main biographical source material on Ramanujan to all the writers/researchers on the life and work of Ramanujan. The CD-ROMs contain biographical and historical parts, in which visuals, audio and video clippings were integrated with the text. The visuals are mostly photographs and the videos are of places and personalities interviewed exclusively for the CD ROM. The NBHM also brought out in March 2000, an edition of the Collected Papers of Srinivasa Ramanujan (Prism Publications, Mysore). Prof. Bruce C. Berndt of the University of Illinois has devoted most of his time, since 1976, to the study of the 3254 Entries (by the reckoning of Prof. R.A. Askey) in the Notebooks of Ramanujan. Bruce Benrdt has published scores of papers and brought out in a series of five volumes entitled Ramanujan's Notebooks, published by Springer-Verlag between 1977 and 1997, a detailed study of every Entry Ramanujan made in his original Notebooks. He was awarded the Steele Medal for this excellent work. This work of Prof. Bruce Berndt is contained as Part-II of the CD ROMs on the Life and Work of Srinivasa Ramanujan. It is a limited edition and as this second CD-ROM contains copyrighted material, the permission of Springer-Verlag was obtained and for 1000 copies of this two-part CD ROM, which was mailed by NMRC, C-DAC, Pune to Dr. V.B. Kamble, Director, Vigyan Prasar (C-24, Qutab International Area, New Delhi 110016; vbkamble@alpha.nic.in) authorized for dissemination of the same.

This successfully completed project preserves for posterity the invaluable Life and Work of Ramanujan; the original Notebooks of Ramanujan, now in the possession of the University of Madras (Part I), and the five-part work of Prof. Bruce Berndt entitled "Ramanujan's Notebooks" (Part—II) was completed and copies of the same were given to the President Dr. A.P.J. Abdul Kalam, at the Rashtrapthi Bhavan on December 26, 2005, at the earliest appointment given by His Excellency, by a delegation led by the author and Dr. V.S. Ramamurthy, DST Secretary; Dr. S. Ramakrishnan, Director C-DAC; Dr. B.D. Acharya, Mathematical Sciences Head of DST; Dr. V.B. Kamble, Director, Vigyan Parasar; and others. The CD ROMs were released earlier at a public function by Dr. V.S. Ramamurthy, Secretary DST, at the Jamia Milia Islamia, on December 22, 2005, itself.

A pilot CD-ROM on the Life of Srinivasa Ramanujan, supported by the Department of Science and Technology, Government of India, was brought out

by this author, on the occasion for the Indian Science Congress Exhibition in January 1999. The pilot CD-ROM received critical acclaim by the public and academics including Drs. R. Chidambaram (Chairman, DAE), A.P.J. Abdul Kalam, V.S. Ramamurthy (Secretary, DST), Prof. Bruce C. Berndt (University of Illinois), Editor of Ramanujan's Notebooks. Biographies were written by Indian authors S.R. Ranganathan (1957); P.K. Srinivasan (1968); and K. Srinivasa Rao (1998).

The completed project resulted in the production of an encyclopaedic version of the life and work of Srinivasa Ramanujan in two CD-ROMs; the first one containing no copyrighted material can be acquired and disseminated by anyone to anyone else. The stupendous work of Ramanujan contained in his Notebooks and their editing by Bruce Berndt in his five-volume publication entitled Ramanujan's Notebooks is the content of a second CD ROM.

Needless to say, the original Notebooks of Ramanujan, with the University of Madras, is a National treasure and a part of the heritage of the country. The stature and achievements of Srinivasa Ramanujan, hailed as one of the greatest Mathematicians of the twentieth century, should be an eternal source of inspiration for generations of students of mathematical sciences of this country. This has been achieved through the creation of digitized CD-ROMs that highlight his life and work.

Professor K. Srinivasa Rao, of IMSc, the domain expert was responsible for providing all the contents for the project. Dr. Dinesh Katre and Mr. Sudhir Patel of The National Multimedia Resource Centre (NMRC)[2] provided the technical support required to produce two CD ROMs. This Centre for Development of Advanced Computing (C-DAC) is sponsored by the Department of Electronics, Government of India, with the objective of developing Low Cost Multimedia Solutions in the Indian context.

The Centre has state-of-the-art infrastructure facilities and has expert teams for implementation of multimedia projects. It is also engaged in developing of tools and templates for multimedia authoring with due regard to Indian language, art and culture. C-DAC has extended these technical capabilities for developing the CD-ROMs on Ramanujan over a period of 2.5 years starting from[3] December 1, 2002, and ended on May 31, 2005. Simultaneously, a website has been created by Dr. K. Srinivasa Rao which contains all the relevant details about Ramanujan including every paper he has published (except the work of his in the "Lost" Notebook of Ramanujan, which is not also in the CD ROMs) at www.imsc.res.in/~rao/ramanujan and has been hailed by Professor G.E. Andrews, the discoverer

[2]Project Planning & Monitoring Group (PPMG) of DST consisting of Dr. B.D. Acharya and Dr. H.K.N. Trivedi of the Mathematical Sciences Division; Prof. K. Paranjape of IMSc; Prof. S. Bhargava of Mysore University; R.J. Hans Gill of Panjab University; Prof. Asha Rani Singhal of Meerut University, besides the two project coordinators: Dr. K. Srinivasa Rao of IMSc and Mr. Dinesh Katre of NMRC.

[3]The day after the superannuation of its coordinator, Dr. K. Srinivasa Rao, as Senior Professor, IMSc, Nov. 30, 2002.

of the "Lost" Notebook of Ramanujan as a significant "contribution to mathematics and mathematicians" (in an e-mail to the author, 2005).

Note: The completion report and all other relevant correspondence details are in the (archived) files of the Institute of Mathematical Sciences, Madras, where the author continued working as a Pensioner without any additional monetary support being taken from the Project for this love of labour.[4]

The Department of Post and Stamps, Government of India, requested the Prime Minister Shri Rajiv Gandhi, to release the Stamp and the First Day Cover, on December 22, 1987, to mark the 75th Birth Anniversary of the mathematical genius Srinivasa Ramanujan and the Prime Minister of India, Dr. Manmohan Singh, released the Stamp and First Day cover of Ramanujan on the occasion of the 125th Birth Anniversary of Ramanujan.[5]

The Government of India announced that December 22, from the year 2012 onwards, will be called as the *Mathematics Day*, in honour of the "natural" mathematical Srinivasa Ramanujan (Dec. 22, 1887–Apr. 26, 1920), who passed away at the young age of 32 years 4 months and 4 days, leaving behind a unique legacy of 4000 Entries or Theorems, in his Notebooks, without Proofs, which enabled many mathematicians of the world devoted their careers to providing a proof for every Entry in the Notebooks.

9.2 Birth Centenary of Ramanujan (1987)

The Birth Centenary of Ramanujan was celebrated with an International Conference at Chennai, by the Institute of Mathematical Sciences, with funds from all the National agencies, on a grand scale. Its inauguration was on December 22, 1987, by the then Prime Minister of India, Sri Rajiv Gandhi. Naturally, Security arrangements were tight around the venue of the Conference, the Kalaivanar Arangam, which had a seating capacity of about 4000. As the members of the host institute, my colleagues and I had the privilege of being seated in the second row in the VIP enclosure and I had the pleasure of being with Professor Sudarshan the Director of the Institute and my senior colleagues. What is more, we were just behind the acting Vice Chancellor of the Madras University, who was the renowned Neurosurgeon Dr. B. Ramamurthy,

[4]This section on CD ROMs appeared in ECR Express, Volume 1: No. 13, January 29–February 4, 2012.

[5]It is unfortunate, that on this occasion, the mirrored image of the authentic passport size photo of Ramanujan appears in the stamp and the first day cover. Similar to the bloomer by the mathematics experts in the committee distributing a conference bag to all the participants of the International Conference to mark the Birth Centenary of Ramanujan with a prominent display of the wrong expression for the renowned τ-conjecture of Ramanujan: viz. $|\tau(p)| \leq 2\sqrt{p}$ instead of the correct expression: $|\tau(p)| \leq 2p^{11/2}$, was proved by Deligne (1974), as a consequence of his proof of the Weil conjectures. Pierre Deligne (Born in 1944) was honoured by the Belgian post office which issued a stamp in his honour in 2007.

who was known to the members of the Institute, especially my senior, Prof. R. Vasudevan. The author had never seen a dais with more number of people seated on the Dais: about 400 important people—including all the MLAs (230), MLCs, Central Government MPs, Ministers of the State Government, special invitees and, of course, a large contingent of the Media. The hall was overflowing with politicians. And only about 400 were Mathematicians from all over the world, including the Indian Mathematicians. The invitation said that it was mandatory for the audience to be seated at 8.15 AM itself[6] though the PM was scheduled to inaugurate the International Conference at 9.15 AM.

The meeting was one of the shortest in duration, as the PM left at 9.45 PM, after releasing the *"Lost" Notebook of Srinivasa Ramanujan*,[7] edited by Professor Andrews, of the Pennsylvania State University, who was honoured for this by the Prime Minister. Professor Andrews presented the first copy of the book to Janaki Ramanujan, the wife of Ramanujan, a special invitee on the Dais. After the brief function, all were requested to remain standing as the Prime Minister and the retinue of VIPs left the hall.

The most surprising aspect of that day was that, unlike in any other international conferences which was inaugurated in the morning (in which I participated), there was no academic programme between 9.45 AM and Lunch time. The audience was requested to reassemble in the afternoon at 2.30 PM for the only public lecture to commemorate the Birth Centenary of Ramanujan. This was to be delivered by none other than the 1983 Nobel Prize winner, Astrophysicist S. Chandrasekar.

To my amazement, he started with the following introductory remark: "What little I know about Ramanujan, I stated at the International Conference at the University of Illinois, Champagne-Urbana. I do not want to repeat myself. So, I am going to talk about 'Newton's Principia' now."

To this day, I consider this as a great disrespect to the native of Madras/Chennai Ramanujan and inexplicable to me as to what made Chandrasekar deviate and disappoint the assembled group of about 400 mathematicians who had come from far and wide to pay their tribute to Ramanujan. And what is more, whether this change of topic, totally irrelevant for the occasion, was made known by him to the

[6]This turned out to be a blessing in disguise for me, as I could have an extensive discussion with the Vice Chancellor about the preservation of the original Notebooks of Ramanujan and my ideas of propagating their contents in an easily accessible form. I quote from memory what he said to me in reply: "you see, Srinivasa Rao, I am a Surgeon, you tell me what I should do as points 1, 2, 3, . . . and I will do it". Since there was plenty of time, I immediately sat down and took a sheet of paper and wrote down my request in the form of a letter addressed to him as the VC of University of Madras. Unfortunately, though he said he had placed the request before the Syndicate at the very next meeting, he had to step down and the person appointed as the Vice Chancellor was one whom I did not want to approach with the request, as I was almost sure of the failure of such a request to him—this is based on the fact that my efforts to get the Wallajah Road renamed to include the name of Ramanujan, as the Ramanujan Institute for Advanced Study was situated on that street, was simply ignored!

[7]See, Chap. 2, about the discovery of this Notebook, by George Andrews.

organizers of the conference. Of course, there was no comment by any reporter in the media about this aspect!

The Number theorist and 1950 Fields Medalist Professor Alte Selberg[8] (1917–2007) was proved independently by Jacques Hadamard and Charles Jean de la Vallée Poussin, in 1896 using the Riemann zeta function. Some of the observations of Selberg, about Ramanujan made extemporaneously to an audience at the Tata Institute of Fundamental Research, Mumbai, at the conclusion of the Centenary Conference there (in Jan. 1988)[9] contains the following observations of his:

- Ramanujan's *Collected Papers* seemed quite like a revelation—a completely new world to me, quite different from any mathematics book I had ever seen—with much more appeal to the imagination.
- Ramanujan's particular talent will seem to be primarily of an algebraic and combinatorial nature... He had on his own, acquired an extraordinary skill of manipulation of algorithms, series, continued fractions and so forth, which certainly is completely unequalled in modern times.
- He seems also to have often taken a very particular delight in the special rather than the general, and if one looks in his *Notebooks*, and also in his Letters to Hardy, for instance, he will often state or choose to state a special case or special cases that seem particularly striking, where very clearly he has much more general results underlying them.
- Ramanujan's recognition of the multiplicative properties of the coefficients of modular forms that we now refer to as cusp forms and his conjectures formulated in this connection, and their generalization, have come to play a more central role in mathematics today, serving as a kind of focus for the attention of quite a large group of the best mathematicians of our time. Other discoveries like the mock-theta functions are only in the very early stages of being undertook and no one can yet assess their real importance. So the final verdict is certainly not in, and it may not be in for a long time, but the estimates of Ramanujan's stature in mathematics certainly have been growing over the years. There is no doubt about that.
- Hardy refers to Ramanujan as one of the few romantic figures in the history of mathematics and also, kin another place, as the one great romance of his life. Other romantic figures one may think of are, for instance, Galois or Abel. They died of course even younger but they did, although they had their difficulties, come from a somewhat more fortunate environment. One may say that none of them seems to have been quite so exclusively committed to mathematics, with no other visible interest in their life as Ramanujan was, and in this, I think, he truly

[8] Alte Selberg and Paul Erdös each obtained independently elementary proofs of the Prime Number Theorem (PNT). The prime number theorem (PNT) describes the asymptotic distribution of the prime numbers among the positive integers. Their asymptotic distribution is given by $\pi(N) \sim N/\log(N)$, where $\pi(N)$ is the prime-counting function and $\log(N)$ is the natural logarithm of N.

[9] Published in the magazine Resonance, in December 1996. Reproduced from the *Alte Selberg: Collected Papers*, Vol. I, Springer-Verlag (1989).

stands completely alone among the mathematicians. At least from what I have read about him, there was no other real interest for him than mathematics.

- The most important lesson that one could draw from Ramanujan's story about the educational system is that allowances should be made for the unusual and perhaps lopsidedly gifted child with very strong interests in one direction, at all stages of the educational system.
- And besides the schools, I think, it also important for the development of possible future Ramanujans, that public libraries should stock a reasonable amount of mathematical books that could inspire and really interest someone who wants to find something outside of his school curriculum. This is one important thing that can be done in the future for making it easier for any future Ramanujan.

The Proceedings of the International Conference were published, in the year 1988, as *Ramanujan Revisited*.[10]

9.3 ICM 2010

Historically, Felix Klein (1849–1925) and Georg Cantor (1845–1918) are accredited with the idea of an International Congress of Mathematicians (ICM) in the 1890s and the first International Congress of Mathematicians was held in Zurich, Switzerland, from Aug. 1897, 121 years ago. It was attended by more than 200 Mathematicians from 16 countries, including 12 from Russia, 7 from the United States (including four women). It is at the 1900 ICM, in Paris, France, that David Hilbert (1862–1943) announced his famous list of 23 unsolved mathematical Problems.[11] It is the largest conference for the topic of mathematics. Organized once every four years, the ICM is hosted by the International Mathematical Union. The Fields Medals, the Nevanlinna Prize, the Gauss Prize and the Chern Medal are awarded during the ICM's opening ceremony.

Invited talks are on current topics of general interest. Delivering an invited talk at the ICM is considered equivalent to the induction to a Hall of Fame. The Proceedings memorialize the academic papers based on invited talks.

For the first time, in India, and only the third time in Asia—ICM 1990 was at Kyoto, Japan, and ICM 2002 was in Beijing, China—the ICM was held at the Hyderabad International Convention Centre (HICC),[12] during the period 19th to 27th August 2010.

In 2012, the 125th Birth Anniversary of Ramanujan was celebrated. The Prime Minister of India, Dr. Manmohan Singh declared December 22 from henceforth

[10] *Ramanujan Revisited*, edited by George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin, Academic Press Inc. (1988).

[11] For the Nature and influence of the problems and for a Table of the 23 Hilbert's problems, we refer the reader to the free online Wikipedia encyclopaedia.

[12] A web page for the ICM 2010 is at http://www.icm2010.org.in.

would be called the National Mathematics Day, and a Commemorative postage stamp was issued on the 125th Birth Anniversary on Srinivasa Ramanujan on 22 December 2012 by India Post.

9.4 The Author's Contributions

The following is my work in the field of Science Popularization:

- Creation of the "πie Pavilion" and the Ramanujan Gallery for the ISCE 1999 and ICM 2010 Hall of Fame; now permanently housed in the Periyar Science and Technology Center, at Kotturpuram, since the year 2000, for which a grant of 70 Lacs has been released by the Government for its enhancement on the lines suggested by the author[13]
- Bringing out three books on the Life and Work of Ramanujan:
  - *Srinivasa Ramanujan: A Mathematical Genius*, Affiliated East-West (India) Ltd., (1999) and its Revised and Enlarged version (2004), with a Foreword by the President of India, Dr. A.P.J. Abdul Kalam;
  - *Mathematical Genius Ramanujan*, a Translation of "Ganithamedhai Ramanujan" by Ragami (T.V. Rangaswamy), K. Srinivasa Rao, with a Foreword by Governor Sardar Surjit Singh Barnala Allied Publishers Private Limited (2008).
  - *A mathematical genius: Srinivasa Ramanujan*, Srinivasa Ramanujan Academy of Maths Talent, Chennai (2012), with a Foreword by K.V. Rangaswami, Advisor to the Chairman, Larsen & Tubro Ltd. (2012), intended for school students.

For over two decades, the author has been associated with the activities of the Srinivasa Ramanujan Academy of Science Talent (SRAMT), Chennai, created by Professor P. Jagadeesan, M.A. (Hist.), M.A. (English), a Former Principal of the Kalakshetra School, and later with the P.S.G. Arts College, Coimbatore, as a Professor of English.

The origin of the Academy is due to a request made to Jagadeesan, at the time of the Birth Centenary of Ramanujan, by Smt. Janaki Ramanujan and the good wishes of distinguished Politician Sri C. Subramanian and the Founder Director of the Institute of Mathematical Sciences, Prof. Alladi Ramakrishnan. Jagadeesan, the

[13]The author received the following awards: The Tamilnadu Academy of Sciences, Mathematical Scientist Award, 2000 of the Tamilnadu State Council, Government of Chennai, Tamilnadu; *Science Popularization Award, 2000*, of the TN State Council, Govt. of Tamilnadu; "Lifetime Achievement Award the Srinivasa Ramanujan Academy of Maths Talent—the citation read "for his significant contributions to Research in Theoretical Nuclear Physics, Quantum Theory of Angular Momentum and Group theoretical methods, and for creating two comprehensive CD ROMs on the Life and Work of Srinivasa Ramanujan; Ramanujan Memorial Award—2013, by the Ramanujan Mathematics Academy and Mathematics Library, Ramachandrapuram, Andhra Pradesh.

Founder Secretary of the SRAMT, since its inception, was obtaining some financial support from the Indian Bank and Mr. N. Ravi, of the Hindu newspaper.

Jagadeesan inducted the author to be its Director (Hon.) in 2010. At present, the SRAMT is in the process of being revamped into a Registered Society to continue its functioning as a non-profit organization to encourage interest in the Secondary and Higher Secondary School students to take to Mathematics and also to propagate the life and work of Srinivasa Ramanujan at these levels. The Founder Secretary and the Director (Hon.) have been together to several acclaimed schools, as members of ad hoc teams of experts to select students/teachers for the annual Best Student, Best Teacher and Best School, Ramanujan Awards of the SRAMT.

Unfortunately, the Founder (Secretary) of the SRAMT is now no more with us. As Prof. P. Jagadeesan (double M.A. in English) who started the SRAMT in Birth Centenary Year 1987, when Janakiammal asked him to "do something for" her husband, breathed his last on April 3, 2019. The author constituted a committee and has started revamping the SRAMT.

9.5 A Plea for a National Science Museum

The stature and achievements of Srinivasa Ramanujan, hailed as one of the greatest mathematicians of the twentieth century, should be an eternal source of inspiration for students of science, engineering and technology. This can best be achieved through the setting up of a National Science Museum. Like the Sistine Chapel of the Vatican in Rome, pointers in such a museum should be to the original Notebooks of Ramanujan in its sanctum sanctorum!

Indian Institute of Science Education and Research (IISER) here has observed 98th Remembrance Day of Ramanujan at its campus on April 26, 2018. On that occasion, a "Ramanujan Mathematics Museum" was inaugurated consisting of 32 standee posters, the contents for which was provided by the author.

The posters give a glimpse into the life and work of the mathematical genius. Ramanujan was a precocious student and recognized in India with the first research scholarship of the University of Madras in 1913.

Ramanujan left India, in 1914, at the invitation of Professor G.H. Hardy as a failed student and returned five years later as a celebrity with the BA degree by research of the Cambridge University and as the first Indian mathematician to become a Fellow of the Royal Society, London, in 1918.

A portrait painted by the Professor in Physics and Dean, graduate studies at IISER, Pune, Dr Gouri Ambika was placed in the museum and she was present on the occasion.

Distinguished applied mathematician and former president of the Indian Mathematical Society Dr P.V. Arunachalam has delivered Srinivasa Ramanujan Remembrance Day lecture while another special lecture was delivered by former professor at Institute of Mathematical Sciences, Dr K. Srinivasa Rao. IISER, Tirupati Director Prof. K.N. Ganesh presided over the proceedings.

Tirupathi Tirumala Devasthanam, Sri Venkateswara Museum Director, Prof. P.V. Ranganayakulu, All India Radio, Tirupati Deputy Director Dr. Nagasuri Venugopal also addressed the gathering.

Setting up a Science Museum, and that too for mathematics, where emphasis is not on the Technological Applications but on the underlying scientific facts and their discovery requires ingenuity, skill and planning, by the best brains in the country interested in popularization of science, to make it a part of a way of life. One can take the examples of museums already set up in the world like the Deutches Museum in Münich, Germany; the Science Museum in London, England; the Smithsonian Museum in Washington D.C.; and the π Room in Paris, France. And, in India, Museums such as the Visvesvaraya Museum in Bangalore, and museums in Calcutta and Pilani, to name a few.

If and when our Government of India establishes such a National Science Museum, to popularize and perpetuate the contributions of our scientists and our scientists and politicians take pride to preserve our heritage, then our government can approach the British Government for the original papers of the Natural Genius Ramanujan.

In such a National Museum, the works of our great scientists:

- Srinivasa Ramanujan, FRS (Dec. 22, 1887–April 20, 1920, known as the greatest self-taught mathematical genius of the twentieth century
- Bharat Ratna Sir C.V. Raman, FRS (Nov. 7, 1988–November 21, 1970), who won the Nobel Prize in Physics for his discovery of the Raman effect, in 1930
- Bharat Ratna M. Visvesvaraya (Sep.15, 1860–Apr. 12, 1962, aged 101), who played a key role in the building of the Krishnarajasagar Dam in Mysore, introduced the steel door and the Brindavan Gardens
- Yellapragada Subba Rao (Jan. 12, 1895–Aug. 8, 1948), Bio-chemist credited with the discovery of a number of life-saving antibiotics and an anti-cancer drug manufactured by him many decades ago continues to be used globally for cancer treatment
- Satyendra Nath Bose (Jan. 1, 1894–Feb. 4, 1974), famous for Bose-Einstein statistics, and an atomic particle called the Boson named after him
- Homi J. Bhabha (Oct. 30, 1909–Jan. 24, 1966), who set up the TIFR and the BARC, and the first Chairman of the Atomic Energy Commission of India
- S. Chandrasekhar, FRS (Oct. 19, 1910–Aug. 21, 1995), won the Nobel Prize in 1983 (with William A. Fowler), for his contributions in the fields of Astrophysics on the physical processes important to the structure and evolution of stars and the Chandrasekar limit
- G.N. Ramachandran, FRS (Oct. 8, 1922–Apr. 7, 2001), Crystallographer and Biophysicist, created Ramachandran plot for understanding peptide structure, nominated for the Nobel Prize
- M.S. Swaminathan (Born Aug. 7, 1925) is a geneticist and international administrator renowned as the "Father of Indian Green Revolution" for his success in introducing and developing high-yielding varieties of wheat in India

- Bharat Ratna A.P.J. Abdul Kalam (Oct. 15, 1931–July 27, 2015), known as the missile man of India, noted for his contribution in the field of vehicle technology and ballistic missiles and who was the President of India
- Salim Moizuddin Abdul Ali (Nov. 12, 1896–June 20, 1987) an Indian ornithologist and naturalist referred to as the "Bird-man of India";
- Harish-Chandra, FRS (Oct. 1923–Oct. 16, 1983), an Indian–American mathematical physicist renowned for his fundamental work in representation theory, especially harmonic analysis on semi-simple Lie groups
- Har Gobind Khorana (Jan. 9, 1922–Nov. 9, 2011), Indian–American Biochemist who shared the 1968 Nobel Prize for Physiology or Medicine (with Marshall W. Nirenberg and Robert W. Holley) for research that showed the order of nucleotides in nucleic acids, which carry the genetic code of the cell and control the cell's synthesis of the proteins
- Sir C.V. Raman (Nov. 7, 1889–Nov. 21, 1970), E.C.G. Sudarshan (Sep. 16, 1931–May 14, 2018), M.N. Saha (Oct. 6, 1893–Feb. 16, 1956), C.S. Seshadri, F.R.S. (Feb. 29, 1932–July 17, 2020), Alladi Ramakrishnan (Aug. 9, 1923–June 7, 2008), and the living centurian legend Dr. C.R. Rao, F.R.S. (D.O.B: Sep. 10, 1920)
- The invaluable original Notebooks of Ramanujan now in the Library of the University of Madras
- The passport of Ramanujan, issued in 1919, in England, now with the family of the foster son (late W. Narayanan) of Smt. Janakiammal;
- The large sized slate used by Ramanujan now with the son of late Mr. S. Narayana Iyer
- The brass vessel and a coffee grinder used by Janakiammal during the last year (March 1919–April 1920) of the life of Ramanujan, now with the family of the foster son of Smt. Janakiammal
- Some original letters of Ramanujan with the Ramanujan Museum in Royapuram—some original letters written by Ramanujan to his friend E. Vinayaka Rao, were handed over by the daughter of Vinayaka Rao, at the instance of the author to the authorities of this Museum
- Other interesting manuscripts connected with the life and times of Ramanujan in the National Archives and the Tamil Nadu Archives

can be preserved for posterity, as these ought to be considered as a part of the National heritage, as they are memorabilia connected with the unique "Swayambhu"[14] Srinivasa Ramanujan.

[14]We owe this most apt nomenclature to Robert Kanigel, who used the same in his book, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Charles Scribner's Sons, New York (1991).

Appendix A
Research Publications of Ramanujan

Papers Written by S. Ramanujan

1. Some Properties of Bernoulli's numbers
   Journal of the Indian Mathematical Society, 3 (1911) 219–234.
2. On Question 330 of Prof. Sanjana
   Journal of the Indian Mathematical Society, 4 (1912) 59–61.
3. Note on a set of simultaneous equations
   Journal of the Indian Mathematical Society, 4 (1912) 94–96.
4. Irregular numbers
   Journal of the Indian Mathematical Society, 5 (1913) 105–106.
5. Squaring the circle
   Journal of the Indian Mathematical Society, 5 (1913) 132.
6. Modular equations and approximations to π
   Quarterly Journal of Mathematics, 45 (1914) 350–372.
7. On the integral $\int_0^x \frac{\tan^{-1}t}{t}\,dt$
   Journal of the Indian Mathematical Society, 7 (1915) 93–96.
8. On the number of divisors of a number
   Journal of the Indian Mathematical Society, 7 (1915) 131–133.
9. On the sum of the square roots of the first n natural numbers
   Journal of the Indian Mathematical Society, 7 (1915) 173–175.
10. On the product $\prod_{n=0}^{n=\infty}\left[1+\left(\frac{x}{a+nd}\right)^3\right]$
    Journal of the Indian Mathematical Society, 7 (1915) 209–211.
11. Some definite integrals
    Messenger of Mathematics, 44 (1915) 10–18.
12. Some definite integrals connected with Gauss's sums
    Messenger of Mathematics, 44 (1915) 75–85.

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13. Summation of certain series
Messenger of Mathematics, 44 (1915) 157–160.
14. New expressions for Riemann's functions $\boldsymbol{\xi}(\mathbf{s})$ and $\boldsymbol{\Xi}(\mathbf{t})$
Quarterly Journal of Mathematics, 46 (1915) 253–260.
15. Highly composite numbers
Proceedings of the London Mathematical Society, 2, 14 (1915) 347–409.
16. On certain infinite series
Messenger of Mathematics, 45 (1916) 11–15.
17. Some formulae in the analytic theory of numbers
Messenger of Mathematics, 45 (1916) 81–84.
18. On certain arithmetical functions
Transactions of the Cambridge Philosophical Society, 22, No.9 (1916) 159–184.
19. A series for Euler's constant γ
Messenger of Mathematics, 46 (1917) 73–80.
20. On the expression of a number in the form $\mathbf{ax^2 + by^2 + cz^2 + du^2}$
Proceedings of the Cambridge Philosophical Society, 19 (1917) 11–21.
21. On certain trigonometrical sums and their applications in the theory of numbers
Transactions of the Cambridge Philosophical Society, 22, No. 13 (1918) 259–276.
22. Some definite integrals
Proceedings of the London Mathematical Society, 2, 17(1918) Records for 17 January 1918.
23. Some definite integrals
Journal of the Indian Mathematical Society, 11, (1919) 81–87.
24. A proof of Bertrand's postulate
Journal of the Indian Mathematical Society, 11 (1919) 181–182.
25. Some properties of $p(n)$, the number of partitions of n
Proceedings of the Cambridge Philosophical Society, 19 (1919) 207–210.
26. Proof of certain identities in combinatory analysis
Proceedings of the Cambridge Philosophical Society, 19, (1919) 214–216.
27. A class of definite integrals
Quarterly Journal of Mathematics, 48 (1920) 294–310.
28. Congruence properties of partitions
Proceedings of the London Mathematical Society, 2, 18 (1920) Records for 13 March 1919.
29. Algebraic relations between certain infinite products
Proceedings of the London Mathematical Society, 2, 18 (1920) Records for 13 March 1919.
30. Congruence properties of partitions
Mathematische Zeitschrift, 9 (1921) 147–153.

Papers Written by S. Ramanujan with G.H. Hardy

1. Une formulae asymptotique pour le nombre des partitions de n *Comptes Rendus*, 2 January 1917.
2. Proof that almost all numbers n are composed of about $\log \log n$ factors *Proceedings of the London Mathematical Society*, 2, 16 (1917) Records for 14 December 1916.
3. Asymptotic formulae in combinatory analysis *Proceedings of the London Mathematical Society*, 2, 16 (1917) Records for 1 March 1917.
4. Asymptotic formulae for the distribution of integers of various types *Proceedings of the London Mathematical Society*, 2, 18 (1920) 112–132.
5. The normal number of prime factors of a number n *Quarterly Journal of Mathematics*, 48 (1917) 76–92.
6. Asymptotic formulae in combinatory analysis *Proceedings of the London Mathematical Society*, 2, 17 (1918) 75–115.
7. On the coefficients in the expansions of certain modular functions *Proceedings of the Royal Society*, A, 95 (1918) 144–155.

These 37 research publications of Ramanujan are reprinted in the *Collected Papers of Srinivasa Ramanujan*, edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson and published first by the Cambridge University Press (1927) and later by Chelsea (1962). The questions posed and the answers provided by Ramanujan, 59 in all, in the Journal of the Indian Mathematical Society follow the research papers in the *Collected Papers*. Besides the Preface, the editors include two biographical *Notices*, one by P.V. Seshu Aiyar and R. Ramachandra Rao and another by G.H. Hardy, at the beginning of the *Collected Papers*. Notes on the research publications and some extracts from Ramanujan's letters to G.H. Hardy are in two Appendices, at the end of the *Collected Papers*.

Appendix B
Wren Library Card Catalogue and Papers of Ramanujan

With the help of Professor Robert A. Rankin, the Wren Library has catalogued the papers of Ramanujan. These have the reference addresses as Add.Ms.a.94, with a superscript which refers to the specific document, and as Add.Ms.b.100–107C.

The papers that are available with reference Add.Ms.a.94^{1-18} are contained in a large sized (A3-size) 2 inches high cardboard box. The reference *Add.Ms.b. 100–107C* refers to the five bound volumes (foolscap size), and, in addition to these, the papers and letters are preserved in two large (A3) sized thick cardboard boxes.

In this Appendix are given the details of the Card Catalogue on Ramanujan, as well as the details of the Ramanujan MSS as maintained in the Archives of the Wren Library of Trinity College, Cambridge University. In fact, a part of the Appendix Card Catalogue is what is contained in Supplement II of Venkatachaliengar's article, as "A list of all the Ramanujan - material at the Trinity College Cambridge given to Professor C.T. Rajagopal by Rankin".

The Card Catalogue in the Wren Library of Trinity College:

1. AIYAR A(P Venkatesvara Seshu)
   see AYYAR (P Venkatesvara Seshu)
2. ANDERSON (Sir Hugh Kerr)
   see HARDY (G.H.)
3. AYYAR (P Venkatesavara Seshu)
   Letter to G.H. Hardy, 23 March 1928
   Add.Ms.a.$94^{1(31)}$
4. AYYAR (P Venkatesavara Seshu)
   Letter to B.M. Wilson, 8 July 1925
   Add.Ms.a.$94^{12(49)}$
5. AYYAR (P Venkatesavara Seshu)
   Letter to B.M. Wilson, 22 April 1926
   Add.Ms.a.$94^{12(58)}$

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6. BERWICK (William Edward Hodgson)
   Letter [to G.N. Watson ?], 24 Oct. 1916
   Add.MS.a.94[5]
7. CAMBRIDGE UNIVERSITY PRESS
   see HARDY (G.H.)
   RAMANUJAN (S.)
8. CHANDRASEKHAR (Subrahmanyan)
   Letter to G.H. Hardy, 4 Aug. 1937 [and 28 Dec. 1937] Add.MS.a.94[8]
9. CLARK (Edward Mellish)
   Letters to G.H. Hardy, 7–25 May 1920
   Add.MS.a.94$^{(12-24)}$
10. DEWSBURY (Francis)
    Letters to G.H. Hardy, 5 March 1918–15 Jan. 1929
    Add.MS.a.94$^{(13-17)}$
11. DEWSBURY (Francis)
    Letter to G.H. Hardy, 13 July 1922
    Add.MS.a.94$^{12(40)}$
12. DEWSBURY (Francis)
    Letter to G.H. Hardy, 2 Aug. 1923
    Add.MS.a.94$^{12(25)}$
13. DEWSBURY (Francis)
    Letter to G.H. Hardy, 6 Aug.–13 Sept. 1923
    Add.MS.a.94$^{(27–29)}$
14. FOWLER (Sir Ralph Howard)
    Letter to G.H. Hardy, 9 June [1922]
    Add.Ms.a.94$^{12(34)}$
15. HARDY (Godfrey Harold) "G.H. Hardy in account with the Syndics of the Cambridge University press: Collected Papers of S. Ramanujan. Statement of Account from 1 January 1928 to 31 December 1928", April 1929.
    Add.Ms.a.94$^{12(77)}$
16. HARDY (Godfrey Harold)
    Letter to S. Ramanujan, 6 Feb. 1918, on summing "partition series" for $\theta_3^r(\tau)(r\ odd)$
    Add.MS.a.94[16]
17. HARDY (Godfrey Harold)
    Letter to G.N. Watson, [April 1929]
    Add.MS.a.94$^{12(76)}$
18. HARDY (Godfrey Harold)
    Letter to G.N. Watson, [n.d.[1]]
    Add.MS.a.94[3]

[1]The abbreviation n.d. stands for not dated.

19. HARDY (Godfrey Harold)
Letter to B.M. Wilson, [5 June 1925]
Add.Ms.a.94$^{12(32)}$
20. HARDY (Godfrey Harold)
Letter to B.M. Wilson, [10 June 1925]
Add.MS.a.94$^{12(41-43)}$
21. HARDY (Godfrey Harold)
Envelope addressed to B.M. Wilson, 14 Oct. 1925
Add.MS.a.94$^{12(56)}$
22. HARDY (Godfrey Harold)
Letter to B.M. Wilson, 15 Oct. [1925]
Add.MS.a.94$^{12(52)}$
23. HARDY (Godfrey Harold)
Letter to B.M. Wilson, [Jan./Feb. 1927]
Add.MS.a.94$^{12(62)}$
24. HARDY (Godfrey Harold)
Letters to B.M. Wilson, [n.d.[2]]
Add.MS.a.94$^{12(5-8)}$
25. HARDY (Godfrey Harold)
Letters to B.M. Wilson, [n.d.]
Add.MS.a.94$^{12(20-21)}$
26. RAMANUJAN (Srinivasa)
Letter on mock theta functions. [This appears to be the letter of 12 January 1920, partly reproduced on pp. 354–355 of the Collected Papers.]
27. HARDY (Godfrey Harold)
"Memorandum of agreement made this 12 August 1925 between Professor G.H. Hardy, New College, Oxford ... and the Syndics of the University Press Cambridge ..." [concerning S. Ramanujan's Collected Papers], signed by G.H. Hardy and Sir H.K. Anderson.
Add.Ms.a.94$^{12(53)}$
28. HARDY (Godfrey Harold)
"Mr. S. Ramanujan's Mathematical Work in England: Report by Mr. G.H. Hardy to the University of Madras" (pr.), Cambridge 1916.
29. HARDY (Godfrey Harold)
Work on hypergeometric series and summation of series of powers of binomial coefficients, [n.d.]
Add.Ms.a.94$^{(17)}$

[2] n.d. stands for not dated document.

30. HARDY (Godfrey Harold)

see	AYYAR (P.V.S.)	KINGSFORD (R.J.L)
	CHANDRASEKHAR (S.)	KOMALATAMMAL
	CLARK (E.M.)	LEHMER (D.H.)
	DEWSBURY (F.)	LITTLEWOOD (J.E.)
	FOWLER (Sir)	MENON(K.R.)
	HENSMAN (J.J.)	NARASIMHAN (S.L.)
	JEANS (Sir)	RAMALINGAM(A.S.)

31. HARDY (Godfrey Harold)

see RAO (K.A.)
RAO (R.R.)
ROBERTS (Sir S.C.)
SPRING (Sir J.E.)
WALLER (A.R.)

32. HEILBRONN (Hans Arnold)
see WATSON (G.N.)
33. HENSMAN (John J)
Letter to G.H. Hardy, 29 April 1920
Add.Ms.a.$94^{1(18)}$
34. JEANS (Sir James Hopwood)
Letter to G.H. Hardy, 15 June 1922
Add.Ms.a.$94^{12(33)}$
35. KINGSFORD (Reginald John Lethbridge)
Letter to G.H. Hardy, 21 August 1925
Add.Ms.a.$94^{12(51)}$
36. KINGSFORD (Reginald John Lethbridge)
Letter to G.H. Hardy, 23 Aug. 1929
Add.Ms.a.$94^{12(78)}$
37. KOMALATAMMAL
Letter to G.H. Hardy, 25 Aug. 1927
Add.Ms.a.$94^{12(78)}$
38. LEHMER (Derrick Henry)
Letter to G.H. Hardy, 2 Nov. 1937
Add.Ms.a.$94^{6(1-2)}$
39. LITTLEWOOD (John Edensor)
Letter to G.H. Hardy, [1913 ?]
Add.Ms.a.$94^{1(2-5)}$
40. LITTLEWOOD (John Edensor)
Letter to G.N. Watson, [June 1948]
Add.Ms.a.$94^{1(1)}$

41. LITTLEWOOD (John Edensor)
Card to B.M. Wilson, 4 March [1927]
Add.Ms.a.94[12(67)]
42. McLEAN (William)
Letter to B.M. Wilson, 13 Aug. 1925
Add.Ms.a.94[12(50)]
43. McLEAN (William)
Letter to B.M. Wilson, 13 Aug. 1925
Add.Ms.a.94[12(54)]
44. McLEAN (William)
Letter to B.M. Wilson, 31 March 1927
Add.Ms.a.94[12(68)]
45. MENON (K Ramunni)
Letter [copy] to G.H. Hardy, 28 July 1920 Add.Ms.a.94[1(25)]
46. NARASIMHAN (S Lakshmi)
Letter [to G.H. Hardy], 29 April 1920
Add.Ms.a.94[1(19)]
47. NEVILLE (Eric Harold)
Card to B.M. Wilson, 28 Dec. 1927
Add.Ms.a.94[12(70)]
48. NEVILLE (Eric Harold)
"The Late Srinivasa Ramanujan", letter pr. in Nature, No. 2673, 20 Jan. 1921, pp. 661–662.
Add.Ms.a.94[12(71)]
49. OPPENHEIM (Tan Sri Sir Alexander)
Letter to B.M. Wilson, 31 Dec. 1926
50. RAM (A.S.)
see RAMALINGAM (A.S.)
51. RAMALINGAM (A.S.)
Letter to G.H. Hardy, 23 June 1918
Add.Ms.a.94[1(7–9)]
52. RAMALINGAM (A.S.)
Letter to G.H. Hardy, 23 June 1918
Add.Ms.a.94[1(10–12)]
53. RAMALINGAM (A.S.)
see RAMANUJAN (S)
54. RAMANUJAN (Srinivasa)
Calculations "copied from the loose papers" by an unknown hand.
Add.Ms.b.104
55. RAMANUJAN (Srinivasa)
Correspondence of G.H. Hardy with Cambridge University Press and others about S. Ramanujan's Collected Papers, 3 Dec. 1920–23, Aug. 1929 and [n.d.]
Add.Ms.a.94[12]

56. RAMANUJAN (Srinivasa)
Displayed formulae for preparation as slides, [n.d.]
Add.Ms.a.94[9]
57. RAMANUJAN (Srinivasa)
Enlarged passport photograph, [1913 ?] [5 copies]
Add.Ms.a.94[7]
58. RAMANUJAN (Srinivasa)
Letters to G.H. Hardy, 28 June 1918 and [n.d.]
Add.Ms.a.94[2]
59. RAMANUJAN (Srinivasa)
Letter to A.S. Ramalingam, 19 June 1918
Add.Ms.a.94[1(16)]
60. RAMANUJAN (Srinivasa)
Letter [incomplete], [n.d.]
Add.Ms.a.94[6(3–8)]
61. RAMANUJAN (Srinivasa)
List of suggested changes in the biographical sketch of S. Ramanujan [for S. Ramanujan's Collected Papers], [1923]
Add.Ms.a.94[12(30)]
62. RAMANUJAN (Srinivasa)
"'Lost' notebook" on q-series and similar topics, [n.d.]
Add.Ms.a.94[15]
63. RAMANUJAN (Srinivasa)
"Minutes of the Congratulatory Meeting held in honour of Mr. S. Ramanujan and Mr. K. Ananda Rao" [copy], 22 March 1928
Add.Ms.a.94[1(29–30)]
64. RAMANUJAN (Srinivasa)
Miscellaneous MSS: highly composite numbers, singular moduli, Rogers-Ramanujan identities, Mersenne numbers, etc. with annotations by G.K. Stanley and G.H. Hardy, and a letter from G.H. Hardy to G.N. Watson, [1930].
Add.Ms.a.94[18]
65. RAMANUJAN (Srinivasa)
"Mr. Ramanujan's accounts" [copy],
28 Nov. 1917–23 Aug. 1919
Add.Ms.a.94[1(20)]
66. RAMANUJAN (Srinivasa)
"Notebook 2", copy by an unknown hand. 4 Vols.
Add.Ms.b.101–104
67. RAMANUJAN (Srinivasa)
"Notes on the biographical sketch by Mr. S. Ramanujan, F.R.S." [copy]
(for S. Ramanujan's Collected Papers), [n.d.]
Add.Ms.a.94[12(26)]

68. RAMANUJAN (Srinivasa)
Papers Add.Ms.a.94
Add.Ms.b.100–107C
69. RAMANUJAN (Srinivasa)
"Properties of $p(n)$ and $\tau(n)$". [This is the paper of which part is published in Collected Papers, No. 30 (pp. 232–238), under the title "Congruence Properties of Partitions"] [n.d.] Add.Ms.a.94[13]
70. RAMANUJAN (Srinivasa)
"Report on the proposed publication of the Collected Mathematical Works of S. Ramanujan" [copy] with annotations by B.M. Wilson [n.d.] Add.Ms.a.94[12(44–46)]
71. RAMANUJAN (Srinivasa)
Reminiscences of, by P. Adinarayana Chetty, copied from the Madras Mail, 17 Apr. 1920
Add.Ms.a.94[3(18)]
72. RAMANUJAN (Srinivasa)

see NEVILLE (E.H.)
RAMALINGAM (A.S.)
SATAGOPAN (T.A.)
STORMER (F.C.)
WATSON (G.N.)
WILSON (B.M.)

Add.Ms.a.94[3(18)]
73. RAO (R Ramachandra)
Letter [to G.H. Hardy], 3 Dec. 1920
Add.Ms.a.94[12(22)]
74. RAO (K Ananda)
Letter to G.H. Hardy, 26 May 1921
Add.Ms.a.94[12(23)]
75. RAO (K Ananda)
Letter to G.H. Hardy, 7 April 1936
Add.Ms.a.94[1(32)]
76. ROBERTS (Sir Sydney Castle)
Letters to G.H. Hardy, 30 Aug. [copy]–23 Sept. 1922
Add.Ms.a.94[12(37–39)]
77. ROBERTS (Sir Sydney Castle)
Letter to G.H. Hardy, 10 April 1923
Add.Ms.a.94[12(24)]
78. ROBERTS (Sir Sydney Castle)
Letter to G.H. Hardy, 6 June 1925
Add.Ms.a.94[12(35)]

79. ROBERTS (Sir Sydney Castle)
Letter to G.H. Hardy, 8 Oct. 1925
Add.Ms.a.94[12(55)]
80. ROBERTS (Sir Sydney Castle)
Letter to G.H. Hardy, 11 Nov. 1925
Add.Ms.a.94[12(57)]
81. ROBERTS (Sir Sydney Castle)
Letter to G.H. Hardy, 7 Jan. 1926 [sic; i.e. 1927]
Add.Ms.a.94[12(63)]
82. ROBERTS (Sir Sydney Castle)
Letter [copy] to G.H. Hardy, 7 Jan. 1927
Add.Ms.a.94[12(65)]
83. ROBERTS (Sir Sydney Castle)
Letter to B.M. Wilson, 12 June 1925
Add.Ms.a.94[12(47)]
84. SATAGOPAN (T.A.)
Copy of S. Ramanujan's "Notebook 1", [n.d.]
Add.Ms.b.100
85. SCOTT (Francis Reginald Fairfax)
Letter to B.M. Wilson, 22 Dec. 1926
Add.Ms.a.94[12(59)]
86. SCOTT (Francis Reginald Fairfax)
Letter to B.M. Wilson, 10 Jan. 1927
Add.Ms.a.94[12(61)]
87. SPRING (Sir Francis Joseph Edward)
Letter to G.H. Hardy, 18 Feb. 1921
Add.Ms.a.94[1(26)]
88. STØRMER (Carl)
see
89. STØRMER (Fredrik Carl Mulertz) "Srinivasa [sic] Ramanujan—et merkelig matemastisk geni", article pr. in Lördagsavisen, 6 Jan. 1934
Add.Ms.a.94[11]
90. TOWLE (F.A.)
Letter to B.M. Wilson, 11 Feb. 1927
Add.Ms.a.94[12(66)]
91. TOWLE (F.A.)
Letter to B.M. Wilson, 1 July 1925
Add.Ms.a.94[12(48)]
92. VIJAYARAGHAVAN (T.)
Letter to B.M. Wilson, 4 Jan. 1928
Add.Ms.a.94[12(72−75)]
93. WALLER (Alfred Rayney)
Letter to G.H. Hardy, 3 June 1922
Add.Ms.a.94[12(36)]

94. WATSON (George Neville)
Letter to H.A. Heilbronn, 19 March 1939
Add.Ms.a.94[4]
95. WATSON (George Neville)
Copy of S. Ramanujan's "Notebook 2", Chapters XII–XXI, with Watson's proofs of Ramanujan's formulae [incomplete], [n.d.]
Add.Ms.b.105–107A
96. WATSON (George Neville)
Copy of S. Ramanujan's three Quarterly Reports to the Board of Studies in Mathematics, 5 Aug. and 7 Nov. 1913 and 9 March 1914
Add.Ms.b.107A
97. WATSON (George Neville)
Notes on S. Ramanujan and his work, [n.d.]
Add.Ms.a.94[10]
98. WATSON (George Neville)

see BERWICK(W.E.H.) LITTLEWOOD (J.E.)
HARDY (G.H.) RAMANUJAN (S.)

99. WILSON (Bertram Martin)
Account of S. Ramanujan and his work, [n.d.]
Add.Ms.b.107*C*[1]
100. WILSON (Bertram Martin)
Comments on Appendix, pp. 225–273, [of S. Ramanujan's Collected Papers], [n.d.]
Add.Ms.a.94[12(3–4)]
101. WILSON (Bertram Martin)
Comments on pp. 1–16, [of S. Ramanujan's Collected Papers [n.d.]
Add.Ms.a.94[12(1–2)]
102. WILSON (Bertram Martin)
General account of S. Ramanujan's life and work, [n.d.]
Add.Ms.b.107*C*[11]
103. WILSON (Bertram Martin)
Investigations on Chapters II–VII of S. Ramanujan's "Notebook 2", [n.d.]
Add.Ms.b.107B
104. WILSON (Bertram Martin)
Letter [copy] (to the guarantors of S. Ramanujan's Collected Papers), 7 Feb. 1927
"Notebook 2", [n.d.]
Add.Ms.a.94[12(64)]
105. WILSON (Bertram Martin)
List of corrections to Chapters I–X of S. Ramanujan's "Notebook 2", with references to published work of other mathematicians, [n.d.]
Add.Ms.b.107*C*[2]

106. WILSON (Bertram Martin)
Notes and proofs of results in Chapters II–XIII of
S. Ramanujan's "Notebook 2", [n.d.]
Add.Ms.b.107$C^{3-10,12}$

107. WILSON (Bertram Martin)
Notes on the biographical sketch of S. Ramanujan
(for Ramanujan's Collected Papers), [1923]
Add.Ms.a.94$^{12(9-19)}$

108. WILSON (Bertram Martin)
"Solutions of questions proposed by other authors" [with reference to S. Ramanujan's Collected Papers], [n.d.]
Add.Ms.a.94$^{12(9-19)}$

109. WILSON (Bertram Martin)

see AYYAR (P.V.S.)
HARDY (G.H.)
LITTLEWOOD (J.E.)
McLEAN (W.)
NEVILLE (E.H.)
OPPENHEIM (Tan Sri Sir A.)
ROBERTS (Sir S.C.)
SCOTT (F.R.F.)
TOWLSE (F.A.)
VIJAYARAGHAVAN (T.)

110. RAMANUJAN (Srinivasa)
see ANDREWS (G.E.)

111. RAMANUJAN (Srinivasa)
see BERNDT (B.C.)

112. RAMANUJAN (Srinivasa)
see HARDY (G.H.) and RAMANUJAN (S.)

113. RAMANUJAN (Srinivasa)
see LITTLEWOOD (J.E.)

114. RAMANUJAN (Srinivasa)
see Newspaper cuttings

115. RAMANUJAN (Srinivasa)

see Newspaper cuttings: Hardy (G.H.)
Neville (E.H.)
Watson (G.N.)
Littlewood (J.E)

116. RAMANUJAN (Srinivasa)
see RANKIN (R.A.)

117. RAMANUJAN (Srinivasa)
see RAMANATHAN (K.G.)

In the Wren Library of Trinity College, Cambridge, the manuscripts of Ramanujan are kept in three large (foolscap size) cardboard boxes. Besides these three

boxes, there are five large bound volumes. Given below are the details of the contents of these Ramanujan Papers accessible with
Add.Ms.a.94 and Add.Ms.b.100–107C: Add.Ms.a.94

1. (1) J.E. Littlewood to [G.N. Watson], [June 1948].
   (2–5) J.E. Littlewood to G.H. Hardy, [? 1913].
   (6) S. Ramanujan to A.S. Ramalingam, 19 June 1918.
   (7–9) A.S. Ramalingam to G.H. Hardy, 23 June 1918.
   (10–12) A.S. Ramalingam to S. Ramanujan, 23 June 1918.
   (13–17) Francis Dewsbury to G.H. Hardy, 1918–20 (5 letters).
   (18) John H. Hensman to G.H. Hardy, 29 April 1920.
   (19) S.L. Narasimhan to [G.H. Hardy], 29 April 1920.
   (20) "Mr. Ramanujan's Accounts" [copy], November 1917–August 1919.
   (21–24) E.M. Clark to G.H. Hardy, May 20 (4 letters).
   (25) K.R. Menon to G.H. Hardy, 28 July 1920 [copy].
   (26) Sir Francis Spring to G.H. Hardy, 18 February 1921.
   (27–28) Komalatammal to G.H. Hardy, 25 August 1927.
   (29–30) Minutes of the Congratulatory Meeting held in honour of Mr. S. Ramanujan and Mr. K. Ananda Rao, 22 March 1928. [copy] (31)P.V.S. Ayyar to G.H. Hardy, 23 March 1928. (32) K.A. Rao to G.H. Hardy, 7 April 1936.
2. S. Ramanujan to G.H. Hardy, 28 June 1918 and n.d. (6 letters of which 4 are from Matlock House and 2 from Fitzroy House).
3. G.H. Hardy to G.N. Watson, n.d. (2 letters).
4. G.N. Watson to H.A. Heilbronn, 19 march 1939.
5. W.E.H. Berwick to [G.N. Watson], 24 October 1916.
6. (1–2) D.H. Lehmer to G.H. Hardy, 2 November 1937.
   S. Ramanujan (incomplete, undated letter).
7. Enlarged passport photograph of S. Ramanujan (5 copies).
8. S. Chandrasekhar to G.H, Hardy, 4 August 1937 and 28 December 1937 [both regarding the Passport sizes photograph of Ramanujan].
9. S. Ramanujan—Displayed formulae for preparation as slides, [n.d.]
10. Notes on Ramanujan and his work by G.N. Watson, [n.d.]
11. Srinivasa [sic] Ramanujan—et merklig matematisk geni article printed in Lördagsavisen by F.C.M. Stormer, 6 January 1934 [Norwegian newspaper]
12. Correspondence of G.H. Hardy with Cambridge University Press and others about S. Ramanujan's Collected Papers, December 1920–August 1929.
13. S. Ramanujan "Properties of $p(n)$ and $\tau(n)$" [This is the paper which part is published in Collected Papers, No. 30 (pp. 232–238), under the title "Congruence Properties of Partitions"] [n.d.]
14. Letter on mock theta functions. [This appears to be the letter of January 1920, partly reproduced on pp. 354–355 of the Collected Papers].
15. S. Ramanujan: "Lost Notebook" on q-series and similar types, [n.d.]
16. G.H. Hardy to S. Ramanujan, 6 February 1918 on summing "partition Series" for $\theta_3^r(\tau)$ (r odd).

17. G.H. Hardy: Work on hypergeometric series and summation of series of powers of binomial coefficients, [n.d.]
18. S. Ramanujan: Miscellaneous MSS including highly composite numbers, singular moduli, Rogers-Ramanujan identities, Mersenne numbers, etc. With annotations by [Miss] G.K. Stanley and G.H. Hardy, and a letter from Hardy to G.N. Watson, [1930].

Add.Ms.b.100

T.A. Satagopan: Copy of S. Ramanujan's "Notebook 1" n.d.[3]

This is Ramanujan's Notebook 1, copied by T.A. Satagopan. The original was published in facsimile edition without a commentary by the Tata Institute of Fundamental Research, Bombay, in 1957, and in 1987, Narosa Publishing House has reprinted it. This Notebook has 350 pages and covers Chapters I to XVI of the original Notebook 1 of Ramanujan.

Add.Ms.b.101–104

T.A. Satagopan: Copy of S. Ramanujan's "Notebook 2" by an unknown hand, n.d.

This is the first of four notebooks into which Ramanujan's Notebook 2 was copied. As in the case of Notebook 1, the original was published in facsimile edition by T.I.F.R., in 1957 and subsequently reprinted by Narosa in 1987. Chapters I to X of the Notebook 2 of Ramanujan are copied in his notebook in 138 pages. Chapter X is continued in Add.Ms.b.102 , Add.Ms.b.103 , and Add.Ms.b.104 , which are, respectively, 138, 139 and 118 pages in length, and Chapter XXI of Ramanujan's Notebook 2 is copied at the end of Add.Ms.b.103 and it is contained in Add.Ms.b.104 , which also contains calculations copied from Ramanujan's "loose papers" by an unknown hand. In Add.Ms.b.102 can be found two letters of Watson to Wilson (the two Mathematicians who undertook the task of editing the original Notebooks of Ramanujan and divided the Notebook 2 into two parts but unfortunately, due to the tragic sudden death of Wilson, this work was never completed).

Add.Ms.b.105–107A

Copy of S. Ramanujan's "Notebook 2", Chapters XII - XXI, with G.N. Watson's proofs of Ramanujan's formulae [incomplete], [n.d.]

These are G.N. Watson's transcripts of Chapters XVI - XXI of the second of Ramanujan's Notebooks together with proofs of Ramanujan's formulae and gaps for proofs to be filled in later (as noted by Prof. Robert A. Rankin, in 1969). In Add.Ms.b.105 Chapters XII–XV are covered. In Add.Ms.106 Chapters XVI–XVIII and in Add.Ms.b.107 Chapters XIX and XX are covered by G.N. Watson.

Add.Ms.b.107A

Copy of Ramanujan's three Quarterly Reports to the Board of Studies in Mathematics, 5 August and 7 November 1913 and 9 March 1914. This manuscript contains 282 sheets which are B.M. Wilson's MSS notes on Ramanujan's Notebooks.

[3] n.d. stands for not dated document.

Add.Ms.b.107B

Investigations on Chapters II–VII of Ramanujan's "Notebook 2" by B.M. Wilson. [n.d.]

Add.Ms.b.107C

Account of S. Ramanujan and his work by B.M. Wilson [n.d.]. List of corrections to Chapters I–X of Ramanujan's "Notebook 2", with references to published work of other mathematicians, by B.M. Wilson, [n.d.]. Notes and proofs of results in Chapters II–XIII of Ramanujan's "Notebook 2" by B.M. Wilson, [n.d.]. General account of Ramanujan's life and work by B.M. Wilson, [n.d.].

Appendix C
Personal File of S. Ramanujan at the National Archives and Papers at the Tamilnadu Archives

This is the "Personal File of S. Ramanujan, F.R.S. (1888–1920)", handed over by the Madras Port Trust authorities to the National Archives of the Government of India at New Delhi in the 1950s. It is a foolscap size volume with 206 pages (133A, 141A, 159A, 23 and 167 are blank sheets). The original documents are de-acidified and laminated (i.e. pasted in between translucent tracing paper) by Mr. MastRam Sharma and dated 30 November 1964 and checked on 22 October 1965. The contents of the volume are not catalogued. The pages are numbered in pencil and the contents are as noted below:

1. p. 1. Letter from S. Ramanujan to the Chief Accountant of Port Trust Madras, dated 9th Feb. [?] (Application for Clerkship).
2. p. 3. Letter of recommendation from E.W. Middlemast, Acting Principal, dated 21-9-1911.
3. p. 5. Unsigned sanction letter (Class III, grade IV in accounts section), dated 9-2-12.
4. p. 7. Appointment order to S. Ramanujan [office copy of the Chief Accountant] (offering Rs. 25/- per month from 1-3-12).
5. pp. 9–11. Letter from C.L.T. Griffith to Sir Francis Spring, 12-11-12.
6. pp. 13–15. Letter from the Director of Public Instruction to Sir Francis Spring, 14-11-12.
7. pp. 17–20. Letter from the Accountant General to C.L.T. Griffith, 27-11-12.
8. pp. 21–22. Letter from C.L.T. Griffith to Sir Francis Spring, 28-11-12.
9. pp. 23–24. Faded and unreadable.
10. p.25. Memo of the Madras Port Trust to Mr. S. Narayana Aiyar, 1-12-[?]
11. p. 26. Faded and unreadable.

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12. pp. 27–29. Letter from M.J.M. Hill.
    It is this letter which contains the following: "One thing however is clear. Mr. Ramanujan has fallen into the pitfalls of the very difficult subject of Divergent series." And:

$$\text{“}1+2+3+\cdots+\infty=-\frac{1}{12}$$

$$1^2+2^2+3^2+\cdots+\infty^2=0$$

$$1^3+2^3+3^3+\cdots+\infty^3=\frac{1}{240}$$

    All these have infinity for their sum." These sums can be understood today in the context of analytic continuation of the Riemann ζ function $\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}$, for negative valued of s. These results find use in the field of String Theory.
13. pp. 30–31. Letter from M.J.M. Hill to Prof. C.L.T. Griffith, 3-12-1912.
14. pp. 32–34. An unsigned letter from the University of London to Mr. C.L.T. Griffith [from M.J.M. Hill ?], dated 7-12-1912.
15. pp. 35–37. Letter from C.L.T. Griffith to Sir Francis Spring, 5-1-13.
16. pp. 39–52. First letter from G.H. Hardy to S. Ramanujan, 8 Feb. 1913 [pp. 40, 42, 44, 46, 48, 50 and 52 are blank].
17. pp. 53–55. Further notes suggested by Mr. J.E. Littlewood [8-2-13 ?].
18. p. 57. Typewritten "chains of acting arrangements", 11-2-13.
19. p. 59. Letter from Mr. Hanumantha Rau to Mr. Narayana Aiyar, 13-3-1913 [calling a meeting of the Board of Studies in Mathematics to discuss "What we can do for S. Ramanujan"].
20. p. 61. Regarding Dr. Gilbert Walker's visit, [31-3-17 ?].
21. Typewritten from S. Ramanujan to the Registrar of the University of Madras, 12-4-1913 [accepting the Rs. 75/- per month scholarship].
22. Letter of Chief Accounts Commissioner (CAC), 11-4-13 [recommending 2 years leave on loss of pay for S. Ramanujan from 1st May 1913].
23. pp. 67–68. Slip Acknowledging letter about scholarship to S. Ramanujan—conveying the same to Mr. Mallet of University Office—Arthur Davies' post card, 7-6-13.
24. p. 69. Mr. S. Narayana Aiyar's note, 31-X-13.
25. pp. 71–73. CAC S. Narayana Aiyar's note 12-3-[1913 ?]
26. pp. 75–78. 4 typewritten (notebook size) pages from Mr. Narayana Aiyer [?] to Mr. Cotterell, 5 Feb. [1914?]
27. p. 79. Letter from the Private Secretary to His Excellency [?] to Sir Francis Spring, 5 Feb. 1914.
28. p. 81. Extract [typewritten] from the "Madras Mail", 13 May 1914.
29. "Notes" of S, Narayana Aiyar, CAC, Madras Port Trust, 15-4-15.
30. Letter to the Registrar from Sir Francis Spring [?, hand written, on his plain paper].
31. p. 87. Notes of S. Narayana Aiyar, 18-12-15.

32. p. 89. Note of S. Narayana Aiyar [to Sir Francis Spring ?], 15-XII-191?.
33. p. 91. Typed copy of the above, 15-XII-191?
34. p. 93. Letter [ref. no. 7923] from Francis Dewsbury to Sir Francis Spring, 16-12-1915.
35. p. 95. Ref. No. 7923 forwarded to the University of Madras by S. Narayana Aiyar, 20-12-15.
36. p. 97. Extract from a letter from S. Ramanujan dated 11 Nov. 1915.
37. p. 99. Letter from Francis Dewsbury to Sir Francis Spring, 17 Jan. 1916.
38. p. 101, Note of S. Narayana Aiyar suggesting extension of leave on loss of pay, 18-X-16.
39. p. 103. Record of a telephone message asking for Ramanujan's date of birth by the Registrar of the University of Madras, 14-3-17 and S. Narayana Aiyar's reply note dated 14-5-17.
40. p. 105. Letter from the Registrar of the University of Madras, regarding Ramanujan's father (whether he was alive and whether he was a District Munsiff), 14-5-17.
41. pp. 109–112. Letter from S. Ramanujan [to Narayana Aiyar ?], 11-11-15.
42. p. 113/1 and 113/2. Letter from S. Ramanujan [to Narayana Iyer ?], 11-11-1915.
43. p. 115. Letter from Nellore, 6 Dec. 1915.
44. p. 121. Note from Sir Francis Spring and reply of S. Narayana Aiyer, 12-3-18.
45. p. 123. Letter from P.V. Seshu AIyer to S. Narayana Aiyar, 17-3-18.
46. p. 125. S. Narayana Aiyar's Notes regarding the FRS felicitation function at the Presidency College during the week, 18-3-18.
47. p. 127. from The Statesman Newspaper cutting from The Statesman, 10-12-1918.
48. p. 129. from The Statesman Newspaper cutting from The Madras Times, March 30, 1919.
49. p. 131. Newspaper cutting from The Madras Mail, with a note of CAC, March 29, 1919.
50. pp. 133–155. Typewritten letter (sd.) Sir Francis Spring, 20 Dec. 1915.
51. p. 157. Note of Mr. S. Narayana Aiyar, CAC, [n.d.]
52. p. 158. Article entitled: "A Famous Madras Mathematician", in the newspaper, The Madras Times, April 6, 1919.
53. p. 161. Letter to S. Ramanujan, in Kumbakonam, from the Chief Accountant, Madras Port Trust, 21-X-19.
54. p. 163. Note granting leave without pay for 5 years to S. Ramanujan and terminating his services on 30-4-1918, dated 21-X-19.
55. p. 165. Letter to the Chief Accountant, Madras Port Trust, reg. Provident Fund + subscription (amounting to Rs. 103—annas 12—paise 8) from S. Ramanujan, 12-1-20.
56. p. 167. Faded and unreadable.
57. From the Chief Accountant, Madras Port Trust, to S. Ramanujan at Chetput, 20-1-[20 ?].
58. From the Chief Accountant, Madras Port Trust, to S. Ramanujan at Chetput, [n.d.]

59. p. 173. Newspaper cutting from The Madras Mail of Monday evening announcing the death of S. Ramanujan, April 26, 1920.
60. pp. 175–176. Letter from Sir Francis Spring in London to S. Narayana Aiyar, 15-2-21.
61. Typewritten letter from S. Narayana Aiyar to Sir Francis Spring, 10-3-21.
62. p. 179. Letter from Ramachandra Rao [asking for details for article], [n.d.]
63. p. 181. Newspaper cutting from The Hindu dated Dec. 11, 1947, announcing the "Death of Hardy".
64. Newspaper cutting from The Hindu dated April 18, 1948—"Tribute to Genius of Ramanujan" and "Portrait Unveiled".
65. p. 187. Newspaper cuttings from The Mail dated June 13, 1948—"Late Mr. Srinivasa Ramanujan" and "Portrait unveiled at Port Trust Office".
66. p. 189. Letter of S. Narayana Aiyer, 19-6-48.
67. p. 191. Article: "Srinivasa Ramanujan: India's Great Mathematician", by S. Krishnan in "Wealth & Welfare", Madras, dated Aug. 1, 1947.
68. p. 195. Typewritten article on "Mr. S. Ramanujan, F.R.S.", by S. Narayana Aiyar, CAC.
69. pp. 197–201. Typewritten address on the occasion of the unveiling of the portrait of S. Ramanujan by the Chief Accountant [of Madras Port Trust ?]. item p. 203. Newspaper cutting from The Hindu dated Nov. 22, 1948—"Tribute to late S. Ramanujan".
70. p. 205. "American Tribute to late Ramanujan", Newspaper cutting dated Dec. 24, 1949—Dr. Marshal Stone's interview with the Globe.

Files in the Tamilnadu Archives

The following are the Madras Government (Educational) Files in the Tamilnadu Archives, Chennai—600008, pertaining to Srinivasa Ramanujan:

S.No.	G.O.No.	Date of G.O.	Regarding	Pages
1.	291	7-4-1913	F. Dewsbury to the Secretary	8
2.	182	12-2-1914	Scholarship sanction	7
3.	289	18-3-1914	C.W.E. Cotton to the Chief Secretary	6
4.	1308	20-11-1914	– do –	3
5.	1441	20-12-1914	F. Dewsbury to the Secretary	5
6.	345	13-3-1917	F. Dewsbury to the Secretary	6
7.	475	11-4-1918	F. Dewsbury to the Secretary	3
8.	408	2-4-1919	F. Dewsbury to the Secretary	6
9.	1072	5-9-1919	Komalatammal to the Secretary	3
10.	1072-73	17-9-1920	Pension matters to the family	56
11.	132	26-1-1921	Pension matters	22
12.	487	13-3-1921	Registrar to the Secretary	2
13.	1160	11-8-1921	E.S. Montagu's grant of allowance.	9

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The principal references are denoted by Roman numerals.

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IV. Ramanujan: Twelve Lectures on subjects suggested by his life and work first published by G.H. Hardy, Chelsea, New York, 1940 by AMS Chelsea Publishing, AMS, Providence, Rhode Island, 2000.

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XII. A Mathematician's Apology,
G.H. Hardy, (with a Foreword by C.P. Snow), Camb. Univ. Press,
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Notes

Biographical Often Ramanujan is compared with Leonhard Euler, Carl Friedrich Gauss or Carl Gustav Jacob (C.G.J.) Jacobi, for *natural genius*.

Euler, Leonhard (1707–1783): Switzerland's foremost scientist and considered as one of the three greatest mathematicians of modern times, along with Gauss and Riemann. His important contributions are to practically every area of pure and applied mathematics. He was the author of more than 200 articles and three treatises on analysis. Though he became totally blind, in 1771, he continued to contribute to optics, algebra and the theory of moon's motion. He made the concept of a function fundamental in analysis. He established the modern form and notation for the trigonometric and logarithmic functions. The equations: $e^{\pm ix} = \cos x \pm i \sin x$ are called as Euler's equations. Through his work, the symbols: e, π and i became common for all mathematicians and they are linked through the fundamental relation: $e^{i\pi} + 1 = 0$, considered as the most beautiful equation in the whole of mathematics, by E.T. Bell. It relates the five fundamental constants 0, the identity for addition; 1, the identity for multiplication; $i = \sqrt{-1}$, the imaginary unit, or indeterminate; π essential in all formulas which involve the conic sections; e essential for all decay or growth phenomena are ubiquitous and unique. The notations for trigonometric functions $\sin x$, $\cos x$, and the use of $f(x)$ for a general function and the symbol $\sum$ for summation are due to Euler. The constant

$$\gamma = \lim_{m->\infty} \left\{ 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{m} - \log m \right\} = 0.5772517 \cdots,$$

is known as Euler's constant, or as Mascheroni's constant.

Ğauss, Carl Friedrich (1777–1855): German mathematician, acknowledged to be one of the three leading mathematicians of all time, the others being Archimedes, and Sir Isaac Newton. His outstanding work includes the discovery of the method of least squares, the discovery of the non-Euclidean geometry and important contributions to the theory of numbers. He surprised his teacher at the age of ten

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K. Srinivasa Rao, *Srinivasa Ramanujan*,
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by summing 1–100 mentally by writing 1–50 in the first row from left to right and the numbers 51–100 from right to left in the second row directly below the first set of numbers. Noted that the sums of the elements in each of the 50 rows is 101 which when multiplied by 50 obtained the result $101 \times 50 = 5050$, from which one can deduce the summation formula for the sum of the first n integers is $S_n = n(n+1)/2$. The teacher expected the students in his class would be occupied with the summing of the first 100 integers for quite some time, but Gauss wrote down the number 5050 on his slate as soon as the problem was given and surprised the teacher. In 1796, Gauss discovered how to construct a regular polygon of 17 sides using only a compass and straight edge. In fact, he proved that a regular polygon with n sides is constructible if and only if n is a distinct prime number of the form

$$p_k = 2^{2^k} + 1.$$

Thus, when $k = 0, 1, 2, 3, \ldots$, the corresponding numbers p_k are $3, 5, 17, 257, \ldots$ are prime numbers and so regular polygons with these number of sides are constructible. This result happens to be the first item in a mathematical diary Gauss kept until 1814. Gauss's diary like the Notebooks of Srinivasa Ramanujan makes it possible to verify the priority of Gauss in many of the discoveries he did not publish. This little booklet of only 19 pages, considered as one of the most precious documents in the history of science, was unnoticed till 1898, and though not lost, was found among family papers with one of Gauss's grandsons. It contains 146 concise statements of the results of his researches, between 1796 to 1814. Among the unpublished discoveries of his are: non-Euclidean geometry and non-commutative algebras.

Gauss's doctoral thesis contained the first proof of the fundamental theorem of algebra, which states that every algebraic equation has a root of the form $a + i\,b$ where a and b are real numbers and i is $\sqrt{-1}$. In general, numbers of the form $a + i\,b$ are called as *complex numbers*, and, in particular, when a and b are integers, the numbers $a + i\,b$ are called as *Gaussian numbers*. Complex numbers are represented in a plane called the Gaussian plane.

Gauss published, in 1801, *Disquisitiones Arithmeticae*, his greatest treatise, in which Gauss created the modern rigorous approach to mathematics. It contains the new algebra of congruence. He introduced the symbols $\equiv$ for congruence and the notation

$$a \equiv b \ (mod\ m)\ ,$$

which has since become a standard in the theory of numbers. The *law of quadratic reciprocity* discovered by Gauss was called the *gem of arithmetic* by him: *if p and q are prime numbers, then either*

$$x^2 \equiv q \ (mod\ p) \text{ and } x^2 \equiv p \ (mod\ q)$$

are both solvable or both unsolvable, unless both p and q are of the form $4n+3$, in which case one is solvable and the other is not.

Gauss's *Arithmetical Investigations* is divided into seven parts: Congruences in general, Congruences of the first degree, Residues of powers, Congruences of the second degree, Quadratic forms, Applications and divisions of the circle. This work (judged extremely hard to read) has been disseminated by his successor at Göttingen, Peter Gustav Dirichlet (1805–1859).

In 1811, Gauss extended the calculus to functions of a complex variable and discovered a fundamental theorem of integration: if a function $f(z)$ is analytic at all points on and within a closed curve C in the Gaussian plane, then the integral of $f(z)$ along C is zero. Since he did not publish this theorem it is known today, after its re-discoverer, as Cauchy's integral theorem. In January 1812, recognizing the need for convergence of infinite series, he published his comprehensive, important work on hypergeometric series (described earlier in this book). Gauss wrote extensively on the theory of errors of measurement in his *Theoria Motus* and subsequent works that the normal frequency distribution which is the familiar bell-shaped curve is often referred to as the Gaussian distribution which is the familiar bell-shaped curve and finds a place on the face of the 10 Deutch Mark (DM) currency note of Germany. Gauss is considered as the effective founder of differential geometry. He introduced the concept of the Gaussian or total curvature of a surface, as a measure of curvature (which remains unchanged under a continuous deformation of a flexible and inextensible surface).

Gauss developed the theory of elliptic functions, though he did not publish his results. The remarkable property of double periodicity of these functions was rediscovered by Niels Abel and Carl Jacobi, between 1827 and 1829.

A unit of magnetism is named after Gauss. The eyepiece used for auto-collimation in spectrometers is designed by him. He invented the heliotrope, the bifilar magnetometer and with Weber devised the mirror galvanometer. In 1834–1835, Gauss and Weber invented an apparatus for transmission of messages. This telegraph system of Gauss and Weber was destroyed by lightning in 1845. There exists a Gauss-Weber, life-size, bronze monument in Göttingen where the period when Gauss was Astronomer Royal marked a golden era of science in Germany. The interested reader may read "Carl Friedrich Gauss: A Biography" by Tord Hall, published by the M.I.T. Press, Cambridge, Massachusetts (1970).

Jacobi, Carl Gustav Jacob (1804–1851): A German mathematician who made important contributions to many branches of mathematics. With Neils Abel, Jacobi was the discoverer of elliptic functions, in 1829, which led him to further generalizations of the concept for Abelian functions, in 1832–1835. Jacobi transformed William Hamilton' theory of mechanics into what is known today as the Hamilton-Jacobi theory, which is considered as the most elegant formulation of classical mechanics. Jacobi's work made determinants generally acceptable to all mathematicians. Jacobi visited Gauss on several occasions to discuss his recent researches with him and *"each time Gauss pulled 30-year-old manuscripts out of his desk and showed Jacobi what Jacobi had just shown him"*! Therefore, it is not surprising that it is believed that Jacobi could never come into close contact with Gauss.

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Algebraic Number A *complex number* a is said to be an algebraic number if it satisfies a polynomial equation over the field of rational numbers. For example, $\sqrt{2}$ is an algebraic number as it satisfies the polynomial equation: $x^2 - 2 = 0$. The complex number i is also an algebraic number, since it is the solution of the algebraic equation $x^2 + 1 = 0$. A number which is not an algebraic number is called a *transcendental number*.

American Mathematical Society The AMS was founded in 1888. About 30,000 individuals, colleges, universities and other institutions are among its members. (The author of this book was a member of the AMS during 1993-1996.) This society publishes the Proceedings of the AMS, the *Memoirs*, the *Notices* and the *Transactions* of the AMS, as well as the extremely useful *Mathematical Reviews*, a journal for reviews and abstracts; the two monographs series: *Colloquium Publications* and *Mathematical Surveys*; and the *Bulletin* of the AMS. Nine sectional meetings, a summer meeting in September and an annual meeting in December are held by the AMS. Early each summer, the society holds a symposium in applied mathematics. The AMS and the London Mathematical Society combined publication, "Ramanujan: Letters and Commentary", by Bruce C. Berndt and Robert A. Rankin (1995) has been adjudged as the "Outstanding Academic Book of 1996" by Choice. An Indian edition published by Affiliated East West Pvt. Ltd., with a *Preface and Additions to the Indian Edition*, was brought out by K. Srinivasa Rao, the author of this book. (Details about the AMS may be obtained from the Society's website, http://www.ams.org.)

Analytic Function A function of the complex variable $f(z)$ is said to be *differentiable* at $z = a$ if the limit

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

exists independent of the path of $h \to 0$ in the complex plane. The limit is denoted by $f'(a)$ and is called the *derivative of* $f(x)$ at a. A function $f(z)$ is said to be analytic, in a region of the complex plane, if it is differentiable in that region. A function $f(z)$ with the property that a power series expansion of the form

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

is valid in some neighbourhood of the point z_0, where $a_n = f^{(n)}(z_0)/n!$, $f^{(n)}(z_0)$ being the n-th derivative of the function at z_0, is said to be analytic at z_0.

Analytic Number Theory It is a branch of number theory. This subject deals with the problems of distribution of primes, the behaviour of number-theoretic functions and the theory of algebraic and transcendental numbers.

Arithmetic Function A real or complex valued function defined on the set of positive integers is called an arithmetic function.

Association of Mathematics Teachers of India (AMTI) This Association was registered under the Society's Act, on Oct. 29, 1965. Its first President was the distinguished Dr. A. Narasinga Rao and he was followed by Prof. P.L. Bhatnagar. The *Mathematics Teacher* is its official journal. A large number of mathematics teachers from all over India are members of AMTI and take part in interactive annual meetings of the association. The AMTI conducts the Mathematics Olympiads in India and helps talented students in several possible ways. The student Kannan Soundararajan (ref. p. 210 of this book) is one of those who has been nurtured and helped by AMTI.

Astrophysics The study of stellar evolution, stellar dynamics and inter-stellar material; or, the Physics of Astronomical bodies like: Stars, Planets, Galaxies, etc.

Asymptotic Formula An identity of the form $f(x) \sim g(x)$ which means that $f(x)$ is equal to $g(x)$ as $x \to \infty$. We say that $f(x)$ is asymptotically equal to $g(x)$. For example, $x^3 + x + 1 \sim x^3$.

Bernoulli Numbers The contemporary definition for Bernoulli numbers, B_n is

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n, \quad \text{(exponential generating function).}$$

Beta Function A function of two variables p and q, which for $p, q > 0$, is defined by

$$B(p, q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$

Black Hole Black holes are regions of space-time where the gravitational filed is so strong that even light cannot escape them. Inevitably, black holes form as a result of the gravitational (matter) collapse of a star, whose mass is sufficiently large (of the order of a few solar masses), once it has burnt its nuclear fuel. They can also be formed by the gravitational collapse of the inner regions of a galaxy, or by collisions.

Carr's Synopsis A copy of the Carr Synopsis is available at the Library of the University of Madras (though not easily displayed, like Newton's *Principia*, or the original *Winnie the Pooh* of A.A. Milne). It is reported that the copy at the Town High School in Kumbakonam, used by Ramanujan, has been lost decades ago!—as per the private communication with a Japanese professor who visited Kumbakonam in Feb. 1998, by the author.

Cauchy's Theorem If f is analytic in a simply connected region Ω, then $f'(z) = f(z)$. In particular,

$$\int_{\gamma} f(z)\, dz \;=\; 0,$$

for a simply closed curve γ in Ω.

Circle Method This method was developed by Ramanujan and Hardy when they were working on the partition problem. The problem is to find an asymptotic formula for $p(n)$, the number of partitions of n. Euler proved that

$$\prod_{j=1}^{\infty}(1 - z^j)^{-1} \;=\; \sum_{n=1}^{\infty} p(n) z^n.$$

Let $f(z)$ denote the infinite product on the left side. The singularities of $f(z)$ are the roots of unity and lie densely on the unit circle $|z| = 1$. Thus, $f(z)$ has the unit circle as its circle of convergence. By Cauchy's theorem, we have

$$p(n) = \frac{1}{2\pi i} \int_{|z|=r} \frac{f(z)}{z^{n+1}} dz, \qquad \text{for } 0 < r < 1.$$

Thus, the problem of estimating $p(n)$ is equivalent to estimating an integral. This beautiful idea has turned out to be applicable to a large class of related problems—for instance, Waring's problem in Additive Number Theory (ref. Some unsolved problems in Number Theory, K. Ramachandra, Resonance, May 1997, vol. 2, No. 5, p. 78).

Combinatorial Analysis It is a branch of mathematics which deals with the problems of choosing and arranging the elements of certain (normally finite) sets according to some prescribed rules.

Complex Numbers a number of the form $x + iy$, where x and y are arbitrary real numbers, is called a complex number. The complex number $x + iy$ is a number pair (x, y) for which elementary algebraic rules of addition, subtraction, multiplication and division are defined as:

$$(x, \pm y) \;=\; x \pm iy$$

$$(x_1, y_1) \pm (x_2, y_2) \;=\; (x_1 \pm x_2, y_1 \pm y_2)$$

$$(x_1, y_1) \cdot (x_2, y_2) \;=\; (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$$

$$\frac{1}{(x, \pm y)} = \frac{(x, \mp y)}{(x, \pm y)(x, \mp y)} = \frac{(x, \mp y)}{x^2 + y^2}$$

$$\frac{(x_1, y_1)}{(x_2, y_2)} = \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}, \frac{-x_1 y_2 + y_1 x_2}{x_2^2 + y_2^2} \right)$$

If $z = z \pm iy$ is a complex number, then x and y are called the real and imaginary parts of z and we write $x = Re(z)$ and $y = Im(z)$.

Composite Number A *natural number* which is not a *prime number* is called a composite number. The number 1 is neither a prime nor composite. Every composite number can be written in only one way as the product of prime numbers, which are called as prime factors. For example, $6 = 2 \times 3, 9 = 3 \times 3$.

Congruence If a number m divides the difference of two numbers a and b, viz. $(a - b)$ or $(b - a)$, exactly, then we say that a is congruent to b modulo m and we write

$$a \equiv b(mod\ m)$$

This relation is called a *congruence* and m is called the modulus of the congruence. The number b is called the residue of a modulo m. The congruence $a \equiv 0(mod\ m)$ means that a is divisible by m, for example: $35 \equiv 0(mod\ 5)$ means 35 is divisible by 5 and 0 is the residue of 35 modulo 5.

Continued Fraction Any expression of the form:

$$C(x) = \frac{1}{1} + \frac{x}{1} + \frac{x^2}{1} + \frac{x^3}{1} + \cdots \equiv \cfrac{1}{1 + \cfrac{x}{1 + \cfrac{x^2}{1 + \cfrac{x^3}{1 + \cdots}}}}$$

for $|x| < 1$, is called a continued fraction. When $x = 1$ it is $\left(1 + \sqrt{5}\right)/2$, the golden ratio.

Convergent Sequence sequence (a_n) is said to converge to the limit a if for every $\varepsilon > 0$, there is a positive integer N such that $n \geq N$ implies $|a_n - a| < \varepsilon$. We write $\lim_{n->\infty} a_n = a$. For example: the sequence $(\frac{1}{n})$ is convergent and converges to 0. A sequence which does not converge is said to *diverge*. For example: the sequence (n) is divergent.

Cosmic Ray Earth is constantly bombarded by extra-terrestrial radiation consisting predominantly of protons and subnuclear particles of high energies. Early workers gave this phenomenon the name Cosmic Rays. Many important discoveries, including the production and identification of several new "elementary" particles called mesons and hyperons, were made from cosmic rays, especially before the advent of man-made high energy particle accelerators.

Diophantine Equation Certain equations in number theory are called Diophantine equation after Diophantus who lived in Alexandria around 250 A.D. It is said that he published a series of books, *Arithmetica*, in 13 volumes, which were lost for more than a thousand years and that they were found around 1570. Apparently, the early volumes introduced, for the first time, negative numbers, algebraic notation and equations, while the later volumes dealt with number theory. His books are in the style of problems and solutions. The earliest equation encountered in the Schools, and perhaps the first theorem of importance known to most students is the Pythagoras theorem, which states that x, y are sides if a right-angled triangle, then the hypotenuse z of this triangle is given by $z^2 = x^2 + y^2$. As examples of the same, $x = 3, y = 4, z = 5$ satisfy the relation: $x^2 + y^2 = z^2$. The author feels that even at this stage, soon after the introduction of the statement and proof of the Pythagoras theorem, children should be taught:

(i) That if the quantities x, y, z which are real numbers, are restricted to integers, then the integers j, k, ℓ which satisfy $j^2 + k^2 + \ell^2 = n^2$ and the equation becomes a Diophantine equation.

(ii) (j, k, ℓ) is called a Pythagorean triple. The smallest such triple is (3,4,5). If (3,4,5) is a Pythagorean triple, then so is $(nj, nk, n\ell)$, for any positive integer n. A primitive Pythagorean triple is one in which j, k, ℓ are coprime. The students should note that non-integer sides of a right-angled triangle do not form Pythagorean triples, such as $(1, 1, \sqrt{2})$ is not a Pythagorean triple because $\sqrt{2} = 1.414\ldots$ is not an integer, but an irrational number. School children may ask themselves the question as to how many Pythogorean triples are there below $c \leq 100$ (say). The answer is that there are 16 and these are: $(3, 4, 5)$, $(5, 12, 13)$, $(7, 24, 25)$, $(8, 15, 17)$, $(9, 40, 41)$, $(11, 60, 61)$, $(12, 35, 37)$, $(13, 84, 85)$, $(16, 63, 65)$, $(20, 21, 29)$, $(28, 45, 53)$, $(33, 56, 65)$, $(36, 77, 85)$, $(39, 80, 89)$, $(48, 55, 73)$, $(65, 72, 97)$.

(iii) It is also possible to introduce the concept of generalization by stating that there are no integer solutions to equations of the type: $j^n + k^n = \ell^n$, for j, k, ℓ integer, $n > 2$, have no integer solutions, which is the statement of the famous Fermat's Last Theorem, which was an unsolved problem for 358 years[1] and solved only recently by the French mathematician Andrew Wiles whose citation for the Fields Medal mentions this.

(iv) The concept of infinity, ∞, can also be introduced at this stage itself (denumerable infinity), instead of waiting for the concept of differentiation to be started at the present day XI standard in most school syllabuses.

Finally, the questions whether Ramanujan was aware of Fermat's Last theorem and whether he would have attempted the solution of this problem are questions for

[1] Simon Singh *Fermat's Last Theorem*, Fourth Estate, ISBN 1-85702-669-1 and e-book (1997); Simon Singh, *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem*, Anchor, ISBN 0-385-49362-2 (1998). *Fermat's Last Theorem*, Simon Singh (P) 2016 Audible, Ltd.

which there are no answers from his mentor Hardy or from any other contemporary or later day ardent followers of Ramanujan. It is in the realm of another pointer to the enigmatic nature of Ramanujan's mathematics.

Dirac's Theory of the Electron Paul Adrian Maurice Dirac (1902–1982), the Physics Nobel Prize winner of 1933, considered one of the greatest theoretical physicists of the twentieth century, applied the techniques of relativistic mechanics to quantum theory to arrive at the relativistic wave equation for the electron, since named after him as the Dirac equation. The solution of this equation predicted the existence of negative (as well as positive) energy states. Dirac interpreted that, in the absence of an external electric field, the negative energy states are always filled. When an appropriate field is applied, Dirac predicted that the transition of a particle from a filled negative energy state into an unfilled positive energy state takes place. This leads to pair production and the resulting positively charged particle (in all other aspects like an electron) is called the positron, the first anti-particle predicted by Dirac from his theory of the electron.

Dirichlet Series A series of the form

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^s},$$

where $f(n)$ is an arithmetical function, is called a Dirichlet series with coefficient $f(n)$.

Doubly Periodic Function A function of a complex variable $f(z)$ is called doubly periodic function of z if $f(z) = f(z + 2\omega_1) = f(z + 2\omega_2)$, where ω_1, ω_2 are complex numbers for which the ratio ω_1/ω_2 is not a real number. $2\omega_1, 2\omega_2$ are called the periods of the function.

Euler-Maclaurin Formula The Euler-Maclaurin formula was indeed a favourite tool if Ramanujan. If the function f has $2n + 1$ continuous derivatives on the closed interval $[\alpha, \beta]$, where α, β are integers, then

$$\sum_{k=\alpha}^{\beta} f(k) = \int_{\alpha}^{\beta} f(t)\, dt + \frac{1}{2}\{f(\alpha) + f(\beta)\} +$$

$$+ \sum_{k=\alpha}^{n} \frac{B_{2k}}{2k!}\{f^{(2k-1)}(\beta) - f^{(2k-1)(\alpha)}\} + R_n,$$

where, for $n \geq 0$,

$$R_n = \frac{1}{(2n+1)!} \int_{\alpha}^{\beta} B_{2n+1}(t - [t])\, f^{(2n+1)}(t)\, dt,$$

where $B_n(x)$, $0 \leq n < \infty$, denotes the n-th Bernoulli number.

False Theta Function They are the *theta functions* with irregular/wrong signs.

F.R.S. Fellow of the Royal Society, London. The Royal Society was started in the year 1645. The FRS is an award granted by the judges of the UK-based Royal Society to those who have made a "substantial contribution to the improvement of natural knowledge, including mathematics, engineering science and medical science". Ramanujan was the second Indian but the first Indian Mathematician to have been honoured with the Fellowship. Among the Indian Fellows of the Royal Society are: C.R. Rao, Statistician, M.S. Swaminathan, the Green Revolution Agricultural Scientist, and mathematicians C.S. Seshadri and M.S. Narasimhan.

Fields Medal This is awarded in the name of John Charles Fields who was Professor of Mathematics at the University of Toronto, Canada. His successful organization of the International Mathematical Congress in Toronto, in 1924, ended with a surplus of funds which provided the initial funding for the Fields Medal. These were later further endowed from his estate. In 1932, the International Mathematical Congress at Zürich adopted his proposal and the Fields Medal awards were first made at the next Congress held at Oslo in 1936. The Medals have been traditionally awarded to Mathematicians under 40 in compliance with Fields' wish that they be an encouragement for further work by the recipients. The gold plated cast is 25 cms in diameter, and the head appearing on the medals minted in the Royal Canadian Mint is that of Archimedes. This award is the equivalent of the Noble Prize, since there is no Fields Medal for Mathematics! Unlike the Nobel Prizes which are awarded every year, the Fields Medals are awarded once in four years, for up to four persons.

A very popular story about why Mathematics is not included by Nobel for an award is due to the fact that the girl who was to marry him chose to marry Mittag-Leffler and the latter was a mathematician! In Sweden, there is a Mittag-Leffler Institute[2]"—an International Centre, of the for Royal Swedish Academy of Sciences, for research and post-doctoral training in Mathematical Sciences, with special attention to the development of mathematical research in Nordic countries. Acta Mathematica and Arkic för Matematik (founded in 1903) are two of the prestigious mathematics research journals of the Institute.

Fundamental Theorem of Arithmetic For every integer $n > 1$ can be represented as a product of prime factors in only one way, apart from the order of the factors.

Gamma Function The gamma function is defined as

$$\Gamma(z) = \int_0^\infty z^{z-1}\, e^{-x}\, dx, \quad for Re(z) > 0.$$

[2]Institut Mittag-Leffler was *founded in 1916 by Professor Gösta Mittag-Leffler and his wife Signe, who donated their magnificent villa with its first-class library, for the purpose of creating the Institute that bears their name*. It is acclaimed as the oldest Institute in the world, with great autonomy.

It is remarkable that Ramanujan deduced the values of a large variety of known and new gamma function integrals.

Gauss Hypergeometric Series A series of the form

$$F(a, b;\ c;\ z) = \sum_{n=0} \infty \frac{(a)_n (b)_n}{n!(c)_n} z^n,$$

where $(a)_n = a(a+1)(a+2)\cdots(a+n-1)$ is called the Pochammer symbol. This series is not defined or meaningful when c is zero or a negative integer. The series is convergent for $|z| < 1$, divergent for $|z| > 1$ and for $|z| = 1$ it is convergent if $Re(c-a-b) > 0$ and divergent if $Re(c-a-b) < 0$. If $a = 1$ and $b = c$, then the series reduces to the geometric series: $1 + x + x^2 + x^3 + \cdots$. Gauss who introduced this series in 1812, considered it as a function of four variables. The modern notation (due to Barnes) for the function on the left hand side of the above equation is ${}_2F_1(a, b;\ c;\ z)$ where the subscripts 2 and 1 represent the fact that the series has two numerator parameters, and one denominator parameter, and x is the variable. Gauss considered a, b, c, z, all the four as complex variables. When any one of the numerator parameters/variables is a negative integer, say $-n$, then the series is terminating and becomes a polynomial of degree n.

Generalized Hypergeometric Series A series of the form

$${}_pF_q(a_1, a_2, \cdots, a_p;\ b_1, b_2, \cdots, b_q;\ z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \cdots (a_p)_n}{(b_1)_n (b_2)_n \cdots (b_q)_n} \frac{z^n}{n!},$$

where $(a)_n = a(a+1)(a+2)\cdots(a+n-1)$ is the Pochammer symbol.

Goldbach's Conjecture Every integer $n > 5$ is the sum of 3 primes. Euler showed that it is equivalent to the statement that every integer $2n > 4$ is the sum of two primes.

Highly Composite Number A number whose number of divisors exceeds that of all its predecessors is defined as a highly composite number, by Ramanujan. Examples of the first few highly composite numbers are: 2, 3, 4, 6, 12, 24, $\cdots$.

Imaginary Numbers While the equation $x^2 - 1 = 0$ has two roots, 1 and -1, the equation $x^2 + 1 = 0$ has no roots in the domain of real numbers. Formally, the roots of the equation $x^2 = -1$ are $\sqrt{-1}$ and $-\sqrt{-1}$. Euler and others before him, denoted $\sqrt{-1}$ by the symbol i, an abbreviation for the word "imaginary" which was meant to indicate that they have no existence in the real world! Products of i follow the rules:

$$i \cdot i = i^2 = -1, \qquad i \cdot i \cdot i = i \cdot i^2 = -i, i \cdot i \cdot i \cdot i = i^2 \cdot i^2 = i^4 = 1.$$

I.M.S The Indian Mathematical Society was founded by V. Ramaswamy Iyer in the year. One of the reasons for its creation was that the contemporary mathematics teachers were unable to come up with suggestions of journals for publishing the prolific output of research of the prodigy Srinivasa Ramanujan.

Infinite Series Let (a_n) be a sequence of real or complex numbers. The formal infinite sum $a_1 + a_2 + a_3 + \cdots$ is called an infinite series or simply series. It is denoted by $\sum_{n=1}^{\infty} a_n$, or simply $\sum a_n$. With (a_n) we associate a finite sequence (s_n) where $s_n = a_1 + a + 2 + \cdots + a_n = \sum_{i=1}^{n} a_i$. (s_n) is called the n-th partial sum of $\sum a_n$. If (s_n) converges to S, we say that the series converges and write $\sum a_n = S$ and S is called the sum of the series. If (s_n) diverges, the series is said to diverge. For example, $\sum \frac{1}{n^2}$ converges. The geometric series $\sum x^n$ converges to $\frac{1}{1-x}$, if $0 \leq x < 1$ and diverges if $x \geq 1$. The series $1 + 1 + 1 + \cdots$ and the series $\sum \frac{1}{n}$ are divergent.

Infinity Infinity is larger than the largest number one can choose. Thus, it is not a number in the usual sense and it is denoted by the symbol ∞.

Integers $0, \pm 1, \pm 2, \pm 3, \ldots$. The set of integers is denoted by $\mathcal{Z}$.

Integral The simplest of the differential equations $\frac{dy}{dx} = f(x)$ has the solution: $y = \int f(x)\, dx + c$, where $\int f(x)\, dx$ is a symbol for the function with derivative of $f(x)$. We can almost always give a meaning to the solution by writing it in the form:

$$y = \int_{x_0}^{x} f(t)\, dt + c,$$

where t is a dummy constant of integration. The crux of the matter is that this definite integral (an integral with specified limits of integration as x_0 and x, respectively, as opposed to an indefinite integral in which the limits are not specified) is a function of the upper limit x which always exists when the integrand is continuous over the range of integration and that its derivative is $f(x)$. This statement is one of the fundamental theorems of calculus.

Interpolation It is a process of estimating the values of a function $y(x)$ for arguments between $x_0, x_1, \ldots x_n$, at which the values $y_0, y_1, \ldots y_n$ are known.

Irrational Number A number which is not rational (i.e. expressible as the ratio of two integers) is called an irrational number. Examples are $\sqrt{2}, \pi, e$. The universal constants π, e are called *transcendental numbers*. A transcendental number is defined as a number which is not the solution of an algebraic equation. The solution of the algebraic equation $x^2 + 1 = 0$ is the complex constant i.

Klein-Nishina Formula The differential cross section for the scattering of a photon by a free electron, called as the Compton scattering (or, Compton effect), is expressed by the Klein-Nishina formula.

Kortweg-deVries Equation The non-linear differential equation:

$$u_t + u \cdot u_x + \delta^2 u_{xxx} = 0,$$

where $u_t = \partial/u\partial t, u_x = \partial u/\partial x$ and $u_{xxx} = \partial^3 u/\partial x^3$ is called the Kortweg-de Vries (KdV) equation. A smooth initial wave turns into a series of several profound pulses, each of which is a solitary wave solution of the KdV equation. These pulses proceed freely, collide mutually, interact non-linearly and then recover their profiles. Zablusky and Kruskal discovered these solutions, in 1965, and named these stable pulses as *solitons*.

London Mathematical Society The London Mathematical Society (LMS) was founded in 1865 for the promotion and extension of mathematical knowledge and is now the major British society for mathematicians. Its principal activities are the publication of periodicals and books and the organization of meetings and conferences. The *Proceedings*, the *Journal* and the *Bulletin* of the LMS are its periodicals. The LMS also publishes the translations of the *Russian Mathematical Surveys, Izvestia: Mathematics* and *Sbornik: Mathematics* in collaboration with the Russian Academy of Sciences and Turpion; and in collaboration with the American Mathematical Society publishes the translation of the *Transactions of the Moscow Mathematical Society*. The LMS publishes a monthly *Newsletter*, except in the month of August. The *LMS monographs* are published by the Oxford University Press. The *LMS Lecture Notes Series* and the *LMS Student Texts* are published by the LMS jointly with the AMS. The book *Ramanujan Letters and Commentary*, by Bruce C. Berndt and Robert A. Rankin [XI] belongs to this series. Details may be obtained from the Society's website: http://www.lms.ac.uk.

Magic Square A magic square is a square array of (usually distinct) natural numbers in which the sum of the numbers in each row, column, diagonal and skew-diagonal are equal to an integer. In some instances, the requirement of the equality of the two diagonal sums with the row and column sums of the elements of the array is dropped.

Master of Trinity College The present Master of Trinity College (since 2012) is the Molecular Biochemist Sir Gregory Paul Winter. Sir J.J. Thomson (1918, 1940), Sir Michael Atiyah (1990, 1997) and Amartya Sen (1998) were also Masters of Trinity College (the term years are indicated in brackets).

Mathematical Induction It is a method of providing proofs to mathematical results based on the principle of mathematical induction which is described as follows: An assertion $A(n)$, depending on a natural number n, is regarded as proved

if $A(1)$ has been proved and if for any natural number n the assumption that $A(n)$ is true implies that $A(n+1)$ is also true. This is usually called the first principle of induction. The second principle of induction is described as follows: An assertion $A(n)$ is regarded as proved if $A(1)$ has been proved and if the assumption $A(k)$, for $1 \leq k < n$, is true implies that $A(n)$ is true.

Natural Numbers $1, 2, 3, 4, \ldots$. These are called as positive integers.

Negative Numbers The equation $x+1=0$ has a negative root, namely -1. Due to the prevalent opposition to the extension of the notion of numbers, some preferred to say, in the seventeenth century, that the equation had no root at all! Thus, the example given shows that the additive inverse of any positive number is a negative number.

Neighbourhood Of a point z is an open set containing that point.

Neutron Star A star that has undergone gravitational collapse where electrons and protons compress to form a neutron star.

Nobel Laureate A winner of the Nobel Prize is called a Noble Laureate. The Prize was instituted by Alfred Nobel, the Swedish Chemist, who discovered dynamite. Every year Nobel Prizes are awarded for outstanding work in Physics, Chemistry, Physiology or Medicine, Literature and Peace. However, the Prize is not awarded for Mathematics. There is a popular story that the decision was due to the fact that the first such prize would have gone to Mittag-Leffler, and Nobel did not want the person who married the girl who was to marry him (Alfred Nobel) be honoured with the Nobel Prize. There is a Mittag-Leffler Institute[3] for research in mathematics.

Nucleus The positively charged core of an atom, consisting of protons and neutrons. The proton carries one positive electric charge while the neutron has no electric charge.

One-Dimensional Heat Equation The equation:

$$a^2 u_{xx} = u_t, \text{ where } u_{xx} = \partial^2 u/\partial^2 x, \quad u_t = \partial u/\partial t,$$

is called the one-dimensional heat equation. It is a partial differential equation.

Ordinary Differential Equation An equation involving one dependent variable and its derivative with respect to one or more independent variables is called an ordinary differential equation (ODE). An ODE is one in which there is only one independent variable so that all the derivatives occurring in it are ordinary

[3]The Mittag-Leffler Institute is a mathematical research institute located in Djursholm, a suburb of Stockholm, Sweden. The Institute's main building was donated by Magnus Gustaf (Gösta) Mittag-Leffler (1846–1927) along with the mathematician's extensive Library. He founded the mathematics journal *Acta Mathematica (1882)*. His villa was donated to the Royal Swedish Academy of Sciences, and became the Mittag-Leffler Institute, in operation since 1969.

derivatives. Examples of ODEs are:

$$\frac{dy}{dt} = -ky;$$

$$m\frac{d^2y}{dx^2} = -ky;$$

$$x^2\,\frac{d^2y}{dex^2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0,$$

where the dependent variable in each of these equation is y, the independent variable is either x or t; and k, m, p are constants.

Partial Differential Equation A partial differential equation (PDE) is one involving partial derivatives of one or more dependent variables with respect to one or more of the independent variables. The Laplace equation:

$$u_{xx} + u_{yy} + u_{zz} = 0,$$

the heat equation:

$$a^2(u_{xx} + u_{yy} + u_{zz}) = u_t,$$

and the wave equation:

$$a^2(u_{xx} + u_{yy} + u_{zz}) = u_{tt},$$

where $u \equiv u(x, y, z, t)$ is a function of the three rectangular coordinates of a point in space and time, are classic examples of partial differential equations.

Partition A partition of a positive number n is a representation of n as the sum of any number of positive integral parts—that is, the number of distinct ways of representing n as a sum of natural numbers (with order irrelevant). For example:

$$5 = 4+1 = 3+2 = 3+1+1 = 2+2+1 = 2+1+1+1 = 1+1+1+1+1.$$

Thus the number 5 has 7 partitions, which is written as $p(5) = 7$, thus denoting the number of partitions of n by $p(n)$ called a *partition function*. Thus, $p(n)$

$$p(1) = 1,\ p(2) = 2,\ p(3) = 3,\ p(4) = 5,\ p(5) = 7,\ p(6) = 11,\ p(7) = 15,$$

$$p(8) = 22,\ p(9) = 30,\ p(10) = 42, \cdots p(100) = 190569292,$$

$$p(1000) = 24061467864032622473692149727991 \sim 2.40615 \times 10^{31},$$

is a rapid increasing function.

Prime Number A *natural number* that is larger than 1 and has no factor other than 1 and itself is called a prime number. The *sequence* of primes are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, ... There are 15 primes less than 50. There are 25 primes less than 100. The number 2 is the only even prime number. It qualifies to be a prime number mainly because it is the smallest number which has 2 divisors. So, even though it is an even number, it qualifies to be a prime number, as all prime numbers have only two divisors (1 and itself). Prime numbers of the form $2^n - 1$ are called *Mersenne numbers*, after the seventeenth-century French Fransiscan monk Marin Mersenne (though they were studied earlier by Euclid).

Prime Number Theorem For every $x > 0$, let $\pi(x)$ denote the prime counting function giving the number of primes less than or equal to x. Then $\pi(x)$ is asymptotically (i.e., as $x \to \infty$) equal to $\frac{x}{\log x}$. Carl Friedrich Gauss was perhaps the first mathematician who had this result as an unproved hypothesis and more than a hundred years later the Belgian Charles de la Vallé-Poussin and the Frenchman Jacques Salomon Hadamard proved independently that

$$\pi(x) = \int_2^x \frac{dt}{\log t} + O\left(xe^{-A\sqrt{\log x}}\right),$$

where $O(\ldots)$ refers to the so-called "big-O" notation: in general, $O(f(x))$ represents an unspecified function of x which grows not faster than some constant times $f(x)$.

Quantum Mechanics Classical mechanics, in general, deals with the mechanics of macroscopic objects. This fails in the microscopic level of the atom and at lower levels (molecular, nuclear, elementary particle levels). The mechanics at the microscopic level requires the quantum mechanics.

Question 330 of Prof. Sanjana Prof. Sanjana in the Journal of Indian Mathematical Society (IV, 1912, 59–61) remarks that it is not easy to evaluate the series:

$$\frac{1}{1^n} + \frac{1}{2}\frac{1}{3^n} + \frac{1}{2}\frac{3}{4}\frac{1}{5^n} + \frac{1}{2}\frac{3}{4}\frac{5}{6}\frac{1}{7^n} + \text{adinf.},$$

if $n > 3$. Ramanujan has summed the series for all values of n. In his paper, Ramanujan [III] quotes *Williamson, Integral Calculus* and *Carr's Synopsis*, 2295.

Ramanujan Mathematical Society Founded in 1985 by a group of Mathematicians of South India, led by Professors R. Balakrishnan (National College, Trichy), E. Sampathkumar (University of Mysore), K.S. Padmanabhan (Ramanujan Institute for Advanced Study in Mathematics of the University of Madras). The *Journal of the Ramanujan Mathematical Society*(RMS) and a *Newsletter* of the (RMS) are its regular publications. They are now brought out from Canada. An annual meeting of the RMS is held in the summer annually.

Rational Number A number of the form m/n, where m and n are integers with $n \neq 0$, is called a *rational number*.

Relatively Prime or Coprime Numbers Two numbers m and n are said to be relatively prime if their greatest common divisor is 1. In this case, m and n are said to be coprime.

Round Numbers A number is said to be a *round number* if it is the product of many complimentary small factors. Thus $1200 = 2^4.3.5^2$ and $2187 = 3^7$ are examples of round numbers—the later more than the former. The question proposed by Hardy and Ramanujan is: *How many prime factors may one expect to occur in a random large number n?*, which they solved (ref. [XII], ch. III).

Riemann Hypothesis It is the conjecture of Bernhard Riemann (1859) that the zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \text{where } s \text{ is a complex number, with } Re(s) > 1,$$

has its zeros only at the negative even integers, that is,

$$\zeta(s) = 0, \text{ when } s = -2, -4, -6, \ldots,$$

and complex numbers with real part $\frac{1}{2}$. It became renowned as the Riemann hypothesis and also used for some closely related to Riemann hypothesis for curves over finite fields. This famous hypothesis was considered as the most important open problem in pure mathematics[4] and some of its generalizations, along with Goldbach's conjecture and the twin prime conjecture, comprise Hilbert's 8th problem, in David Hilbert's list of 23 unsolved problems. It was also one of the Clay Mathematics Institute's Millennium Prize problems. Sir Michael Atiyah, Fields Medalist 19, explained his proof of the Riemann Hypothesis at the Heidelberg Laureate Forum on Sept. 24, 2018, but there is still some scepticism about the proof.

Sequence By a sequence is meant the values of a function f defined on the set $\mathbf{Z}_+$ of all positive integers. If $f(n) = a_n$ for $n \epsilon Z_+$, we use the notation (a_n) to denote the sequence whose elements are $a_1, a_2, a_3, \ldots$ The elements of the sequence are called the terms of the sequence. The number a_n is called the nth term of the sequence. Note that the terms need not be distinct.

Set A set is a collection of well-defined objects. For example: {1,2,3}, {a,b,c,d}, etc.

Singularity If a function of a complex variable $f(z)$ possesses a derivative at some point z_0 and its neighbourhood, in a complex z plane, then the function is said to

[4]Enrico Bombieri, *The Riemann Hypothesis-official problem description*, Clay Mathematics Institute, retrieved by Wikipedia on 25 Oct. 2008, and reprinted in Peter Borwein, Stephen Choi, Berndan Rooney, Andrea Weirathmueller, (Eds) in *The Riemann Hypothesis: A Resource for the Afficionado and Virtuoso Alike, CMS Books in Mathematics, New York: Springer, doi:10.1007/978-0-387-72126-2, ISBN 978-0-387-72125-5.*

be *regular* at z_0 and z_0 is said to be a *regular point*. Functions which have at least one regular point are called as *analytic functions*. A point at which a function is not regular is said to be a *singular point*, or merely *singularity*. For example, the function

$$f(s) = \frac{s^2}{s^2 - 1} = \frac{s^2}{(s-1)(s+1)},$$

has $s = +1, -1$ as its singular points. The function $f(s)$ is an analytic function which is singular at 1 and -1 and regular at all other points.

Solar Mass The Sun is the smallest star that is closest to the Earth. Its mass, 1.985×10^{30} kg, is taken as the unit of mass in astrophysical studies.

Solitons Solitons are essentially elementary excitations in non-linear dissipative systems (see Kurtweg-de Vries equation above).

Statistical Mechanics This subject is specially devised for the treatment of complicated (many-body or many-particle) systems having many degrees of freedom. The principles of statistical mechanics permit reasonable predictions of the future condition of such a complex system to hold on the average, starting from an incomplete knowledge of its initial state.

Stellar Evolution Stars are resources of abundant energy which is produced by nuclear reactions taking place in their interior—the carbon-nitrogen cycle or the hydrogen-helium fusion reaction cycle. The study of what happens to a star as it goes on emitting radiation is the subject matter of stellar evolution. In some cases the star ends up as a white dwarf and in some others as a black hole! There is a limit called the Chandrasekhar limit of 1.44 Solar Masses which determines whether a star in its final stages, after its gravitational collapse, will end up as a black hole or not.

String The string in High Energy Physics is a one-dimensional object, a mathematical curve. Both open strings, which have end points, and closed strings, which form a topological view point are circles, are considered. Since a string is free to oscillate it has an infinite number of harmonics, which correspond to an infinite number of particle states.

Superstring Theory The fundamental constituents of matter are considered as tiny strings in High Energy Physics. The theory which treats Bosonic (integral spin) and Fermionic (half-integral spin) strings as members of the same multiplet, on an equal footing, is called Superstring Theory.

Symmetric Function A function that remains unchanged under any permutation of its independent variables. For example, $f(x, y) = xy$ and $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \frac{(-1)^n x^{2n}}{(2n)!}$ are symmetric functions, in two variables and one variable, respectively.

Theta Functions The four θ-functions of Jacobi are:

$$\theta_1(x,\omega) = i\sum_{n=-\infty}^{\infty}(-1)^n\, q^{(n-1/2)^2}\, e^{i\pi(2n-1)x}$$

$$\theta_2(x,\omega) = \sum_{n=-\infty}^{\infty} q^{(n-1/2)^2}\, e^{i\pi(2n-1)x}$$

$$\theta_3(x,\omega) = \sum_{n=-\infty}^{\infty} q^{n^2}\, e^{i\pi(2n)x}$$

$$\theta_4(x,\omega) = \sum_{n=-\infty}^{\infty}(-1)^n\, q^{(n-1/2)^2}\, e^{i\pi(2n)x}$$

with $q = e^{i\pi\omega}$, Im ω>0. All the four θ-functions are entire functions of x and are all periodic. The period of θ_1 and θ_2 is 2 and that of θ_3 and θ_4 is 1.

Transcendental Number A number which is not algebraic is said to be transcendental. Examples, e, π, etc.

Trigonometric Series A series of the form

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx),$$

is called a trigonometric series, with a_n and b_n called the coefficients of the series.

Unit Circle A circle of unit radius around the origin, in a two-dimensional real or complex plane, is called the unit circle.

Waring's Problem For every $k \geq 2$, there is a number $r \geq 1$ such that every natural number is a sum of at most r k-th powers.

White Dwarf A star that has exhausted most of its nuclear fuel and has shrunk to a very small size. This is sometimes the final stage of evolution of a star.

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